Measurement of the solar neutrino capture rate with gallium metal. III. Results for the 2002–2007 data-taking period

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The Russian-American experiment SAGE began to measure the solar neutrino capture rate with a target of gallium metal in December 1989. Measurements have continued with only a few brief interruptions since that time. In this article we present the experimental improvements in SAGE since its last published data summary in December 2001. Assuming the solar neutrino production rate was constant during the period of data collection, combined analysis of 168 extractions through December 2007 gives a capture rate of solar neutrinos with energy more than 233 keV of $65.4^{+3.1}_{-3.0}$ (stat) $^{+2.6}_{-2.8}$ (syst) SNU. The weighted average of the results of all three Ga solar neutrino experiments, SAGE, Gallex, and GNO, is now 66.1 ± 3.1 SNU, where statistical and systematic uncertainties have been combined in quadrature. During the recent period of data collection a new test of SAGE was made with a reactor-produced ³⁷Ar neutrino source. The ratio of observed to calculated rates in this experiment, combined with the measured rates in the three prior ⁵¹Cr neutrino-source experiments with Ga, is 0.87 ± 0.05 . A probable explanation for this low result is that the cross section for neutrino capture by the two lowest-lying excited states in 71Ge has been overestimated. If we assume these cross sections are zero, then the standard solar model including neutrino oscillations predicts a total capture rate in Ga in the range of 63 SNU to 66 SNU with an uncertainty of about 4%, in good agreement with experiment. We derive the current value of the neutrino flux produced in the Sun by the proton-proton fusion reaction to be $\phi_{op}^{\circ} = (6.0 \pm 0.8) \times$ 10^{10} /(cm² s), which agrees well with the pp flux predicted by the standard solar model. Finally, we make several tests and show that the data are consistent with the assumption that the solar neutrino production rate is constant in time.

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I. INTRODUCTION

The SAGE experiment was built to measure the capture rate of solar neutrinos by the reaction ${}^{71}\text{Ga} + \nu_e \rightarrow {}^{71}\text{Ge} + e^-$ and thus to provide information to aid in understanding the deficit of neutrinos observed in the ${}^{37}\text{Cl}$ experiment [1], in which only about one-third of the solar neutrino capture rate predicted by the standard solar model was detected. The feature that distinguishes the Ga experiment from all other past or present solar neutrino detectors is its sensitivity to the proton-proton fusion reaction, $p + p \rightarrow d + e^+ + \nu_e$, which generates most of the Sun's energy. Ga experiments have provided the only direct measurement of the current rate of this reaction.

A full description of the SAGE experiment and the results of each measurement from its inception to December 1997 was presented in Ref. [2]. Part II of this series, although not called by this name, described the changes to the experiment and gave the results for the period January 1998 to December 2001 [3]. In Secs. II and III of the present article we do the same for the 6-year period January 2002 to December 2007. We then discuss the four neutrino source experiments with Ga in Sec. IV, give the present interpretation of the SAGE results in Sec. V, derive the *contemporary* value of the neutrino flux produced by the proton-proton fusion reaction in Sec. VI and present a brief consideration of the question of possible time variation in the data in Sec. VII.

In addition to SAGE, there also existed a second Ga solar neutrino experiment called Gallex. It contained 30 tons of gallium in a solution of $GaCl_3$ and measured the solar neutrino capture rate from 1991 to 1997. In 1998 this experiment was reconstituted under the name of GNO and it took data until 2003. We give the results of these experiments and combine them with the SAGE data in Sec. III.

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II. EXPERIMENTAL PROCEDURES

A. Overview

The SAGE experiment is in a dedicated deep-underground laboratory excavated into the side of Mt. Andyrchi in the northern Caucasus mountains of Russia. The rock overburden is equivalent to 4700 m of water and the measured muon flux at the location of the experiment is $(3.03 \pm 0.10) \times 10^{-9}/(\text{cm}^2 \text{ s}).$

The mass of gallium used in SAGE at the present time is 50 tonnes. It is contained in seven chemical reactors that are heated to 30°C so the gallium metal remains molten. A measurement of the solar neutrino capture rate begins by adding to the gallium a stable Ge carrier. The carrier is a Ga-Ge alloy with a known Ge content of approximately $350 \mu g$ and is distributed equally among all reactors. The reactor contents are stirred thoroughly to disperse the Ge throughout the Ga mass. After a typical exposure interval of 1 month, the Ge carrier and ⁷¹Ge atoms produced by solar neutrinos and background sources are chemically extracted from the Ga. The final step of the chemical procedure is the synthesis of germane (GeH₄), which is used as a proportional counter fill gas with an admixture of 90-95% Xe. The total efficiency of extraction is the ratio of mass of Ge in the germane to the mass of initial Ge carrier and is typically $95 \pm 3\%$.

B. Extraction of Ge from Ga

The extraction procedures from 1990 to 1997 are described in Ref. [2]. At the beginning of 1998 some minor modifications were made as described in Ref. [4].

Beginning with the December 2005 extraction, the carrier used to measure the extraction efficiency was isotopically enriched in either ⁷²Ge or ⁷⁶Ge. At the end of each extraction a sample was taken from the final extraction solution and this sample was analyzed with an inductively-coupled plasma mass spectrometer to determine the fractional content of the various Ge isotopes. The efficiency of Ge extraction from the Ga metal was then calculated using the method outlined in Appendix A. This procedure for determining the extraction efficiency has the advantage that it gives a direct measure of any Ge that may enter the sample from unknown sources.

C. Counting of ⁷¹Ge

⁷¹Ge decays to ⁷¹Ga by pure electron capture with a halflife of 11.4 days. Two peaks are observed in the proportional counter—the *K* peak at 10.4 keV and the *L* peak at 1.2 keV. The counter containing the GeH₄ from the extraction is placed in the well of a NaI detector that is within a large passive shield and is counted for a typical period of 6 months. To reduce the influence of ²²²Rn, the volume inside the shield around the counters is purged with boil-off gas from a dewar filled with liquid nitrogen.

A completely redesigned proportional counter [5] began to be used with the extraction of April 2001 and has been used for all but two extractions since that time. In contrast to the usual counters with a solid cathode, the cathode of the new counters is made from vapor-deposited carbon, thus eliminating the usual dead volume behind the cathode. The dead volume is further reduced and end effects are nearly eliminated by curving inwards the regions of the counter where the cathode ends. The cathode and anode leads are sealed into the counter body with Mo ribbon that makes the counter leak free and ensures excellent gain stability. The cathode is so thin that the counter body is transparent, making it possible to visually inspect all the internal counter parts.

During 2004–2005 an extensive series of measurements of the efficiency of these new counters was made. The methods of measurement were described in Ref. [2] and counter fillings of ⁶⁹Ge, ⁷¹Ge, and ³⁷Ar were used. The measured volume efficiency of the new counters was 96% with a spread in efficiency of only $\pm 1\%$ for all counters of this type. This should be compared with an average volume efficiency of 89% for our original counter design. Further, the fraction of events that is degraded in energy was found to be significantly less than in the old design. These decreases in degraded fraction combined with the increase in volume efficiency lead to a quite dramatic increase in efficiency for these new counters compared to the old type, approximately 25% in the *K* peak and 10% in the *L* peak.

Another innovation in the new counter design is that the Suprasil counter body is etched in hydrofluoric acid to a thickness of \sim 0.2 mm. This permits calibration of the counter with our standard ⁵⁵Fe source over nearly its entire volume. As an undesired side effect, however, the thin body, combined with the very thin cathode, makes these counters sensitive to low-energy x rays from local radioactivity. To eliminate this response, a graded shield consisting of an outer layer of 1 mm of Cu and an inner layer of 3 mm of low-background acrylic (to absorb Cu x rays) is placed over the counter body during measurement with ⁷¹GeH₄.

The pulses from the proportional counter are sent to a fast transient analyzer where they are digitized for 800 ns after pulse onset at two different gains, one chosen for the L peak and the other for the K peak. The transient digitizer serves to differentiate fast-rising ⁷¹Ge pulses from generally slowerrising background pulses. This can be seen by comparing the upper and lower panels of Fig. 1, which show the pulses from the 77 extractions that have been measured in the new proportional counters. The upper panel is for all events that pass the time cuts for Rn (see Sec. II D), are not high-voltage breakdown, do not have a NaI coincidence, and occur during the first 30 days of counting. The total live time is 1999.8 days and there are 2063 events. The lower panel of this figure shows the 1545 events that occurred between days 100.0–130.1 (the same live time duration as in the upper panel). The fast-rising ⁷¹Ge events in the L and K peaks are evident in the upper panel but missing in the lower panel because the ⁷¹Ge has decayed away.

Aside from replacing some modules that failed, no changes were made to the counting system electronics since their description in Ref. [2].

D. Data analysis

Based on criteria described in Ref. [2], a group of events is selected from each extraction that are candidate ⁷¹Ge decays.



FIG. 1. (Upper panel) Count rate vs. energy and rise time for events during the first 30 days of counting. Regions where the *L* and *K* peaks are predicted to occur based on ⁵⁵Fe calibrations are shown darkened. There are 427 counts in the *L*-peak region and 287 counts in the *K*-peak region. The counts in both regions are a combination of events from ⁷¹Ge decay and background. (Lower panel) Equivalent graph for all events that occurred during an equal live time interval beginning at day 100 after extraction. There are 226 counts in the *L*-peak region and 94 counts in the *K*-peak region.

These events are fit to a maximum likelihood function [6], assuming that they originate from an unknown but constantrate background and the exponentially decaying rate of ⁷¹Ge. Because only a few ⁷¹Ge counts are detected from each extraction, a single run result has a large statistical uncertainty and thus little significance.

Several minor changes in the methods of analysis have occurred since Part II of this series. These include the following:

- (i) As discussed in Ref. [2], a small fraction of the decays of ²²²Rn are occasionally misidentified as pulses from ⁷¹Ge. To reduce this effect for ²²²Rn located external to the counter we now delete all data that is acquired within 2.6 h after counting begins. In our initial analysis this time cut was for only 1 h.
- (ii) To reduce the influence of ²²²Rn that may enter the counter when it is filled, we delete all data from 15 min before an event that saturates the energy scale to 3 h after each saturated event. The SAGE measurements before September 1992, however, were measured in

counting systems that did not have the capability to recognize saturated events [2]. To reduce the number of these false ⁷¹Ge events produced by ²²²Rn in these early runs we determine for all subsequent runs the difference in capture rate in the *K* peak between the data analyzed with and without this time cut and then subtract this difference from the result of each of the runs before September 1992.

- (iii) The predicted position and resolution of the *L* peak from calibrations was changed slightly from previous work. These changes were indicated by the results of a set of new calibrations made with counters filled with 71 Ge.
- (iv) The *L* and *K*-peak shapes were changed from pure Gaussian to Gaussian plus a degraded term. The new functional form for the line shape as a function of energy is

$$F(E) = he^{-[(E-C)/(\sqrt{2}\sigma)]^2} + hd\sqrt{\frac{\pi}{2}}\frac{\sigma}{C}\operatorname{erfc}\left(\frac{E-C}{\sqrt{2}\sigma}\right), \qquad (1)$$

where h, C, and σ are the peak height, center, and width and d is a parameter related to the fraction of degraded events. The error function term here is the integral of the Gaussian from energy E to ∞ ; it is essentially flat below the peak, monotonically decreases in the peak region, and is zero above the peak. This new line shape only makes a very small change to the counting efficiency in the L peak for a few runs whose energy window width is obliged to be less than 2 full widths at half maximum.

(v) For all runs after August 1992 the likelihood function was modified to include a factor that weights each event according to its measured energy. This requires knowledge of the energy distribution for ⁷¹Ge pulses and for background events, both of which can be determined from the long duration of counting data that we have accumulated. When this method is applied, it is found that the overall statistical uncertainty decreases by 0.1-0.2 SNU, but the systematic uncertainty increases by ~0.1 SNU.

These changes in analysis methods have been applied to all data.

III. RESULTS

⁷¹Ge has been extracted from the Ga target to measure the solar neutrino capture rate every month from January 2000 to the present time. We even were able to make six solar extractions during the time of the ³⁷Ar neutrino source experiment in 2004 by sending the samples to Gran Sasso. In a cooperative effort [7], the GNO Collaboration synthesized GeH₄ and measured the samples in their counting system.

The results for each individual extraction are tabulated in Appendix B and the combined result of each year of SAGE data since its beginning is shown in Fig. 2.

The systematic uncertainties in the experiment have been considered in detail in Refs. [2,3] and the most recent values are given in Table I. The only significant changes from our previous articles are due to the new proportional counters. Their high stability and efficiency have led to a considerable reduction of the uncertainties associated with counting.

In radiochemical experiments the capture rate has been conventionally expressed in "SNU units," defined as one neutrino capture per second in a target that contains 10^{36} atoms of the neutrino-absorbing isotope, in our case ⁷¹Ga.

TABLE I. Summary of known systematic effects and their uncertainties. SNU values for extraction and counting efficiency are based on a rate of 65.4 SNU.

Origin of uncertainty	Uncertair	nty
	In percentage	In SNU
Extraction efficiency		
Ge carrier mass	$\pm 2.1\%$	± 1.4
Mass of extracted Ge	$\pm 2.5\%$	± 1.6
Residual Ge carrier	$\pm 0.8\%$	± 0.5
Ga mass	$\pm 0.3\%$	± 0.2
Total (extraction)	±3.4%	± 2.2
Counting efficiency		
Volume efficiency	$\pm 1.0\%$	± 0.7
End losses	$\pm 0.5\%$	± 0.3
Monte Carlo interpolation	$\pm 0.3\%$	± 0.2
Shifts of gain	-1.1%	+0.7
Resolution	+0.5%, -0.7%	-0.3, +0.5
Rise time limits	$\pm 1.0\%$	± 0.7
Lead and exposure times	$\pm 0.8\%$	± 0.5
Total (counting)	+1.8%, -2.1%	-1.2, +1.4
Nonsolar neutrino production of	⁷¹ Ge	
Fast neutrons		< -0.02
²³² Th		<-0.04
²²⁶ Ra		<-0.7
Cosmic-ray muons		<-0.7
Total (nonsolar)		<-1.0
Background events that mimic ⁷¹	Ge	
Internal ²²² Rn		<-0.2
External ²²² Rn		0.0
Internal ⁶⁹ Ge		<-0.6
Total (background events)		<-0.6
Energy weighting in analysis		± 0.1
Total		-2.8, +2.6

For all SAGE data from January 1990 through December 2007 (168 runs and 310 separate counting sets) the global best fit capture rate is $65.4^{+3.1}_{-3.0}$ SNU, where the uncertainty is statistical only. If one considers the *L*-peak and *K*-peak data separately, the results are $67.2^{+4.8}_{-4.6}$ SNU and $64.0^{+4.1}_{-4.0}$ SNU, respectively. The agreement between the two peaks serves as a strong check on the robustness of the event selection



FIG. 2. Combined SAGE results for each year. Shaded band is the combined best fit and its uncertainty for all years. Vertical error bars are statistical with 68% confidence.

criteria. Including the systematic uncertainty, our overall result is $65.4^{+3.1}_{-3.0}(\text{stat})^{+2.6}_{-2.8}(\text{syst})$ SNU.

As further evidence that we are truly counting ⁷¹Ge, we can allow the decay constant during counting to be a free variable in the maximum likelihood fit, along with the combined ⁷¹Ge production rate and all the background rates. The best fit halflife to all selected events in both *L* and *K* peaks is then 11.5 \pm 0.9 (stat) days, in agreement with the measured value [8] of 11.43 \pm 0.03 days.

The waveform data from the Gallex experiment has recently been re-evaluated by Kaether using a new pulse-shape analysis method [9] and the result is $73.1^{+6.1+3.7}_{-6.0-4.1}$ SNU. The result of the GNO experiment was $62.9^{+5.5+2.5}_{-5.3-2.5}$ SNU [10]. If we combine the statistical and systematic uncertainties in quadrature, then the weighted combination of all the Ga experiments, SAGE, Gallex, and GNO, is

$$66.1 \pm 3.1$$
SNU. (Present Ga experiment result.) (2)

IV. SOURCE EXPERIMENTS

The experimental procedures of the SAGE and Gallex experiments, including the chemical extraction, counting, and analysis techniques, have been checked by exposing the gallium target to reactor-produced neutrino sources whose activity was close to 1 MCi. SAGE has irradiated about 25% of their target with a ⁵¹Cr source [11] and an ³⁷Ar source [4,12,13] and Gallex has twice used ⁵¹Cr sources to irradiate their entire target [14]. The results, expressed as the ratio R of the measured ⁷¹Ge production rate to that expected due to the source strength, are shown in Fig. 3. The weighted average value of the ratio for the four experiments is $R = 0.87 \pm 0.05$, more than two standard deviations less than unity. Although the distribution of results is somewhat unusual, with none of the central values from the four measurements lying within the 1σ band around the weighted average, the quality of fit to the average value is quite high (χ^2 /DOF = 1.9/3, GOF = 59%).

We can suggest several possibilities for the unexpectedly low result in the source experiments:



FIG. 3. Results of all neutrino source experiments with Ga. Gallex results are from the recent pulse shape analysis of Kaether [9]; SAGE results are from Refs. [11] and [4]. Hashed region is the weighted average of the four experiments.

- (i) We do not correctly know the various efficiency factors that enter into the calculation of the production rate, namely the extraction efficiency and the counting efficiency. Both SAGE and Gallex have, however, made many ancillary experiments [3,14] that have established with high probability that these efficiencies and their accompanying systematic uncertainties are well determined. These tests have also proven that there are no substantial errors in the methods used to select ⁷¹Ge events or in the methods of analysis. Further, the ⁷¹As experiment of Gallex [15] has ruled out any "hot-atom" chemical effects that might make the ⁷¹Ge atoms produced by neutrino capture difficult to extract. We thus very strongly doubt that the low average result of the source experiments is due to incorrect knowledge of efficiencies, errors in event selection, improper functioning of the counting systems, or errors in analysis.
- (ii) A statistical fluctuation. A χ^2 test of the compatability of the four source experiments to R = 1.0 gives $\chi^2/\text{DOF} = 7.7/3$, whose probability is 5.3%. The probability is small but still quite possible.
- (iii) Electron neutrinos disappear due to a real physical effect of unknown origin. Some possibilities that have been suggested are a transition to sterile neutrinos [16] or quantum decoherence in neutrino oscillations [17].
- (iv) The production rate from the source is not as great as has been assumed. It is our opinion that this is the most likely cause of the apparently low result in the source experiments. As suggested by Haxton [18] it is quite possible that the cross sections for neutrino capture to the two lowest excited states in ⁷¹Ge, both of which can be reached using either ⁵¹Cr or ³⁷Ar sources, have been overestimated. 95% of the capture rate with these sources arises from the ⁷¹Ga to ⁷¹Ge ground-state transition with 5% due to transitions to the two excited states. If the contribution of the excited states to the predicted rate were to be zero then $R = p(\text{measured})/p(\text{predicted}) = 0.92 \pm 0.06$ and the fit to the expected value of 1.0 becomes quite reasonable $(\chi^2/\text{DOF} = 4.58/3, \text{GOF} = 21\%).$

A concern in this context is the implication of the apparently low result of the source experiments on the solar neutrino result given in Eq. (2). It is difficult to address this concern because we do not understand why the source experiments give a lower result than expected. If we suppose that the cause is item (i) in the list above, then the rate in Eq. (2) should be divided by the factor 0.87, i.e., we should add 15% to the systematic uncertainty. But, as stated above, we consider that explanation for the apparent discrepancy in the source experiments to be very unlikely. However, if we suppose that the cause of the low result in the source experiments is any of the other items in the list above, then the source experiments have no bearing on the solar neutrino result and the rate in Eq. (2) should not be changed. Because we do not know why the source experiments appear to be low, we can only caution the reader to accept the result in Eq. (2) on a provisional basis, subject to the caveats that not all effects in the emission of neutrinos

from the Sun and the capture of neutrinos by ⁷¹Ga may be fully understood.

Based on the information given in the definitive article of Bahcall [19] on the neutrino capture cross section of 71 Ga, we have approximately calculated the cross section if we assume zero strength for capture to the first two excited states of 71 Ge. These results are given in Appendix C and will be used as a working hypothesis in what follows.

V. INTERPRETATION OF RESULTS

In contrast to all other present or past solar neutrino experiments, the radiochemical Ga experiment, because of its low threshold of 233 keV, is sensitive to all components of the solar spectrum, from the low-energy pp neutrinos to the high-energy neutrinos produced in the decay of ⁸B. In Table II we give the flux of the various solar neutrino components at their production regions in the Sun as calculated by Bahcall and Collaborators [20,21]. In this section we will estimate the neutrino capture rate from each flux component and compare their total to the measured rate.

The total capture rate R of solar neutrinos in a radiochemical experiment such as Ga is given by

$$R = \int_{E_{\text{threshold}}}^{\infty} \sigma(E) \Phi^{\dagger}(E) dE, \qquad (3)$$

where $\sigma(E)$ is the cross section of the neutrino-capture reaction and $\Phi^{\diamond}(E)$ is the total flux of electron neutrinos at the Earth, which can be expressed as

$$\Phi^{\diamond}(E) = \sum_{i} \phi_{i}^{\diamond} S_{i}^{\diamond}(E).$$
(4)

In this expression the index *i* refers to the various nuclear reactions in the Sun that produce neutrinos $(pp, {}^{7}\text{Be}, pep, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F}, {}^{8}\text{B}, \text{and } hep), \phi_{i}^{\circ}$ is the amplitude of flux component

TABLE II. Solar neutrino fluxes calculated by the standard solar model of Bahcall, Peña-Garay, and Serenelli [21] for two different conservative choices of heavy element composition, labeled GS98 [22] (high metallicity) and AGS05 [23] (low metallicity). The spectrum components refer to the nuclear reaction from which they originate. Units of flux are 10^{10} (*pp*), 10^{9} (⁷Be), 10^{8} (*pep*, ¹³N, ¹⁵O), 10^{6} (⁸B, ¹⁷F), and 10^{3} (*hep*) cm⁻²s⁻¹. The uncertainty values are at 68% confidence.

Spectrum component i	$\mathrm{Flux} \ \phi_i^\odot$					
	GS98	AGS05				
рр	$5.97(1^{+0.006}_{-0.006})$	$6.04(1^{+0.005}_{-0.005})$				
рер	$1.41(1^{+0.011}_{-0.011})$	$1.45(1^{+0.010}_{-0.010})$				
⁷ Be	$5.07(1^{+0.06}_{-0.06})$	$4.55(1^{+0.06}_{-0.06})$				
¹³ N	$2.88(1^{+0.15}_{-0.15})$	$1.89(1^{+0.14}_{-0.13})$				
¹⁵ O	$2.15(1^{+0.17}_{-0.16})$	$1.34(1^{+0.16}_{-0.15})$				
¹⁷ F	$5.82(1^{+0.19}_{-0.17})$	$3.25(1^{+0.16}_{-0.15})$				
⁸ B	$5.94(1^{+0.11}_{-0.11})$	$4.72(1^{+0.11}_{-0.11})$				
hep	$7.90(1^{+0.15}_{-0.15})$	$8.22(1^{+0.15}_{-0.15})$				

i at the Earth, and $S_i^{\diamond}(E)$ is the spectrum of the *i*th neutrino component at the Earth, each of which is normalized such that $\int_0^{\infty} S_i^{\diamond}(E) dE = 1$. The neutrino spectrum at the Earth is related to the spectrum produced in the Sun $S_i^{\odot}(E)$ by

$$S_i^{\diamond}(E) = A_i S_i^{\odot}(E) P_i^{ee}(E), \tag{5}$$

where A_i is a constant of normalization and $P_i^{ee}(E)$ is the probability that an electron neutrino produced in the Sun by reaction *i* with energy *E* will reach the Earth without a change of flavor, commonly called the survival factor. The physical origin for the reduction of the electron component of the solar neutrino flux is the now well-established mechanism of MSW neutrino oscillations [24]. $P_i^{ee}(E)$ is different for each flux component as the neutrinos are produced at different locations in the Sun and thus pass through regions of different electron density during their travel to the Earth. $P_i^{ee}(E)$ can only be calculated if one knows where in the Sun the neutrinos are made and thus requires a solar model.

We integrate Eq. (5) and obtain $A_i = 1/\langle P_i^{ee} \rangle$ where

$$\left\langle P_i^{ee} \right\rangle = \int_0^\infty S_i^{\odot}(E) P_i^{ee}(E) dE \tag{6}$$

is the spectrum-weighted average value of P_i^{ee} . The physical interpretation of $\langle P_i^{ee} \rangle$ is as the ratio of the solar neutrino amplitudes at the surface of the Earth and at the production point in the Sun:

$$\left\langle P_i^{ee} \right\rangle = \frac{\phi_i^\circ}{\phi_i^\circ}.\tag{7}$$

Combining these equations, and expressing *R* as the sum of its spectral components, $R = \sum_{i} R_{i}$, we have

$$R_i = \phi_i^{\odot} \langle \sigma_i^{\odot} \rangle, \tag{8}$$

where

$$\left\langle \sigma_{i}^{\odot} \right\rangle = \int_{E_{\text{threshold}}}^{\infty} \sigma(E) S_{i}^{\odot}(E) P_{i}^{ee}(E) dE, \qquad (9)$$

or, equivalently, if it is the flux at the Earth that is assumed known,

$$R_i = \phi_i^{\circ} \langle \sigma_i^{\circ} \rangle, \tag{10}$$

where

$$\langle \sigma_i^{\circ} \rangle = \frac{\langle \sigma_i^{\circ} \rangle}{\langle P_i^{ee} \rangle}.$$
 (11)

In Table III we give values of $\langle P_i^{ee} \rangle$ and $\langle \sigma_i^{\delta} \rangle$ for each neutrino component. These were calculated assuming threeneutrino mixing to active neutrinos with parameters from Ref. [25]: $\Delta m_{12}^2 = (7.65^{+0.23}_{-0.20}) \times 10^{-5} \text{ eV}^2$, $\theta_{12} = 33.46^{+1.36}_{-1.00}$ degrees, and $\theta_{13} = 5.7^{+3.5}_{-5.7}$ degrees. The approximate formulae given in Ref. [26] were used for the survival probability $P_i^{ee}(E)$. As we show in Sec. VII there is no appreciable difference between the day and night capture rates in Ga and thus regeneration in the Earth was neglected. The cross sections $\sigma(E)$ were taken from Appendix C for Ga and Ref. [27] for Cl. The neutrino spectra $\phi_i^{\odot}(E)$ are from Refs. [19] $(pp, {}^{13}N, {}^{15}O, {}^{17}F), [27] ({}^8B)$, and [28] (hep).

Exp.	Spect.	$\langle P_i^{ee} \rangle$	Percentage u	uncertainty in	$\langle P_i^{ee} \rangle$ due to	Total unc.	$\langle \sigma_{i}^{\dagger} \rangle$	Perce	entage uncerta	inty in $\langle \sigma_i^{\circ} \rangle$ of	lue to	Total unc.
	comp.		Δm_{12}^2	θ_{12}	θ_{13}	$\ln \langle P_i^{ee} \rangle (\%)$	$(10^{-40}\mathrm{cm}^2)$	σ	Δm_{12}^2	θ_{12}	θ_{13}	$\ln \langle \sigma_i^{\circ} \rangle (\%)$
⁷¹ Ga	pp	0.561	+0.0, -0.0	+2.2, -2.8	+2.0, -3.1	+3.0, -4.2	11.75	+2.4, -2.3	+0.0, -0.0	+0.0, -0.0	+0.0, -0.0	+2.4, -2.3
	рер	0.521	+0.3, -0.3	+1.9, -2.3	+1.9, -2.9	+2.7, -3.7	194.4	+17, -2.4	+0.0, -0.0	+0.0, -0.0	+0.0, -0.0	+17, -2.4
	⁷ Be	0.542	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+2.9, -4.0	68.21	+7.0, -2.3	+0.0, -0.0	+0.0, -0.0	+0.0, -0.0	+7.0, -2.3
	¹³ N	0.545	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+2.9, -4.0	56.83	+9.8, -2.3	+0.0, -0.0	+0.1, -0.0	+0.0, -0.0	+9.8, -2.3
	¹⁵ O	0.535	+0.2, -0.2	+2.0, -2.5	+1.9, -3.0	+2.8, -3.9	107.1	+13, -2.3	+0.0, -0.0	+0.1, -0.1	+0.0, -0.0	+13, -2.3
	¹⁷ F	0.535	+0.2, -0.2	+2.0, -2.5	+1.9, -3.0	+2.8, -3.9	107.7	+13, -2.3	+0.0, -0.0	+0.1, -0.1	+0.0, -0.0	+13, -2.3
	⁸ B	0.365	+0.7, -0.6	+3.2, -2.2	+1.8, -2.7	+3.8, -3.5	21400	+32, -14	+0.2, -0.2	+2.1, -1.7	+0.1, -0.1	+32, -15
	hep	0.337	+0.5, -0.5	+4.8, -3.4	+1.8, -2.8	+5.2, -4.4	66000	+33, -15	+0.2, -0.2	+1.4, -1.1	+0.1, -0.1	+33, -16
³⁷ Cl	рер	0.521	+0.3, -0.3	+1.9, -2.3	+1.9, -2.9	+2.7, -3.7	16.00	+2.0, -2.0	+0.0, -0.0	+0.0, -0.0	+0.0, -0.0	+2.0, -2.0
	⁷ Be	0.542	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+2.9, -4.0	2.397	+2.0, -2.0	+0.0, -0.0	+0.0, -0.0	+0.0, -0.0	+2.0, -2.0
	¹³ N	0.545	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+2.9, -4.0	1.686	+2.0, -2.0	+0.1, -0.1	+0.1, -0.1	+0.0, -0.0	+2.0, -2.0
	¹⁵ O	0.535	+0.2, -0.2	+2.0, -2.5	+1.9, -3.0	+2.8, -3.9	6.662	+2.0, -2.0	+0.1, -0.1	+0.2, -0.1	+0.0, -0.0	+2.0, -2.0
	¹⁷ F	0.535	+0.2, -0.2	+2.0, -2.5	+1.9, -3.0	+2.8, -3.9	6.710	+2.0, -2.0	+0.1, -0.1	+0.2, -0.1	+0.0, -0.0	+2.0, -2.0
	⁸ B	0.365	+0.7, -0.6	+3.2, -2.2	+1.8, -2.7	+3.8, -3.5	10140	+3.7, -3.7	+0.2, -0.2	+2.4, -1.9	+0.1, -0.1	+4.4, -4.1
	hep	0.337	+0.5, -0.5	+4.83.4	+1.82.8	+5.2, -4.4	40910	+3.73.7	+0.20.2	+1.51.2	+0.10.1	+4.03.9

TABLE III. Factors needed to compute the solar neutrino capture rate in ⁷¹Ga and ³⁷Cl solar neutrino experiments. The uncertainty values are at 68% confidence. The parameter $\langle \sigma_i^{\dagger} \rangle$ is defined in Eq. (11).

Now that all the terms have been calculated, we can use the fluxes in Table II, combined with Eqs. (8) and (11), to predict the capture rate in Ga from each of the solar neutrino components. The individual rates and the total rate are given in Table IV for two recent solar models from Table II. For both models there is good agreement between the calculated total rate and the observed capture rate of 66.1 ± 3.1 SNU. The major contribution to the uncertainty in the predicted total rate is from the neutrino capture cross section, with smaller contributions from the solar model flux, θ_{12} , and θ_{13} .

In this analysis we have used the cross sections in Appendix C, in which the contribution of the two lowest-lying excited states in ⁷¹Ge has been set to zero. If instead we use the original Bahcall cross sections, then the total rate increases by 1.2 SNU with the GS98 composition and by 1.1 SNU with the AGS05 composition. Whatever cross sections are assumed is thus not a significant factor in the interpretration of the total rate in the Ga experiment. The cross sections are, however, of vital importance in understanding the origin of the unexpectedly low result in the source experiments.

The attentive reader may be concerned that there is a logical inconsistency in the argument presented here: the predicted capture rates we derive for the Ga experiment depend on the neutrino oscillation parameters, but the measured total rate of the Ga experiment is itself one of the inputs used to determine the oscillation parameters. Although this is true in a strict sense, the neutrino oscillation parameters derived from a global fit of all experiments are for all practical purposes independent of the rate in the Ga experiment. Rather, the parameter θ_{12} is principally determined by the SNO experiment, θ_{13} by the CHOOZ experiment, and Δm_{12}^2 by the KamLAND experiment. Although it was not true in the past, the result of the Ga experiment is at present only a very minor input to the determination of these parameters.

Incidentally, if we carry out the same analysis for the ³⁷Cl solar neutrino experiment, the total calculated rate is $3.09(1^{+0.094}_{-0.091})$ SNU using fluxes based on GS98 and $2.53(1^{+0.091}_{-0.089})$ SNU using fluxes based on AGS05. These should be compared with the experimental rate of 2.56 ± 0.16 (stat) ± 0.16 (syst) SNU [1].

VI. THE pp NEUTRINO FLUX FROM THE SUN

In this section we will use the Ga measurement given in Eq. (2) and the results of other solar neutrino experiments to determine the pp flux from the Sun. The conventional way to make this calculation is by a combined fit to all experiments, as for example is presented in Ref. [29] and [30]. Here we give an alternate approach that successively decomposes the

TABLE IV. Capture rates R_i for Ga experiments calculated with fluxes from Ref. [21].

Spect.		With GS98 composition							With AGS05 composition					
comp.	Cap. rate	rate Percentage uncertainty in rate due to					Total unc. Cap. rate		Percentage u	ncertainty in r	ate due to		Total unc.	
	(SNU)	φ	σ	Δm_{12}^2	θ_{12}	θ_{13}	in rate (%)	(SNU)	φ	σ	Δm_{12}^2	θ_{12}	θ_{13}	in rate (%)
рр	39.35	+0.6, -0.6	+2.4, -2.3	+0.0, -0.0	+2.2, -2.8	+2.0, -3.1	+3.9, -4.8	39.81	+0.5, -0.5	+2.4, -2.3	+0.0, -0.0	+2.2, -2.8	+2.0, -3.1	+3.9, -4.8
рер	1.43	+1.1, -1.1	+17.0, -2.4	+0.3, -0.3	+1.9, -2.3	+1.9, -2.9	+17.2, -4.6	1.47	+1.0, -1.0	+17.0, -2.4	+0.3, -0.3	+1.9, -2.3	+1.9, -2.9	+17.2, -4.5
⁷ Be	18.73	+6.0, -6.0	+7.0, -2.3	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+9.7, -7.5	16.81	+6.0, -6.0	+7.0, -2.3	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+9.7, -7.5
^{13}N	0.89	+15.0, -15.0	+9.8, -2.3	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+18.1, -15.7	0.58	+14.0, -13.0	+9.8, -2.3	+0.1, -0.1	+2.1, -2.6	+2.0, -3.0	+17.3, -13.8
¹⁵ O	1.23	+17.0, -16.0	+12.9, -2.3	+0.2, -0.2	+2.0, -2.4	+1.9, -3.0	+21.5, -16.6	0.77	+16.0, -15.0	+12.9, -2.3	+0.2, -0.2	+2.0, -2.4	+1.9, -3.0	+20.7, -15.6
¹⁷ F	0.03	+19.0, -17.0	+12.9, -2.3	+0.2, -0.2	+2.0, -2.4	+1.9, -3.0	+23.1, -17.6	0.02	+16.0, -15.0	+12.9, -2.3	+0.2, -0.2	+2.0, -2.4	+1.9, -3.0	+20.8, -15.6
^{8}B	4.64	+11.0, -11.0) +31.8, -14.4	+0.5, -0.4	+5.4, -3.9	+1.8, -2.8	+34.1, -18.7	3.68	+11.0, -11.0	+31.8, -14.4	+0.5, -0.4	+5.4, -3.9	+1.8, -2.8	+34.1, -18.7
hep	0.02	+15.0, -15.0) +32.7, -15.4	+0.3, -0.3	+6.2, -4.5	+1.9, -2.9	+36.5, -22.2	0.02	+15.0, -15.0	+32.7, -15.4	+0.3, -0.3	+6.2, -4.5	+1.9, -2.9	+36.5, -22.2
Total	66.31	+1.9, -1.9	+3.3, -1.8	+0.1, -0.1	+1.5, -1.8	+1.3, -2.0	+4.3, -3.8	63.16	+1.8, -1.8	+3.1, -1.8	+0.1, -0.0	+1.5, -1.9	+1.4, -2.1	+4.1, -3.8

total measured rate into the components from each neutrino source. The final result is identical to what one obtains in a combined fit and has the advantage that the argument is simple and transparent.

The rate in Eq. (2) is the sum of the rates from all the components of the solar neutrino flux, which we denote by

$$[pp + {}^{\prime}Be + CNO + pep + {}^{8}B|Ga] = 66.1(1 \pm 0.047) \text{ SNU.}$$
(12)

We ignore the tiny *hep* contribution and combine the ¹³N, ¹⁵O, and ¹⁷F components into a single value, called here "CNO."

In an experiment of great technical difficulty, the ⁷Be flux has been directly measured by Borexino and they report the result as $\phi_{7Be}^{\odot} = 5.18(1 \pm 0.098) \times 10^9$ neutrinos/(cm² s) [30]. Using Eqs. (8) and (11), we multiply this flux by the electron neutrino survival factor for ⁷Be and by the cross section of ⁷Be on Ga (the values of these factors and their uncertainties are given in Table III) and obtain the rate of ⁷Be in Ga of

$$[^{7}Be|Ga] = 19.1(1^{+0.12}_{-0.11}) \text{ SNU.}$$
(13)

The ⁸B flux at the Earth has been directly measured by SNO to be $\phi_{^8B}^{^{\dagger}} = (1.67 \pm 0.05) \times 10^6$ electron neutrinos/(cm² s) [31]. In a similar way to ⁷Be, we multiply this flux by the spectrum-integrated cross section for ⁸B neutrinos on Ga and obtain the ⁸B contribution to the Ga experiment of

$$[^{8}B|Ga] = 3.6 (1^{+0.32}_{-0.16}) SNU.$$
(14)

Subtracting these measured rates of ^{7}Be and ^{8}B from the total Ga rate in Eq. (12) gives

$$[pp + \text{CNO} + pep|\text{Ga}] = 43.3(1^{+0.087}_{-0.094}) \text{ SNU.}$$
(15)

We can obtain an approximate value for the contribution of CNO and *pep* to the Ga experiment from the measured capture rate in the Cl experiment [⁷Be + CNO + *pep* +⁸B|Cl] = 2.56(1 ± 0.088) SNU [1]. As in the case of Ga, we use the ⁷Be flux measured by Borexino, the ⁸B flux measured by SNO, and the cross sections in Table III to determine [⁷Be|Cl] = $0.67(1^{+0.105}_{-0.108})$ SNU and [⁸B|Cl] = $1.73(1^{+0.068}_{-0.067})$ SNU. We subtract these values from the total Cl rate and are left with [CNO + *pep*|Cl] = $0.19(1^{+1.36}_{-1.00})$ SNU.

If we attribute this entire rate to the neutrinos from *pep* then, using the cross sections for *pep* on Cl and Ga, we calculate a rate of $[pep| \text{ Ga}]_{\text{test}} = 2.35(1^{+1.37}_{-1.00})$ SNU. On the other hand, if we attribute this entire rate to CNO, we obtain in the same manner a rate of $[\text{CNO}|\text{Ga}]_{\text{test}} = 3.11(1^{+1.37}_{-1.00})$ SNU. The upper extreme of these two test rates is $3.11 \times (1 + 1.37) = 7.37$ SNU. As a reasonable estimate we can thus set the sum of CNO and *pep* rates at half this upper limit with an uncertainty of 100%:

$$[\text{CNO} + pep|\text{Ga}] = 3.68 (1^{+1.00}_{-1.00}) \text{ SNU.}$$
(16)

We subtract this estimate for the CNO plus *pep* rate from the rate in Eq. (15) and obtain the result for the measured *pp* rate in the Ga experiment

$$[pp|Ga] = 39.7(1^{+0.13}_{-0.14}) \text{ SNU.}$$
(17)

Dividing this capture rate by the cross section for capture of pp neutrinos from Table III gives the measured electron neutrino pp flux at Earth of

$$\phi_{pp}^{\circ} = 3.38 (1_{-0.14}^{+0.14}) \times 10^{10} / (\text{cm}^2 \text{ s}).$$
 (18)

If we use Eq. (7) and the value of $\langle P_i^{ee} \rangle = 0.561(1^{+0.030}_{-0.042})$ from Table III then the *pp* flux produced in the Sun is

$$\phi_{pp}^{\odot} = 6.0(1 \pm 0.14) \times 10^{10} / (\text{cm}^2 \text{ s}).$$
 (19)

Our present result for the pp flux is in good agreement with the previous estimates that we have made during the past 6 years [3,32,33], with the major change being a gradual reduction of the uncertainty. In the future, as Borexino continues to collect data, and as direct measurements are made of the CNO and *pep* fluxes, the uncertainty in this flux should be further reduced and eventually may be dominated by the uncertainty in the Ga rate itself. By that time, however, there will hopefully be direct experiments that measure the *pp* flux in real time.

For comparison, we see from Table II that the predicted pp flux from the two recent solar models with different composition is $\phi_{pp}^{\odot} = 5.97 \pm 0.04$ and 6.04 ± 0.03 , both in units of $10^{10} v_e/(\text{cm}^2 \text{ s})$. There is good numerical agreement between these flux values and the result in Eq. (19), but, as made clear by Bahcall and Peña-Garay [34], there is a large difference in interpretation: the result in Eq. (19) was derived from contemporary solar neutrino experiments and is the pp neutrino flux at the present time. In contrast, energy generation in the solar model is highly constrained by the measured solar luminosity and thus, when the luminosity constraint is imposed, as is the case for the models given in Table II, the calculated *pp* flux is what the Sun was producing some 40 000 years ago. The agreement between the present *pp* flux, as measured by the Ga experiment, and the past flux, as inferred from the solar model with the luminosity constraint, implies that the pp flux from the Sun has not altered (within our 14%) uncertainty) during the last 40 000 years.

VII. CONSIDERATION OF TIME VARIATION

In a plot of the SAGE results as a function of time there is a slight visual hint of a long-term decrease, as illustrated in Fig. 2. The average rate prior to 1996 is somewhat higher than after 1996. A plot of the Gallex-GNO data shows a similar behavior [10]. When examined quantitatively, however, the evidence for a long-term decrease in the capture rate is unconvincing. A χ^2 test applied to these yearly SAGE data points assuming the rate is constant at 65.4 SNU gives $\chi^2/\text{DOF} = 12.0/17$, which has a probability of 80%. The fit to a constant rate is thus quite good.

In previous articles we have demonstrated the agreement between the assumption of a constant production rate and the SAGE measurements by use of the cumulative distribution of the capture rate C(p), defined as the fraction of data sets whose capture rate is less than p. Figure 4 shows this distribution for the data and the expected distribution derived from 100 simulations of all 168 runs, where it is assumed in the simulations that the production rate is constant and has a value of 65.4 SNU. For each run the rates from the separate L and



FIG. 4. Measured capture rate for all 310 SAGE data sets (jagged curve) and the expected distribution derived by 100 Monte Carlo simulations of each set (smooth curve).

K peaks are used in this figure, not the rate from the L + K combination. To ensure that the simulations parallel the real data as closely as possible, all parameters of the simulation, such as background rates, efficiencies, exposure times, and counting times, were chosen to be the same as for the real data. Only the number of counts in each run and the times when these counts occurred were allowed to vary.

The data spectrum and the simulated spectrum are very similar to each other, indicating that the distribution of capture rates is what one would expect if the rate is constant. A quantitative comparison can be made by calculating the Nw^2 test statistic for the data distribution and comparing it to the distribution from simulations using the method described in Ref. [35]. The fraction of simulated spectra whose Nw^2 was larger than for the data distribution is 43%, which shows that the assumption of a constant capture rate is in good agreement with our measurements.

A standard method to look for periodic signals in unevenly sampled data, such as we have in SAGE, was devised by Lomb and Scargle. Application of this method, using the implementation of Press *et al.* [36], to all runs from the SAGE experiment yields the power spectrum shown in Fig. 5. The frequency range considered is from nearly zero up to slightly







FIG. 6. Histogram of powers in spectrum of Fig. 5. The bin size is 0.07 power units. The solid line is the expected distribution if there is no time variation, i.e., the number of frequencies $\times \exp(-\text{power})$, integrated over the limits of each bin.

less than twice the Nyquist frequency. The maximum Lomb power is 6.10 and it occurs at a frequency of 8.47 cycles/year.

A simple way to assess the significance of a peak in such a spectrum is to make a histogram of the number of frequencies as a function of power. In the absence of any time variation this distribution is an exponential; if there were any peak present with significant power it would appear at the upper end of the distribution and be clearly separated from the exponential trend. This distribution for the spectrum of Fig. 5 is shown in Fig. 6. As this distribution visually shows, there is no evidence for exceptionally high power in the data spectrum at any frequency.

A quantitative way to prove that no frequency has exceptionally high power is with a shuffle test. In this test the SNU results are randomly reassigned to the different runs, the power spectrum is recalculated, and the maximum power is found. The maximum power in the spectra from 1000 such shuffles is plotted in Fig. 7. The observed maximum power for the SAGE data of 6.10 occurs very near the center of this distribution. Of the shuffles, 44% have a greater power than for the data



FIG. 7. Histogram of maximum power found in Lomb power spectrum analysis of 1000 random shuffles of the 168 SAGE runs. The bin size is 0.13 power units.

and 56% a lesser power, thus showing that the observed power distribution is consistent with the assumption of a constant rate.

The same result is obtained if power spectra are produced over a wider frequency range. For ranges up to 19 per year and 38 per year the maximum power remains 6.10 at 8.47 cycles per year and is not statistically significant.

The three tests in this section do not find any evidence for periodic variation in the SAGE data. The frequency range to which these tests are sensitive is about one cycle per month to about one cycle per 10 years. For a frequency to be detected within this range the amplitude of the periodic oscillation must, as shown by Pandola [37], exceed the statistical uncertainty of a single run, i.e., must be $\gtrsim 30$ SNU.

We end this section by noting that the winter minus summer difference in SAGE capture rate is $R_W - R_S = 5.8^{+6.2}_{-6.1}$ SNU where the stated uncertainty is only statistical. For this calculation summer was defined as the ±1/4-year interval centered on 21 June and winter as the rest of the year. In our method of data analysis [2] we remove the known change in rate caused by the Earth's orbital eccentricity. If, rather than using the above solstice-based definition, we define summer as the ±1/4-year interval centered on 5 July, the time of the aphelion, then $R_W - R_S = 4.2^{+6.2}_{-6.1}$ SNU. With both of these definitions $R_W - R_S$ is consistent with zero, indicating that there is no appreciable difference between the day and night capture rates in Ga, as is expected for the currently determined values of the neutrino oscillation parameters [38]).

VIII. SUMMARY AND DISCUSSION

In 18 years of operation the SAGE experiment has carried out 168 measurements of the solar neutrino capture rate. Analysis of the times of occurrence of all events in windows centered on the L and K peaks and with 2 FWHM width ascribes 853.9 of these events to the decay of ⁷¹Ge. To compare these values with other radiochemical solar neutrino experiments, the Cl experiment collected solar data for a longer time (108 runs with rise-time counting during 24 years), but the number of detected events within an energy window of 2 FWHM that were ascribed to ³⁷Ar was 875 [1], quite comparable to what we now have in SAGE. So far as we are aware, neither Gallex nor GNO have reported their total number of detected ⁷¹Ge events, but because the mass of their Ga target was less (30 tonnes), and their number of runs was less (123 during 12 years), this number must be considerably less than we now have in SAGE.

The measured best-fit capture rate in the SAGE experiment is $65.4^{+3.1}_{-3.0}$ (stat) $^{+2.6}_{-2.8}$ (syst) SNU. Combining this with the results of Gallex and GNO gives the rate as 66.1 ± 3.1 SNU, where statistical and systematic uncertainties have been combined in quadrature. The solar-model prediction for the Ga experiment is 66.3 SNU for a model with high metallicity and 63.2 SNU for a model with low metallicity, where the uncertainty of both estimates is $\sim 4\%$. There is thus excellent agreement between theory and experiment for the Ga experiment. Further, both the experimental measurement and the theoretical prediction are known to about the same accuracy.

By use of the results of other solar neutrino experiments and neutrino oscillation theory we have derived the contemporary value of the *pp* flux from the Sun to be $(3.40^{+0.46}_{-0.47}) \times 10^{10}/(\text{cm}^2 \text{ s})$ at the Earth and $(6.0 \pm 0.8) \times 10^{10}/(\text{cm}^2 \text{ s})$ at the Sun. The latter is in good agreement with standard solar model predictions of 5.97 ± 0.04 (high metallicity) and 6.04 ± 0.03 (low metallicity), both in units of $10^{10} v_e/(\text{cm}^2 \text{ s})$. Gallium experiments have thus proven that the overwhelming fraction of solar neutrinos that reach the Earth are the low-energy neutrinos from the *pp* reaction.

We have assumed in these calculations that the cross section for capture to the two lowest-lying excited states in ⁷¹Ge is zero, as is implied by the four neutrino source experiments with gallium. This assumption is in contradiction to the standard interpretation of the two experiments that have attempted to measure the Gamow-Teller strength of these low-lying excited states in ⁷¹Ge. These experiments were made by (p, n) scattering [19,39] and (³He,t) scattering [40–42]. If all the events observed in these experiments at low excitation energy are attributed to Gamow-Teller strength the results of these experiments are in reasonably good agreement. It is, however, not evident that these experiments solely measure Gamow-Teller strength—as emphasized by Haxton [18], for very weak transitions, such as is the case in these experiments, there may be an appreciable (perhaps dominant) contribution to the cross section from the spin-tensor interaction. New experimental data is needed to settle this question and to definitively determine the magnitude of the matrix elements for neutrino capture to these two low-lying excited states. We strongly encourage any new experiments that might shed light on this question. As part of our future experimental program we intend to pursue a new measurement that will use a very intense neutrino source in an optimized detector geometry.

The SAGE experiment continues to collect data on the solar neutrino capture rate with a gallium target. Up to now it is only the Ga experiment that has measured the low-energy *pp* solar neutrinos. As we continue to monitor the solar neutrino flux we will increase our statistical accuracy and further reduce our systematic uncertainties.

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APPENDIX A: CALCULATION OF EXTRACTION EFFICIENCY FROM ISOTOPIC ANALYSIS

It is assumed the extracted Ge consists of a combination of

- (i) Ge from the carrier added for the current extraction,
- (ii) residual Ge that remained from the carrier added for the two preceding extractions, and
- (iii) additional Ge with natural isotopic composition, such as may be dissolved from the surfaces of the extraction system or the vessels that contain the Ga.

According to this model the predicted mass of isotope *i* obtained in extraction *n*, called $M_n^p(i)$, is thus

$$M_n^p(i) = \varepsilon_n \{ C_n(i) + (1 - \varepsilon_{n-1}) [C_{n-1}(i) + (1 - \varepsilon_{n-2}) C_{n-2}(i)] \} + E_n I(i),$$
(A1)

where ε_m is the efficiency of Ge removal in extraction m, $C_m(i)$ is the mass of isotope i of carrier added to extraction m (where m can take on the values n, n - 1, or n - 2), and E_n is the mass of additional Ge with natural isotopic composition I(i) that is removed in extraction n. It is assumed that I(i), $C_n(i)$, $C_{n-1}(i)$, and $C_{n-2}(i)$ are known and the variables ε_n , ε_{n-1} , ε_{n-2} , and

 E_n are to be determined. For N extractions there are thus 2N variables (the extraction efficiency and mass of extra natural Ge for each extraction) and 5N equations that relate these variables, one for each of the naturally occurring Ge isotopes, ⁷⁰Ge, ⁷²Ge, ⁷³Ge, ⁷⁴Ge, and ⁷⁶Ge. Because there are more relationships than unknowns, the problem is solved by finding the set of variables that minimizes the function

$$\chi^{2} = \sum_{n=1}^{N} \sum_{i=1}^{5} \left(\frac{M_{n}^{p}(i) - M_{n}^{e}(i)}{\sigma_{n}(i)} \right)^{2},$$
(A2)

where $M_n^e(i)$ is the measured mass of isotope *i* in extraction *n* and $\sigma_n(i)$ is the total uncertainty in the knowledge of the predicted and measured masses in extraction *n*.

APPENDIX B: RESULTS FOR EACH SAGE EXTRACTION

The capture rate for each SAGE extraction is given in Table V. The statistic Nw^2 in this table measures the goodness of fit between the observed sequence of events in time and the time distribution predicted by the model used in analysis, viz., that the events are produced by the sum of two processes: the decay of a fixed initial number of ⁷¹Ge atoms and background events at a constant rate. The probability to obtain a value of Nw^2 larger than in the observed time distribution is given in the last column. It is derived by 1000 simulations for each extraction using the method in Ref. [35] and has an uncertainty of ~1.5%. The time that the exposure interval for each extraction began and ended can be obtained from the entries in columns 2 and 3 by use of transformation equations (6.4) in Ref. [2].

TABLE V.	Results of	analysis o	f SAGE	extractions.
INDEL V.	Results of	anarysis 0	IOL	extractions.

Exposure date	Median exposure date	Exposure time (days)	Ga mass (tons)	Number of candidate events	Number fit to ⁷¹ Ge	Best fit (SNU)	68% conf. range (SNU)	Nw^2	Probability (%)
Jan. '90	1990.040	42.0	28.67	8	0.0	0	0–65	0.532	2
Feb. '90	1990.139	30.0	28.59	2	1.5	74	19–160	0.167	25
Mar. '90	1990.218	26.0	28.51	10	1.0	40	0-211	0.040	83
Apr. '90	1990.285	19.0	28.40	11	0.0	0	0-157	0.119	35
July '90	1990.540	21.0	21.01	13	0.0	0	0–252	0.080	50
June '91	1991.463	53.0	27.43	10	0.0	0	0–120	0.188	20
July '91	1991.539	23.0	27.37	1	0.6	34	0-116	0.163	33
Aug. '91	1991.622	26.3	49.33	16	9.4	395	247-584	0.036	85
Sep. '91	1991.707	27.0	56.55	11	2.0	42	9-123	0.023	97
Nov. '91	1991.872	26.0	56.32	31	3.1	61	9-162	0.173	12
Dec. '91	1991.948	26.8	56.24	10	8.8	159	100–219	0.061	79
Feb. '92–1	1992.138	24.5	43.03	14	0.0	0	0-43	0.108	44
Feb. '92–2	1992.138	24.5	13.04	1	0.8	80	0-193	0.084	87
Mar. '92	1992.214	20.9	55.96	24	11.7	285	203-414	0.077	36
Apr. '92	1992.284	23.5	55.85	15	1.4	34	13-112	0.143	20
May '92	1992.383	27.5	55.72	5	0.0	0	0-86	0.142	33
Sep. '92	1992.700	116.8	55.60	11	6.5	84	52-125	0.120	24
Oct. '92	1992.790	27.2	55.48	18	3.3	31	7–63	0.093	37
Nov. '92	1992.871	26.7	55.38	28	6.9	90	45-145	0.143	13

Exposure date	Median exposure date	Exposure time (days)	Ga mass (tons)	Number of candidate events	Number fit to ⁷¹ Ge	Best fit (SNU)	68% conf. range (SNU)	Nw^2	Probability (%)
Dec. '92	1992.945	24.3	55.26	27	17.6	174	121–229	0.063	57
Jan. '93	1993.039	32.3	55.14	17	9.9	122	74–176	0.093	33
Feb. '93	1993.115	23.0	55.03	3	0.8	18	0–56	0.090	47
Apr. '93	1993.281	26.6	48.22	7	2.3	56	15-106	0.038	90
May '93	1993.364	30.9	48.17	8	0.6	28	0-122	0.115	41
June '93	1993.454	30.4	54.66	18	5.1	63	22-116	0.426	0
Julv '93	1993.537	27.9	40.44	28	6.7	198	100-312	0.041	84
Aug. '93–1	1993.631	34.0	40.36	4	2.7	73	28-125	0.051	81
Aug. '93–2	1993.628	63.8	14.09	1	1.0	120	0-230	0.093	75
Oct. '93–1	1993.749	13.0	14.06	0	0.0	0	0-158	NA	NA
Oct. '93–2	1993.800	34.7	14.10	4	3.1	144	71-246	0.052	86
Oct. '93–3	1993.812	24.6	14.02	6	2.9	132	64–231	0.049	82
July '94	1994.551	31.3	50.60	20	4.5	63	29–108	0.018	100
Aug. '94	1994.634	31.0	50.55	25	3.6	42	14–79	0.031	95
Sep. '94-1	1994.722	33.2	37.21	30	5.9	101	42-174	0.100	36
Oct. '94	1994.799	28.8	50.45	44	0.0	0	0-128	0.269	12
Nov. '94	1994.886	31.0	50.40	23	8.0	115	68-172	0.015	100
Dec. '94	1994.951	21.0	13.14	9	0.0	0	0–236	0.184	20
Mar. '95	1995.209	42.5	24.03	23	3.6	145	48-264	0.042	84
July '95	1995.538	19.9	50.06	33	7.3	106	53-168	0.108	28
Aug. '95	1995.658	46.7	50.00	21	7.5	105	62-158	0.081	43
Sep. '95	1995.742	28.8	49.95	33	1.3	29	0-126	0.058	75
Oct. '95	1995.807	18.7	49.83	25	5.8	148	62-254	0.037	89
Nov. '95	1995.875	25.8	49.76	31	10.6	131	83-188	0.028	94
Dec. '95–2	1995.962	32.7	41.47	39	1.6	39	0–117	0.093	50
Jan. '96	1996.045	29.7	49.64	34	0.0	0	0–42	0.095	53
May '96	1996.347	49.9	49.47	16	4.7	70	25-127	0.028	98
Aug. '96	1996.615	45.0	49.26	21	4.9	77	31–134	0.075	49
Oct. '96	1996.749	45.8	49.15	21	5.9	82	46-127	0.053	70
Nov. '96	1996.882	48.7	49.09	28	1.6	22	0–64	0.097	45
Jan. '97	1997.019	49.8	49.04	24	2.8	37	6–79	0.197	13
Mar. '97	1997.151	44.9	48.93	23	1.6	19	0–55	0.457	1
Apr. '97	1997.277	42.9	48.83	22	3.2	41	12–79	0.049	79
June '97	1997.403	45.6	48.78	26	10.3	140	91–199	0.073	43
July '97	1997.537	45.9	48.67	22	1.6	22	0–56	0.445	1
Sep. '97	1997.671	46.4	48.56	15	3.9	62	25-110	0.036	91
Oct. '97	1997.803	45.0	48.45	25	4.6	63	28-108	0.127	23
Dec. '97	1997.940	47.0	48.34	22	4.7	78	34–135	0.054	66
Apr. '98	1998.225	44.9	48.05	38	5.8	82	35-140	0.048	77
May '98	1998.347	30.0	51.17	21	4.4	57	24–98	0.036	90
July '98	1998.477	45.6	51.06	21	5.7	72	36-118	0.076	46
Aug. '98	1998.611	45.7	50.93	31	4.1	52	20-95	0.047	82
Oct. '98	1998.745	45.8	50.81	38	4.7	56	18-103	0.027	96
Nov. '98	1998.883	45.8	50.68	30	5.2	59	20-107	0.078	51
Jan. '99	1999.014	44.7	50.54	21	2.3	29	0–72	0.084	51
Feb. '99	1999.130	38.7	50.43	15	2.3	34	4–76	0.096	42
Apr. '99	1999.279	51.7	50.29	9	1.5	33	5-74	0.054	76
June '99	1999.417	46.7	50.17	14	14.0	185	140-239	0.031	98
July '99	1999.551	45.7	50.06	17	6.0	111	54-182	0.100	32

TABLE V. (Continued.)

Exposure date	Median exposure date	Exposure time (days)	Ga mass (tons)	Number of candidate events	Number fit to ⁷¹ Ge	Best fit (SNU)	68% conf. range (SNU)	Nw^2	Probability (%)
Sep. '99	1999.685	45.7	49.91	20	3.3	42	5-93	0.250	5
Oct. '99	1999.801	38.7	49.78	15	9.6	134	81–196	0.082	49
Ian '00	2000 035	28.8	49 59	23	73	84	46_129	0 101	30
Feb '00	2000.033	20.0	49.39	23	7.5	04 92	40-129 55-138	0.101	80
Mar '00	2000.127	28.8	49.40	18	93	106	70-150	0.044	72
May '00	2000.207	30.7	49.72	12	1.6	16	0_43	0.031	85
June '00	2000.557	33.7	49.18	16	0.8	13	0-59	0.324	6
July '00	2000.131	32.0	49.12	27	6.2	66	33-107	0.083	42
Aug '00	2000.510	31.3	49.06	14	5.2	74	41-116	0.088	37
Sep '00	2000.020	27.7	49.00	30	9.0	107	62-160	0.000	36
Oct. '00	2000.796	30.7	48.90	14	0.3	4	0-31	0.090	56
Nov. '00	2000.876	28.7	48.84	22	1.0	11	0-41	0.166	24
Dec. '00	2000.958	30.7	48.78	25	7.4	78	43–119	0.066	64
Feb '01	2001 122	20.8	41.11	20	6.5	80	47 123	0.100	20
Mar '01	2001.122	23.0	41.11	17	2.3	26	47-125	0.100	29 55
Apr 01	2001.214	22.4 22.7	48.33	16	67	20 70	41 107	0.077	40
May '01	2001.290	31.7	48 37	20	12.0	118	41–107 85–158	0.087	35
June '01	2001.373	31.7	48 27	19	7.2	66	38_99	0.070	55 77
July '01	2001.407	23.7	48.17	7	3.0	36	17-65	0.047	98
Aug '01	2001.517	28.7	48 11	17	5.0 7.0	117	66-180	0.020	41
Sep '01	2001.024	20.7	48.06	10	2.5	24	4-52	0.002	22
Oct '01	2001.701	30.7	47.96	12	2.5	63	39_94	0.120	23
Nov '01	2001.887	34.8	47.91	19	47	39	17-67	0.120	29
Dec. '01	2001.955	22.8	47.86	20	4.4	47	22-80	0.056	72
Ian '02	2002 043	29.7	47 75	31	23.2	201	153_254	0 162	18
Feb '02	2002.045	27.7	41.01	12	73	78	48_114	0.102	24
Mar '02	2002.120	27.7	47.62	12	6.2	53	28_84	0.121	35
Apr $^{\circ}02$	2002.199	30.7	47.51	13	2.7	25	10-46	0.020	28
May '02	2002.251	20.7	47.45	23	6.0	63	31-104	0.024	99
June '02	2002.448	36.8	47.40	30	77	67	39-101	0.089	38
July '02	2002.541	29.7	47.30	16	2.0	20	0-50	0.009	60
Aug. '02	2002.619	27.7	47.24	18	12.7	126	91–168	0.027	97
Sep. '02	2002.698	28.7	47.18	14	7.8	74	48-107	0.035	91
Oct. '02	2002.790	30.8	47.07	16	4.7	42	19–70	0.030	96
Nov. '02	2002.868	27.8	42.54	48	6.0	61	24-106	0.073	59
Dec. '02	2002.947	28.8	49.58	25	4.7	46	19–81	0.044	84
Jan. '03	2003.040	30.8	49.51	15	6.9	59	37–86	0.106	27
Feb. '03	2003.117	27.7	49.44	20	5.9	53	27-86	0.071	50
Mar. '03	2003.199	29.8	49.38	21	8.0	70	44-103	0.093	31
Apr. '03	2003.284	27.7	49.27	22	4.7	54	27-89	0.122	23
May '03	2003.366	29.7	49.21	13	7.1	66	37-102	0.084	40
June '03	2003.448	29.7	49.16	17	10.4	114	77–159	0.077	46
July '03	2003.538	29.8	49.05	21	10.1	106	67–154	0.068	48
Aug. '03	2003.628	32.7	48.94	19	2.9	32	4–67	0.029	96
Sep. '03	2003.713	30.7	48.94	11	0.0	0	0–15	0.065	74
Oct. '03	2003.793	24.5	48.83	20	10.0	104	69–147	0.057	64
Nov. '03	2003.866	26.7	35.64	18	3.6	47	20-85	0.135	23
Nov. '03–1	2003.875	26.2	13.11	10	2.4	84	11–187	0.066	71
Dec. '03	2003.945	27.5	35.61	11	5.9	78	43-123	0.113	27
Dec. '03–1	2003.960	26.9	13.07	10	2.1	77	13-170	0.023	99

TABLE V. (Continued.)

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Exposure date	Median exposure date	Exposure time (days)	Ga mass (tons)	Number of candidate events	Number fit to ⁷¹ Ge	Best fit (SNU)	68% conf. range (SNU)	Nw^2	Probability (%)
Jan. '04	2004.037	30.8	35.54	19	0.0	0	0–24	0.045	89
Jan. '04–1	2004.053	17.3	13.00	7	2.4	132	44-278	0.077	70
Feb. '04	2004.131	34.8	35.43	14	5.1	62	30-102	0.098	41
Feb. '04–1	2004.145	18.3	13.01	10	3.5	151	43-287	0.091	46
Mar. '04	2004.212	28.8	35.37	22	4.6	59	28-101	0.097	39
Mar. '04–1	2004.226	30.8	12.99	8	0.0	0	0-163	0.200	28
Apr. '04	2004.289	23.6	48.29	19	4.3	67	30-116	0.076	41
May '04	2004.354	23.4	22.03	9	3.6	78	25-148	0.076	46
June '04	2004.455	38.5	22.00	10	4.1	98	34-182	0.091	32
July '04	2004.544	24.9	21.95	14	2.0	43	0-118	0.051	74
Aug. '04	2004.623	29.3	21.93	12	5.2	139	80-218	0.048	82
Sep. '04	2004.712	32.9	42.42	14	0.1	0	0–25	0.103	42
Oct. '04	2004.800	28.8	47.67	11	1.9	22	1–50	0.074	56
Nov. '04–1	2004.881	29.7	47.62	17	7.3	73	43-111	0.037	87
Dec. '04	2004.954	25.6	47.57	45	25.3	305	238-381	0.025	96
Jan. '05	2005.047	31.7	47.47	14	1.4	12	0-30	0.083	52
Feb. '05	2005.148	21.0	47.39	11	7.8	89	55-131	0.242	10
Mar. '05	2005.221	27.6	47.34	10	3.6	35	16-60	0.054	70
Apr. '05	2005.283	20.6	47.29	22	7.7	90	52-137	0.053	73
May '05	2005.373	31.6	47.19	18	4.9	43	22-69	0.079	53
June '05	2005.474	20.6	45.99	19	1.7	19	1–47	0.227	13
July '05	2005.545	26.7	45.93	14	1.7	16	3–39	0.048	84
Aug. '05	2005.626	24.0	45.81	19	5.0	52	22-91	0.118	25
Sep. '05	2005.702	26.0	45.74	19	4.2	40	16-72	0.081	47
Oct. '05	2005.781	27.0	45.67	12	7.6	80	49–119	0.170	14
Nov. '05	2005.872	30.8	45.57	22	11.7	101	69–140	0.045	77
Dec. '05	2005.953	27.0	45.50	25	12.3	106	74–143	0.089	32
Jan. '06	2006.046	34.8	45.45	13	3.7	32	15–54	0.102	33
Feb. '06	2006.138	29.7	45.36	30	6.9	71	35-114	0.059	64
Mar. '06	2006.214	24.7	45.27	17	2.2	20	2–46	0.084	47
Apr. '06	2006.281	23.7	45.22	25	13.6	137	98-182	0.045	78
May '06	2006.370	33.7	45.14	16	6.4	59	33-92	0.043	83
June '06	2006.461	32.7	45.08	16	5.8	56	33-86	0.159	16
July '06	2006.546	30.7	45.06	28	7.1	74	34-121	0.108	25
Aug. '06	2006.637	29.7	44.98	21	1.6	18	0–54	0.129	31
Sep. '06	2006.717	28.7	44.94	20	8.7	91	53-137	0.051	71
Oct. '06	2006.796	28.7	44.89	25	5.7	57	31–91	0.067	59
Nov. '06	2006.873	23.7	50.88	30	17.0	152	111–199	0.056	65
Dec. '06	2006.948	27.6	50.83	30	6.2	69	31–114	0.056	70
Jan. '07	2007.043	35.6	50.77	25	10.8	89	57-126	0.082	36
Feb. '07	2007.138	30.6	50.66	25	6.8	63	31–103	0.093	36
Mar. '07	2007.214	26.6	50.60	19	4.5	41	19–70	0.207	7
Apr. '07	2007.279	22.7	50.55	22	2.4	23	3–50	0.133	28
May '07	2007.368	30.7	50.45	19	7.4	70	38-108	0.069	52
July '07	2007.544	28.7	50.34	21	3.7	36	13–66	0.044	83
Aug. '07	2007.637	30.8	50.24	22	0.0	0	0–25	0.068	73
Sep. '07	2007.715	27.5	50.19	24	15.0	130	93-172	0.131	19
Oct. '07	2007.796	29.7	50.13	18	4.7	37	19–62	0.020	99
Nov. '07	2007.886	29.7	50.02	22	7.9	68	42-100	0.068	55
Dec. '07	2007.964	26.7	49.96	20	8.8	70	44-101	0.043	79

TABLE V. (Continued.)

TABLE VI. Approximate cross section for neutrino capture by ⁷¹Ga if the contribution of the first two excited states is set to zero.

v energy	Cros	ss section (10^{-46})	cm ²)	v energy	Cross	section (10^{-46} c)	cm ²)	v energy	Cros	Cross section (10^{-46} cm^2)		
(MeV)	Best	-1σ	$+1\sigma$	(MeV)	Best	-1σ	$+1\sigma$	(MeV)	Best	-1σ	$+1\sigma$	
0.240	1.310×10^{1}	1.280×10^{1}	1.340×10^{1}	1.445	1.944×10^{2}	1.897×10^{2}	2.274×10^2	9.500	4.749×10^{4}	4.053×10^{4}	6.275×10^{4}	
0.250	1.357×10^{1}	1.326×10^{1}	1.388×10^{1}	1.500	2.153×10^{2}	2.104×10^2	2.495×10^2	10.000	5.653×10^4	4.820×10^4	7.474×10^4	
0.275	1.499×10^{1}	1.465×10^{1}	1.533×10^{1}	1.600	2.451×10^2	2.395×10^{2}	2.859×10^2	10.500	6.638×10^4	5.650×10^4	8.785×10^4	
0.300	1.662×10^{1}	1.624×10^{1}	1.700×10^{1}	1.700	2.771×10^2	2.707×10^2	3.252×10^2	11.000	7.703×10^4	6.548×10^4	1.020×10^5	
0.325	1.836×10^{1}	1.794×10^{1}	1.878×10^{1}	1.750	2.939×10^2	2.871×10^2	3.458×10^2	11.500	8.848×10^4	7.510×10^4	1.173×10^{5}	
0.350	2.018×10^{1}	1.972×10^{1}	2.064×10^{1}	2.000	3.932×10^2	3.702×10^2	4.712×10^2	12.000	1.007×10^5	8.535×10^4	1.336×10^{5}	
0.375	2.208×10^{1}	2.157×10^{1}	2.259×10^{1}	2.500	6.428×10^2	5.986×10^2	7.826×10^2	12.500	1.137×10^5	9.621×10^{4}	1.508×10^5	
0.400	2.406×10^{1}	2.351×10^{1}	2.461×10^{1}	3.000	9.806×10^2	9.043×10^{2}	1.211×10^3	13.000	1.274×10^5	1.077×10^5	1.692×10^{5}	
0.425	2.606×10^{1}	2.546×10^{1}	2.799×10^{1}	3.500	1.449×10^{3}	1.323×10^{3}	1.811×10^3	13.500	1.418×10^5	1.197×10^5	1.884×10^5	
0.450	2.810×10^{1}	2.746×10^{1}	3.029×10^{1}	4.000	2.108×10^{3}	1.905×10^{3}	2.662×10^{3}	14.000	1.569×10^{5}	1.322×10^{5}	2.086×10^5	
0.500	3.231×10^{1}	3.157×10^{1}	3.516×10^{1}	4.500	3.043×10^{3}	2.722×10^{3}	3.880×10^{3}	14.500	1.728×10^5	1.454×10^{5}	2.298×10^{5}	
0.600	4.109×10^{1}	4.015×10^{1}	4.585×10^{1}	5.000	4.336×10^{3}	3.842×10^{3}	5.573×10^{3}	15.000	1.893×10^{5}	1.590×10^{5}	2.520×10^{5}	
0.700	5.027×10^{1}	4.912×10^{1}	5.776×10^{1}	5.500	6.072×10^{3}	5.337×10^{3}	7.853×10^{3}	15.500	2.064×10^{5}	1.731×10^{5}	2.749×10^{5}	
0.800	6.478×10^{1}	6.329×10^{1}	6.924×10^{1}	6.000	8.350×10^{3}	7.290×10^{3}	1.086×10^4	16.000	2.241×10^{5}	1.875×10^{5}	2.988×10^{5}	
0.900	7.829×10^{1}	7.649×10^{1}	8.470×10^{1}	6.500	1.133×10^{4}	9.832×10^{3}	1.479×10^{4}	18.000	3.010×10^{5}	2.495×10^{5}	4.025×10^{5}	
1.000	9.299×10^{1}	9.085×10^{1}	1.017×10^{2}	7.000	1.515×10^{4}	1.309×10^{4}	1.985×10^4	20.000	3.860×10^{5}	3.162×10^{5}	5.184×10^{5}	
1.100	1.168×10^{2}	1.142×10^{2}	1.306×10^{2}	7.500	1.989×10^{4}	1.712×10^{4}	2.613×10^4	22.500	5.013×10^{5}	4.028×10^{5}	6.778×10^{5}	
1.200	1.372×10^{2}	1.341×10^{2}	1.549×10^{2}	8.000	2.550×10^{4}	2.188×10^{4}	3.357×10^4	25.000	6.233×10^{5}	4.883×10^{5}	8.501×10^{5}	
1.300	1.593×10^{2}	1.557×10^{2}	1.814×10^{2}	8.500	3.198×10^4	2.737×10^{4}	4.216×10^4	30.000	8.701×10^{5}	6.354×10^{5}	1.217×10^{6}	
1.400	1.831×10^2	1.789×10^2	2.100×10^2	9.000	3.928×10^4	3.357×10^4	5.186×10^4					

APPENDIX C: MODIFIED CROSS SECTION FOR NEUTRINO CAPTURE

The cross section for neutrino capture by ⁷¹Ga was calculated by Bahcall [19] and is given in his Table II (best estimate), Table III (3σ lower limit), and Table IV (3σ upper limit). Based on information on the contributions of the excited states given in the text of Bahcall's article, if we assume the matrix element for neutrino capture to the first two excited states of ⁷¹Ge to be zero, but that the matrix elements of the other excited states are unchanged, we can approximate the best-estimate cross section in various energy regions as

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follows:

 $(13.10 + 91.29(E_v - 0.24)^{1.157})$

		for	$0.24 < E_{\nu} < 0.733$
~	$0.946 \sigma_{\text{best}}$	for	$0.733 < E_{\nu} < 1.033$
y = q	$0.953 \sigma_{\text{best}}$	for	$1.033 < E_{\nu} < 1.483$
	$0.96 \sigma_{\text{best}}$	for	$1.483 < E_{\nu} < 1.983$
	$0.99 \sigma_{\text{best}}$	for	$1.983 < E_{\nu} < 30.0$

where σ_{best} is the value in Table II of Ref. [19], σ is in 10^{-46} cm², and the neutrino energy E_{ν} is in MeV. The results are given in our Table VI. The $\pm 1\sigma$ limits in Table VI were obtained in a similar manner from the 3σ limits given in Ref. [19].

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