Bound state problem of S-wave heavy quark meson-antimeson systems

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We investigated systematically whether the S-wave $(\bar{Q}q)$ meson and the $(Q\bar{q})$ meson may form S-wave bound states in a chiral SU(3) quark model by solving the resonating group method equation. Here Q = c or b and q = u, d, or s. Our preliminary calculation disfavors the existence of $I = \frac{1}{2}(\bar{Q}l)-(Q\bar{s})$ molecules (l = u, d) but favors the existence of isoscalar $B\bar{B}$, $B^*\bar{B}^*(J = 2)$, and $B\bar{B}^*(C = +)$ molecules. The existence of isovector (charm-anticharm) and (charm-bottom) molecules is also disfavored. Therefore the resonance-like structure $Z^+(4051)$ is unlikely to be an S-wave $D^*\bar{D}^*$ molecule.

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I. INTRODUCTION

The molecular picture was widely used in discussing the strange states, such as $f_0(980)$, $a_0(980)$ [1–6], and $\Lambda(1405)$ [7–9]. Although it is still difficult to identify an exotic state as a hadronic molecule, the exploration for possible molecules in more systems is an interesting topic. Such a study may help us to understand the strong interactions. There have been dynamical studies of whether the possible molecules exist in the light quark systems. In Refs. [10–13] various meson-baryon systems are investigated, and in Refs. [14–20] the $\overline{\Omega}N$ and $N\overline{N}$ systems are studied.

For heavy quark systems, the formation of molecules is easier due to the relatively small kinetic term. The relevant study can be traced back to 30 years ago [21,22]. Ten years later, Törnqvist studied possible deuteron-like meson-meson states bound by pions in Refs. [23] and [24] that were called deusons [25]. In Ref. [26], Ericson and Karl investigated the critical mass for molecule formation. In recent years, the renaissance of hadron spectroscopy, especially the observation of exotic heavy quark mesons [27–30], has triggered extensive discussions involving the molecular picture.

The charmed meson $D_{sJ}(2317)$ [31,32], whose mass is much smaller than the quark model prediction, was once proposed as a *DK* molecule [33]. Similarly, $D_{sJ}(2460)$ [32] was suggested as a D^*K state. However, their $c\bar{s}$ nature is strongly favored after considering the significant contributions from the *DK* continuum [34].

The discovery of X(3872) [35–38] ignited physicists' great interest. It is almost on the threshold of $D^0\bar{D}^{*0}$ and very close to the thresholds of $\rho J/\psi$, $\omega J/\psi$, and D^+D^{*-} . The most popular interpretation for this intriguing state is a hadronic molecule dominated by $D^0\bar{D}^{*0}$ [39–43]. However, this picture is questioned in Ref. [44]. Very recently, the BaBar Collaboration measured a relatively large branching fraction for $X(3872) \rightarrow \psi(2S)\gamma$, which indicates that X(3872) is possibly a mixing state of $c\bar{c}$ and $D^0\bar{D}^{*0}$ [45].

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For the interpretations of the exotic Y(4260) [46–48] in the molecular picture, Liu *et al.* [49] suggested it is a $\chi_{c1}\rho$ state, while Yuan *et al.* [50] proposed it is a $\chi_{c1}\omega$ state. There are also other molecular proposals such as a Λ_c pair [51] and a $D_0\bar{D}^*$ or $D_1\bar{D}$ bound state [52,53]. In fact, the most popular opinion is that Y(4260) is a hybrid state [54–56], although this interpretation is also inconclusive [57,58]. We still require detailed investigations to answer whether these interpretations are correct.

Recently, the Belle Collaboration observed a charged charmonium-like state $Z^+(4430)$ in the $\pi^+\psi'$ invariant mass distribution [59]. This state is an excellent candidate of heavy quark molecules. The dynamical calculation also indicates $Z^+(4430)$ may be interpreted as a D_1D^* (D'_1D^*) molecule [60–64]. Not long ago, the Belle Collaboration [65] announced two more charged charmonium-like resonances $Z^+(4051)$ and $Z^+(4248)$ in the $\pi^+\chi_{c1}$ mass distribution, which gives us the hope that heavy quark molecules do exist. Unfortunately, the BaBar data do not support the existence of $Z^+(4430)$ [66]. Cross-checks for the other two charged resonances are also desired.

Therefore, none of the heavy quark molecules has been established yet. With the development of experimental measurements, more and more exotic states in the heavy quark region will be found. It is worthwhile to study in which systems heavy molecules can exist. Motivated by the observation of new exotic states and the possibility of forming heavy quark molecules, Wong [43] explored the combinations of heavy mesons and heavy antimesons in a quark-based model and found many molecular states. Voloshin and Dubynskiy [67,68] suggested the possible resonance at the $D^*\bar{D}^*$ threshold. Zhang *et al.* [69] studied possible *S*-wave bound states of two pseudoscalar mesons using the vector meson exchange potential. In Ref. [70], a $D_s D^*$ molecule was proposed.

In a previous work [71], we studied the *S*-wave $D\bar{D}/B\bar{B}$, $D^*\bar{D}^*/B^*\bar{B}^*$, and $D^*\bar{D}/B^*\bar{B}$ systems in a meson exchange model at hadron level, where we considered scalar, pseudoscalar, and vector meson exchanges. In this article, we explore similar systems in a chiral SU(3) quark model (χ QM) [72] and calculate the binding energies by solving the resonating group method (RGM) equation [73]. All the mesons

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below 1.1 GeV are considered. The study can be used to test different model approaches.

The chiral quark model is a useful tool in connecting the QCD theory and the experimental observables. It has been proven successful in studying the baryon-baryon interactions and the meson-baryon interactions. For the mechanism of the short-range quark-quark interaction, it is still controversial whether one-gluon exchange (OGE) or vector meson exchange dominates. Dai *et al.* [74] extended the chiral SU(3) quark model to include the vector meson exchange part and named the model the extended chiral SU(3) quark model ($E_{\chi}QM$), which was also successful in reproducing the energies of the baryon states, the binding energy of the deuteron, and the *NN* scattering phase shifts.

It is interesting to study whether this phenomenological approach is applicable to the heavy quark systems. We have applied this model to the $D^0 \bar{D}^{*0}$ system in Ref. [75] and we will continue to perform similar studies to other systems. One may test this model by comparing the predictions with future measurements.

The article is organized as follows. After the Introduction, we present a brief discussion about the systems we study in Sec. II. In Sec. III, we present the ingredients of the model. Then in Sec. IV, we give the essential parameters for the calculation. We show numerical results for different systems in Sec. V. The last section is the discussion and summary.

II. HEAVY QUARK MESON-ANTIMESON SYSTEMS

The S-wave single heavy quark mesons are pseudoscalar type (D, D_s, B, B_s) and vector type (D^*, D^*_s, B^*, B^*_s) . For simplicity, P(V) represents the heavy quark pseudoscalar (vector) meson. We investigate whether the hadronic molecules can be found in the combinations of these mesons and their antiparticles. From the flavor SU(3) symmetry, the multiplets are $3 \times \overline{3} = 8 + 1$. One may consult Ref. [71] for the explicit flavor wave functions. The largely broken SU(3) symmetry must be taken into account for possible hadronic molecules. In the numerical evaluation, we first consider the isospin symmetric case. Because isospin symmetry breaking (ISB) is probably important, we also discuss the case of large isospin breaking.

In the isospin symmetric case, we need consider only four possibilities: (1) $I = 1/2(\bar{Q}u)-(Q\bar{s})$, (2) $I = 1(\bar{Q}u)-(Q\bar{d})$, (3) $I = 0(\bar{Q}s)-(Q\bar{s})$, and (4) $I = 0(\bar{Q}l)-(Q\bar{l})$, where Q is a charm or bottom quark and l represents an up or down quark. We call them I = 1/2, I = 1, I = 0(s) and I = 0(l) states, respectively, in the following parts.

When studying the possible heavy molecule composed of a heavy meson and an antimeson, we take a simple picture where only color-singlet mesons are involved. The OGE and the confinement interactions occur inside the mesons, while the meson exchange interaction occurs between light quarks of different mesons. To make the description accurate, we label the heavy quarks 1 and 3 and the light quarks 2 and 4. The quarks 1 and 2 are bound to the meson and the quarks 3 and 4 to the antimeson. We do not consider the flavor-singlet meson exchange between heavy quarks or between a heavy quark and a light quark in the present investigation. The consideration is as follows. In the chiral quark model, the constituent mass of the light quark is related to the spontaneous vacuum breaking while the breaking gives small effects to the masses of the heavy quarks, which indicates the coupling of the σ meson and the heavy quarks is weak.

III. HAMILTONIAN

The details of the chiral SU(3) quark model can be found in Refs. [72] and [74]. Here we just present essential constituents for the calculation. The Hamiltonian for the meson-antimeson system has the form

$$H = \sum_{i=1}^{4} T_i - T_G + V^{\text{OGE}} + V^{\text{conf}} + \sum_M V^M, \quad (1)$$

where T_i is the kinetic term of the *i*th quark or antiquark and T_G is the kinetic energy operator of the center of mass motion. *M* is the exchanged meson between light quarks.

The potential of the OGE part reads

$$V_{\bar{q}Q}^{OGE} = g_q g_Q \mathbf{F}_{\bar{q}}^c \cdot \mathbf{F}_Q^c \left\{ \frac{1}{r} - \frac{\pi}{2} \delta^3(\mathbf{r}) \times \left[\frac{1}{m_q^2} + \frac{1}{m_Q^2} + \frac{4}{3} \frac{1}{m_q m_Q} (\boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_Q) \right] \right\}, \quad (2)$$

where $\mathbf{F}_{Q}^{c} = \frac{\lambda}{2}$ for quarks and $\mathbf{F}_{\bar{q}}^{c} = -\frac{\lambda^{*}}{2}$ for antiquarks and m_{q} (m_{Q}) is the light (heavy) quark mass. The linear confinement potential is

$$V_{\bar{q}Q}^{\text{conf}} = -4\mathbf{F}_{\bar{q}}^c \cdot \mathbf{F}_Q^c \left(a_{qQ}^c r + a_{qQ}^{c0} \right).$$

There are similar expressions for $V_{q\bar{Q}}^{\text{OGE}}$ and $V_{q\bar{Q}}^{\text{conf}}$.

For a molecule formed with $(Q\bar{q})$ and $(\bar{Q}q)$ mesons, the light meson exchange occurs only between \bar{q} and q. From Refs. [72] and [74], one gets

$$V^{\sigma_a}(\mathbf{r}_{ij}) = -C(g_{ch}, m_{\sigma_a}, \Lambda) X_1(m_{\sigma_a}, \Lambda, r_{ij}) [\lambda_a(i)\lambda_a(j)]$$

$$(a = 0, 1, 2, \dots, 8),$$

$$m^2$$
(3)

$$V^{\pi_a}(\mathbf{r}_{ij}) = C(g_{ch}, m_{\pi_a}, \Lambda) \frac{m_{\pi_a}}{12m_2m_4} X_2(m_{\pi_a}, \Lambda, r_{ij}) \\ \times [\boldsymbol{\sigma}(i) \cdot \boldsymbol{\sigma}(j)] [\lambda_a(i)\lambda_a(j)],$$
(4)

$$V^{\rho_a}(\mathbf{r}_{ij}) = C(g_{\text{chv}}, m_{\rho_a}, \Lambda) \left\{ X_1(m_{\rho_a}, \Lambda, r_{ij}) + \frac{m_{\rho_a}^2}{6m_2m_4} \right. \\ \left. \times \left[1 + \frac{f_{\text{chv}}}{g_{\text{chv}}} \frac{m_2 + m_4}{M_N} + \left(\frac{f_{\text{chv}}}{g_{\text{chv}}}\right)^2 \frac{m_2m_4}{M_N^2} \right] \right. \\ \left. \times X_2(m_{\rho_a}, \Lambda, r_{ij})[\boldsymbol{\sigma}(i) \cdot \boldsymbol{\sigma}(j)] \right\} [\lambda_a(i)\lambda_a(j)],$$

$$(5)$$

$$V_{q\bar{q}}^M = G_M V_{qq}^M. ag{6}$$

Where G_M is the G parity of the exchanged meson and

$$C(g_{\rm ch}, m, \Lambda) = \frac{g_{\rm ch}^2}{4\pi} \frac{\Lambda^2 m}{\Lambda^2 - m^2},\tag{7}$$

$$X_1(m,\Lambda,r) = Y(mr) - \frac{\Lambda}{m}Y(\Lambda r), \qquad (8)$$

$$X_2(m,\Lambda,r) = Y(mr) - \left(\frac{\Lambda}{m}\right)^3 Y(\Lambda r), \qquad (9)$$

$$Y(x) = \frac{e^{-x}}{x}.$$
(10)

Here we do not present the tensor term and the spin-orbital term in the potentials because we consider only *S*-wave interactions. We use the same cutoff Λ for various mesons. Its value is around the scale of chiral symmetry breaking (~1 GeV).

By solving the RGM equation, one gets the energy of the system and the relative motion wave function. From the definition of the binding energy, $E_0 = M_{\bar{Q}q} + M_{Q\bar{q}} - M_{\text{system}}$, one judges whether a system would be bound.

IV. THE PARAMETERS

There are several parameters in the Hamiltonian and the wave functions: the OGE coupling constants g_q and g_Q ; the confinement strengths a_{qQ}^c ; the zero-point energies a_{qQ}^{c0} ; the quark masses m_Q and m_q ; the harmonic-oscillator width parameter b_u ; the quark-meson coupling constants g_{ch} , g_{chv} and f_{chv} ; the cutoff Λ ; and the mixing angle for the I = 0 mesons. The mass of the phenomenological σ meson is also treated as an adjustable parameter. For other meson masses, we use the experimental values.

The σ meson does not have a definite mass. In the light quark systems, this mass parameter was adjusted to fit the mass of the baryons, the binding energy of the deuteron, and the *NN* phase shifts. When extending the application of this model to the heavy quark systems, we use the values determined in the light quark systems. If the vector meson exchanges are not included, the mass is 595 MeV, whereas $m_{\sigma} = 535$ and 547 MeV was used in the E χ QM.

For the up and strange quark masses, we use the values given in the previous work [14,72,74], $m_u = 313$ MeV and $m_s =$ 470 MeV. To investigate the heavy quark mass dependence, we take several typical values $m_c = 1430$ MeV [76], $m_c =$ 1870 MeV [77], $m_b = 4720$ MeV, which is close to the value in Ref. [78], and $m_b = 5259$ MeV [77].

The chiral coupling constant g_{ch} is related to $g_{NN\pi}$ through

$$\frac{g_{\rm ch}^2}{4\pi} = \frac{9}{25} \frac{g_{NN\pi}^2}{4\pi} \frac{m_u^2}{m_N^2},\tag{11}$$

with $g_{NN\pi}^2/(4\pi) = 13.67$ determined experimentally. From this relation, one gets $g_{ch} = 2.621$. In the extended SU(3) chiral quark model, one also needs the vector coupling constants. We adopt two sets of the values used in the previous work, $(g_{chv}, f_{chv}) = (2.351, 0.0)$ and (1.972, 1.315) [74]. The corresponding σ mass is also presented in Table I. One notes each set of parameters can reproduce the masses of the ground state baryons, the binding energy of the deuteron, and the *NN* and *YN* scattering observables.

The values of g_q , g_Q , a_{qQ}^c , and a_{qQ}^{c0} can be derived from the masses of the ground state baryons and the heavy mesons. The

TABLE I. Three sets of model parameters. Other meson masses are $m_{\sigma'} = 984.7$ MeV, $m_{\epsilon} = 980$ MeV, $m_{\pi} = 138$ MeV, $m_{\eta} = 547.8$ MeV, $m_{\eta'} = 957.8$ MeV, $m_{\rho} = 775.8$ MeV, $m_{\omega} = 782.6$ MeV, and $m_{\phi} = 1020$ MeV.

	χQM	EχQ	М
	Set 1	Set 2	Set 3
$\overline{b_u}$ (fm)	0.5	0.45	0.45
m_{μ} (MeV)	313	313	313
m_s (MeV)	470	470	470
m_{σ} (MeV)	595	535	547
g _{chv}		2.351	1.972
$f_{\rm chv}/g_{\rm chv}$		0	2/3

binding energy for a system of two color-singlet mesons is irrelevant to the internal potentials of the color-singlet meson because of the cancellation [75]. Therefore these four values will not give effects to the final results of E_0 . We do not present them here.

Isoscalar states with the same J^{PC} will mix. The mixing angle for pseudoscalar mesons η_1 and η_8 , θ^{PS} , is taken to be -23° . Because the mixing angle θ^S for scalar mesons is still unclear and controversial, we use three values in the numerical evaluation: 0.0, 35.264°, and -18° . The second number corresponds to the ideal mixing while the last one is taken from Ref. [79]. We use the ideal mixing angle $\theta^V =$ 35.264° for the vector mesons. In the scalar and pseudoscalar meson exchange potentials, we have adopted $\lambda_0 = \mathbb{I}$, where \mathbb{I} is the unit matrix. To investigate its effects, we also use

 $\lambda_0 = \sqrt{\frac{2}{3}}\mathbb{I}$ to calculate the binding energies.

To consider the dependence of the binding energy on the cutoff, we use two values $\Lambda = 1100$ MeV and $\Lambda = 1500$ MeV.

V. NUMERICAL RESULTS

When performing the numerical evaluations, we calculate the binding energies with all possible combinations of the parameters presented in the previous section. Only when all the results for a system indicate it is unbound, do we conclude the system is unbound. On the contrary, we say a molecule is possible only when all the results indicate the system is bound.

A. $P\bar{P}$ systems

The quantum numbers for the neutral states are $J^{PC} = 0^{++}$. The pseudoscalar mesons do not exchange in such systems because the coupling of three pseudoscalar mesons is forbidden.

For I = 1/2 states, we investigate $\overline{D}{}^{0}D_{s}^{+}$, $B^{+}\overline{B}_{s}^{0}$, and $B^{+}D_{s}^{+}$. Such systems are possibly bound by only scalar mesons σ and ϵ . After solving the RGM equation, we find these systems are unbound with various parameters presented in the previous section.

For I = 1 systems, we calculate the binding energies of $\overline{D}^0 D^+$, $B^+ \overline{B}^0$, and $B^+ D^+$. Vector mesons ρ and ω are

TABLE II. The binding energies for $P\bar{P}$ states. An \times means the system is unbound.

Isospin	System	χQM	EχQM
I = 1	$ar{D}^0 D^+$	×	×
	$B^+ar{B}^0$	×	≤1.9
	B^+D^+	×	×
I = 0(s)	$D_s^- D_s^+$	×	≤10.4
	$B^0_s \bar{B}^0_s$	≤13.3	$2.5 \sim 43.7$
	$B_s^{ m 0}D_s^+$	≼3.0	≤22.6
I = 0(l)	$D^-D^+ + ar{D}^0D^0$	≼4.9	$13.7 \sim 52.9$
	$B^0ar{B}^0+B^+B^-$	$10.1\sim26.8$	$47.2 \sim 102.3$
	$B^0D^+ + B^+D^0$	$0.4 \sim 12.8$	$23.1 \sim 72.8$
ISB	$ar{D}^0 D^0$	×	≤12.8
	B^+B^-	$0.1 \sim 10.3$	$11.7 \sim 43.3$
	B^+D^0	≤2.0	$0.5\sim24.2$

permitted, but ρ exchange interaction is repulsive while ω is attractive. Their contributions are almost canceled. σ and ϵ provide attractive force while σ' gives repulsive interaction. By exploring different cases of parameters, we get the binding energies for these systems. The final results are given in Table II. If all the numerical values indicate the system is unbound, we mark it with an "×". If the system is unbound with some parameters and bound with other parameters, we give the upper limit of the binding energy. If all the results indicate the system is bound, we collect the binding energies in a range. From the table, one knows there are no bound states in \bar{D}^0D^+ and B^+D^+ .

The hidden strange I = 0(s) states we investigate include $D_s^- D_s^+$, $B_s^0 \bar{B}_s^0$, and $B_s^0 D_s^+$. They may be bound mainly by the attractive σ , ϵ , and ϕ . According to our model calculation, it is difficult to draw a definite conclusion on whether the bound states may form (see Table II).

The I = 0(l) systems we study are $\frac{1}{\sqrt{2}}(D^-D^+ + \bar{D}^0D^0)$, $\frac{1}{\sqrt{2}}(B^0\bar{B}^0 + B^+B^-)$, and $\frac{1}{\sqrt{2}}(B^0D^+ + B^+D^0)$. In comparison with the I = 1 systems, σ' and ρ exchange interactions are both attractive now. The amplitudes of the potentials are also larger. From the results in Table II, we find the bound states containing bottom quarks exist, even if only scalar mesons can exchange.

In the real world, the isospin symmetry is also broken. The mass difference between D^0 and D^{\pm} is around 5 MeV and it will affect the conclusion of whether hadronic molecules exist. In this study, we also calculate preliminarily the extreme cases $\bar{D}^0 D^0$, $B^+ B^-$, and $B^+ D^0$. Such cases get the minimum contributions from σ' and ρ . Our results indicate the hidden bottom molecule $B\bar{B}$ is still possible. Table II shows relevant results.

B. $V\bar{V}$ systems

The quantum numbers are $J^{PC} = 0^{++}$, 1^{+-} , or 2^{++} for the neutral states. The pseudoscalar mesons, scalar mesons, and

vector mesons can all be exchanged in such systems. In our model, the amplitudes for scalar meson exchange interactions are the same as those for the $P\bar{P}$ case.

Similar to the former case, we first investigate the $\bar{D}^{*0}D_s^{*+}$, $B^{*+}\bar{B}_s^{*0}$, and $B^{*+}D_s^{*+}$ systems with I = 1/2. Here the vector meson exchanges are forbidden. The contributions from η and η' cancel largely and the pseudoscalar mesons give finally small contributions. The σ and ϵ do not have enough attractive force to bind the heavy mesons and these systems are unbound for the angular momentum J = 0, 1, and 2.

We explore three I = 1 systems, $\bar{D}^{*0}D^{*+}$, $B^{*+}\bar{B}^{*0}$, and $B^{*+}D^{*+}$. Comparing with I = 1 $P\bar{P}$ case, the exchanges of pseudoscalar mesons π , η , and η' are permitted. The contributions from η and η' reduce that from π . For J = 0 and J = 1, the interaction due to pseudoscalar mesons is attractive and for J = 2, it is repulsive. From the resulting binding energies, we conclude that $\bar{D}^{*0}D^{*+}$ and $B^{*+}D^{*+}$ are not bound while $B^{*+}\bar{B}^{*0}$ is not excluded. We present our results in Table III.

The hidden strange states (I = 0) include $D_s^{*-}D_s^{*+}$, $B_s^{*0}\bar{B}_s^{*0}$, and $B_s^{*0}D_s^{*+}$. The contributions from η and η' exchange interactions have the same sign. For J = 0 and J = 1, they are repulsive. For J = 2, they are attractive. Our numerical results are also presented in Table III. $D_s^{*-}D_s^{*+}$ is not bound in χ QM.

For I = 0(l) systems, we calculate the binding energies of $\frac{1}{\sqrt{2}}(D^{*-}D^{*+} + \overline{D}^{*0}D^{*0})$, $\frac{1}{\sqrt{2}}(B^{*0}\overline{B}^{*0} + B^{*+}B^{*-})$, and $\frac{1}{\sqrt{2}}(B^{*0}D^{*+} + B^{*+}D^{*0})$. The π , η , and η' exchange interactions have like signs. For J = 0 and J = 1, they are repulsive, whereas they are attractive for J = 2. We find there are no binding solutions for these systems in χ QM if J = 0, whereas the formation of molecules is possible if J = 2. Table III shows our results.

Similar to the $P\bar{P}$ isospin breaking case, we study whether $\bar{D}^{*0}D^{*0}$, $B^{*+}B^{*-}$, and $B^{*+}D^{*0}$ may be bound. According to our calculation, bound states in χ QM do not exist if J = 0 and the hidden bottom molecule is still possible if J = 2. We also present the results for this extreme case in Table III.

C. $P\bar{V} \pm V\bar{P}$ systems

The components $P\bar{V}$ and $V\bar{P}$ do not have definite *C* parity, whereas the neutral $P\bar{V} \pm V\bar{P}$ states do. For a state with given *C* parity, two conventions for the relative sign have been used in the literature. The plus sign for the $C = +D\bar{D}^*$ system corresponding to the X(3872) was widely used, whereas the minus sign was adopted in Refs. [80] and [81]. Recently, Stancu [82] analyzed the charge conjugation in multiquark systems in detail and she also obtained a minus sign. In fact, the convention of the relative sign depends on the phase between *P* and \bar{P} as well as *V* and \bar{V} under the charge conjugation transformation. But the final result is irrelevant with the convention. For example, for the $C = +D^0\bar{D}^{*0}$ state, one gets $X = \frac{1}{\sqrt{2}}(D^0\bar{D}^{*0} - D^{*0}\bar{D}^0)$ with the convention $D^0(D^{*0}) = c\bar{u}$ and $\bar{D}^0(\bar{D}^{*0}) = u\bar{c}$. The resulting matrix element $\langle X | \sigma_2 \cdot \sigma_4 | X \rangle$ is +1. If the conventions $D^0(D^{*0}) = c\bar{u}$ and the same element $\langle X | \sigma_2 \cdot \sigma_4 | X \rangle = +1$. In the following calculation, we

Isospin	System	χQM				EχQM		
		J = 0	J = 1	J = 2	J = 0	J = 1	J = 2	
I = 1	$ar{D}^{*0}D^{*+}$	х	×	×	×	×	×	
	$B^{*+}ar{B}^{*0}$	≤3.5	≤1.2	×	≼7.9	≼4.7	×	
	$B^{*+}D^{*+}$	×	×	×	×	×	×	
I = 0(s)	$D_{s}^{*-}D_{s}^{*+}$	×	×	×	≤10.2	≤10.3	≤10.4	
	$B_{s}^{*0}\bar{B}_{s}^{*0}$	≤10.4	≤11.8	≤15.0	$3.3 \sim 43.7$	$2.9\sim43.7$	$2.1 \sim 43.7$	
	$B_{s}^{*0}D_{s}^{*+}$	≤1.8	≤2.3	≤3.8	≤22.5	≤22.6	≤22.6	
I = 0(l)	$D^{*-}D^{*+} + ar{D}^{*0}D^{*0}$	×	×	$3.6 \sim 22.4$	≤25.6	$3.5 \sim 38.7$	$17.7 \sim 67.9$	
	${B^{*0}}{ar B^{*0}} + \ {B^{*+}}{B^{*-}}$	×	≼5.8	$34.5\sim 59.2$	$22.6 \sim 66.4$	$34.7 \sim 84.2$	$60.2 \sim 120.8$	
	$B^{*0}D^{*+}+\ B^{*+}D^{*0}$	×	≼0.4	13.9 ~ 36.7	$5.6 \sim 41.4$	13.6 ~ 56.7	33.4 ~ 89.5	
ISB	$ar{D}^{*0}D^{*0}$	×	×	≤1.1	≼6.7	≤9.6	≤16.2	
	$B^{*+}B^{*-}$	×	≤3.5	$5.3 \sim 20.2$	$5.6\sim 32.7$	$8.6\sim 37.9$	$14.9 \sim 48.7$	
	$B^{*+}D^{*0}$	×	×	≼7.5	≤16.0	≤20.0	$2.3\sim 28.6$	

TABLE III. The binding energies for $V\bar{V}$ states. An \times means the system is unbound.

adopt the latter convention which is consistent with the PDG assignment. So the quantum numbers for the neutral states are $J^{\text{PC}} = 1^{\pm}$ corresponding to $P\bar{V} \pm V\bar{P}$.

use $P\bar{V} + cV\bar{P}$ to denote these wave functions where c is equivalent to the C parity of the neutral state.

From the flavor SU(3) symmetry, it is easy to get the wave functions of other systems in the same multiplet. One may

In this pseudoscalar vector case, the numerical results may be found in the $P\bar{P}$ systems or the $V\bar{V}$ systems. We explain this fact with I = 1/2 states.

 $P\bar{V} + cV\bar{P}$ c = -1Isospin c = +1 $\bar{D^0}D^{*+}_{c} + c\bar{D}^{*0}D^+_{c}$ $\bar{D}^{*0}D_{s}^{*+}$ (J = 2) $\bar{D}^{*0}D_{s}^{*+}$ (J = 1) $I = \frac{1}{2}$ $B^+ \bar{B}_s^{*0} + c B^{*+} \bar{B}_s^0$ $B^{*+}\bar{B}^{*0}_{s}$ (J = 2) $B^{*+}\bar{B}^{*0}_{s}$ (*J* = 1) $\bar{D^0}D^{*+} + c\bar{D}^{*0}D^+$ $\bar{D}^{*0}D^{*+}$ (J = 2) $\bar{D}^{*0}D^{*+}$ (J = 1) I = 1 $B^{*+}\bar{B}^{*0} (J=1)$ $B^+ \bar{B}^{*0} + c B^{*+} \bar{B}^0$ $B^{*+}\bar{B}^{*0}$ (J=2) $D_s^- D_s^{*+} + c D_s^{*-} D_s^+$ $D_s^{*-} D_s^{*+} (J = 2)$ $D_s^{*-} D_s^{*+} (J = 1)$ I = 0 $B_{s}^{0}\bar{B}_{s}^{*0} + cB_{s}^{*0}\bar{B}_{s}^{0}$ $B_{s}^{*0}\bar{B}_{s}^{*0}$ (J = 2) $B_{s}^{*0}\bar{B}_{s}^{*0}$ (J = 1) $(\bar{D}^0 D^{*0} + c\bar{D}^{*0} D^0) + (D^- D^{*+} + cD^{*-} D^+)$ $\bar{D}^{*0}D^{*0} + D^{*-}D^{*+}$ (J = 2) $\bar{D}^{*0}D^{*0} + D^{*-}D^{*+}$ (J = 1) $(B^+B^{*-} + cB^{*+}B^-) + (B^0\bar{B}^{*0} + cB^{*0}\bar{B}^0)$ $B^{*0}\bar{B}^{*0} + B^{*+}B^{*-}$ (J = 2) $B^{*0}\bar{B}^{*0} + B^{*+}B^{*-}$ (J = 1) $B^{*+}D_s^+/B^+D_s^{*+}$ $B^+ D_{\epsilon}^+$ $I = \frac{1}{2}$ B^+D^+ $B^{*+}D^+/B^+D^{*+}$ I = 1I = 0 $B_{s}^{*0}D_{s}^{+}/B_{s}^{0}D_{s}^{*+}$ $B_{s}^{0}D_{s}^{+}$ $(B^{*0}D^+ + B^{*+}D^0)/(B^0D^{*+} + B^+D^{*0})$ $B^0 D^+ + B^+ D^0$ $\bar{D}^{*0}D_{*}^{+}$ $\bar{D}^0 D_s^+ (I = \frac{1}{2})$ ISB $B^{*+}\bar{B}^{0}_{-}$ $B^+ \bar{B}^0_{\rm s} (I = \frac{1}{2})$ $\bar{D}^{*0}D^+$ $\bar{D}^0 D^+ (I=1)$ $B^{*+}\bar{B}^0$ $B^{+}\bar{B}^{0} (I = 1)$ $B^{*+}D^0/B^+D^{*0}$ B^+D^0 (ISB) $\bar{D}^0 D^{*0} + c \bar{D}^{*0} D^0$ $\bar{D}^{*0}D^{*0}$ (ISB) (J = 2) $\bar{D}^{*0}D^{*0}$ (ISB) (J = 1) $B^+B^{*-} + cB^{*+}B^ B^{*+}B^{*-}$ (ISB) (J = 2) $B^{*+}B^{*-}$ (ISB) (J = 1)

TABLE IV. The correspondence for the numerical results between $P\bar{V} + cV\bar{P}$ and $V\bar{V}$ or $P\bar{P}$. Here c means the C parity of the neutral state of the multiplet.

TABLE V. Summary of possible bound states in $P \bar{P}$ systems.

Isospin	(\bar{c}, c)	(\bar{b},b)	(\bar{b}, c)
$I = \frac{1}{2}$	×	×	×
$I = \tilde{1}$	Х	*	×
I = 0(s)	*	?	?
I = 0(l)	?	\checkmark	\checkmark
ISB	*	\checkmark	?

We investigate $\frac{1}{\sqrt{2}}(\bar{D^0}D_s^{*+}\pm\bar{D}^{*0}D_s^+)$ and $\frac{1}{\sqrt{2}}(B^+\bar{B}_s^{*0}\pm B^{*+}\bar{B}_s^0)$. By comparing the binding energies with the I = 1/2 $V\bar{V}$ case, one finds the results for the c = +1 (c = -1) states are the same as those for J = 2 (J = 1) $\bar{D}^{*0}D_s^{*+}$ or $B^{*+}\bar{B}_s^{*0}$. Therefore these systems are also unbound.

Therefore these systems are also unbound. It is unnecessary to consider $\frac{1}{\sqrt{2}}(B^+D_s^{*+}\pm B^{*+}D_s^+)$ because the mass difference between $B^+D_s^{*+}$ and $B^{*+}D_s^+$ is around 100 MeV and their mixing should be very small. For the system $B^+D_s^{*+}$ or $B^{*+}D_s^+$, the results are the same as those of $B^+D_s^+$ of the $P\bar{P}$ case.

Similarly, for the I = 1 case, the results for the $c = +1(c = -1)\bar{D}^0 D^{*+}$ and $B^+ \bar{B}^{*0}$ are the same as $J = 2(J = 1)\bar{D}^{*0}D^+$ and $B^{*+}\bar{B}^0$, respectively. The results for $B^+ D^{*+}$ or $B^{*+}D^+$ are the same as those for the B^+D^+ case. One can also get the results for I = 0 cases and large ISB cases from J = 2(J = 1) $V\bar{V}$ or $P\bar{P}$. The correspondence for the numerical results between $P\bar{V} \pm V\bar{P}$ and $V\bar{V}$ or $P\bar{P}$ may be found in Table IV.

Such a feature is not difficult to understand. The difference between the $V\bar{V}$ case and the $P\bar{V} \pm V\bar{P}$ case comes from the spin-spin parts of the potentials. The matrix element for the $P\bar{V} \pm V\bar{P}$ is $\langle \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \rangle = \pm 1$ while that for the $V\bar{V}$ is $\langle \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \rangle =$ -2, -1, and +1 corresponding to J = 0, J = 1, and J = 2,respectively. Therefore the results for the $c = +1(c = -1)P\bar{V}$ case are similar to those for the $J = 2(J = 1)V\bar{V}$ case. If pseudoscalar meson exchanges are forbidden, the results for the $P\bar{V} \pm V\bar{P}$ systems are similar to those for the $P\bar{P}$ system.

VI. DISCUSSIONS AND CONCLUSION

From the numerical results in the previous section, we know the binding energy is always larger in the extended chiral quark model than in the chiral quark model. This is partly because the vector mesons provide attractive and relatively important interactions. Another reason is that the σ mass in E_XQM is

TABLE VII. Summary of possible bound states in $P\bar{V} \pm V\bar{P}$ systems.

Isospin	<i>C</i> =	= +	C =	= -		
	(\bar{c}, c)	(\bar{b}, b)	$\overline{(\bar{c},c)}$	(\bar{b}, b)		
$I = \frac{1}{2}$	х	×	×	х		
$I = \overline{1}$	×	×	×	?		
I = 0(s)	*	?	*	?		
I = 0(l)	\checkmark	\checkmark	*	?		
ISB	*		*	?		

smaller than that in χ QM. This makes the attraction from σ stronger and thus the binding energy is larger even if the contributions from vector mesons can be canceled.

To make a clearer picture for the possible hadronic molecules, we summarize our conclusions in Tables V, VI, and VII. In those tables, an "×" means that a bound state does not exist. An "*" means a bound state does not exist in χ QM while it is possible or not excluded in E χ QM. A "?" means we cannot draw a conclusion even in χ QM and the system needs further study. A " $\sqrt{}$ " means a bound state is possible.

From the tables, we know that the I = 1/2, I = 1 charmanticharm and the I = 1 bottom-charm hadronic molecules do not exist. Therefore our conclusion for the $D^{*-}D_s^+ \pm D^-D_s^{*+}$ system is inconsistent with Ref. [70]. Our calculation also indicates that the resonance-like structure $Z^+(4051)$ in the $\pi^+\chi_{c1}$ invariant mass [65] could not be an *S*-wave $D^*\bar{D}^*$ molecule.

However, the isoscalar hidden bottom molecules $B\bar{B}$, $J = 2 B^*\bar{B}^*$, and $C = +B\bar{B}^*$ are very likely to form regardless of whether the isospin symmetry is largely violated or not. All these states should be rather stable because *B* is the lowest bottom meson and B^* does not decay via strong interaction. The experimental search for these states may be used to test our model.

There are so many systems we cannot draw a conclusion, most of which are I = 0 states. Whether the effects due to coupled channels, the annihilation, and the possible mixing between S-wave and D-wave interactions may help is an open question. More detailed studies are necessary.

In the extended chiral SU(3) quark model, the gluon, pseudoscalar, scalar, and vector mesons bind together the light quarks to baryons. The strength of OGE interaction in the SU(3) chiral quark model is greatly reduced because of the existence of vector mesons. But it is still controversial whether

TABLE VI. Summary of possible bound states in $V\bar{V}$ systems.

Isospin		(\bar{c}, c)			(\bar{b},b)	(\bar{b}, b)		(\bar{b}, c)	
	$\overline{J=0}$	J = 1	J = 2	$\overline{J=0}$	J = 1	J = 2	$\overline{J=0}$	J = 1	J = 2
$I = \frac{1}{2}$	х	×	×	×	×	×	×	×	х
$I = \tilde{1}$	х	х	х	?	?	х	х	х	×
I = 0(s)	*	*	*	?	?	?	?	?	?
I = 0(l)	*	*	\checkmark	*	?	\checkmark	*	?	\checkmark
ISB	*	*	?	*	?		*	*	?

OGE or vector meson exchange dominates the short-range quark-quark interaction. If the later mechanism is not suitable, one finds more systems without binding solutions.

In summary, we have performed a systematic study for the bound state problem of S-wave heavy quark meson-antimeson systems in a chiral quark model. The exchanged mesons below 1.1 GeV have all been taken into account. Because we considered just color-singlet meson-meson configuration and several approximations were used, our investigation is preliminary. Our crude calculation disfavors the existence of I = 1/2, I = 1 charm-anticharm and I = 1 charmbottom hadronic molecules but favors the existence of I = 0 $B\bar{B}, B^*\bar{B}^*$ (J = 2), and $B\bar{B}^*$ (C = +) bound states. Whether the consideration of other effects, such as the coupling with hidden-color configuration and the coupling with possible Dwave, supports these conclusions or not will be further studied. In our model, the σ meson exchange interaction plays an important role in the bound state problem of the light quark systems. When extending the model to the heavy quark sector,

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the possibility of the σ meson exchange between heavy quarks or between a heavy quark and a light quark is not excluded. Because no mass factor in the potential may suppress the σ meson contributions, the value of the coupling constant $g_{QQ\sigma}$ is crucial in discussing whether or not such interactions are important. Although the coupling is expected to be weak, a small value may have big effects, which is also an open problem in the present approach.

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