

Analysis of deuteron breakup reactions on ${}^7\text{Li}$ for energies up to 100 MeV

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Inclusive nucleon spectra from deuteron breakup reactions on ${}^7\text{Li}$ are analyzed in terms of the continuum discretized coupled channels theory for the elastic breakup process and the Glauber model for the nucleon stripping process. Both theoretical models use the same phenomenological nucleon optical potential of ${}^7\text{Li}$ and have no other free parameters. The calculations reproduce well a prominent bump observed around half the incident energy in experimental inclusive spectra of 40-MeV (d, xn) and 100-MeV (d, xp) reactions at forward angles. The analysis shows that the stripping process is more important than the elastic breakup process in deuteron breakup reactions on ${}^7\text{Li}$.

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I. INTRODUCTION

In recent years, deuteron breakup reactions have attracted considerable attention in the study of projectile breakup of exotic and halo nuclei as well as in applications associated with accelerator-driven neutron sources. The $\text{Li}(d, xn)$ reaction is regarded as one of the most promising reactions to produce intense neutron beams [*e.g.*, at the International Fusion Material Irradiation Facility (IFMIF)] [1]. To design such a neutron source, experimental data of energy and angular distributions of produced neutrons are indispensable over a wide incident energy range. However, experimental data of the $\text{Li}(d, xn)$ reaction are scarce, and the cross-section data measured with a thin Li target are currently available for only two incident energies: 17 MeV [2] and 40 MeV [3]. Therefore, it is required that a reliable theoretical model be established to create nuclear data necessary for designing the $d + \text{Li}$ neutron source.

Neutron production from $d + \text{Li}$ interactions takes place via the following processes: deuteron elastic (or diffractive) breakup and proton stripping processes, sequential neutron emission from highly excited compound and residual nuclei, and so on. Here, the proton stripping process means that the proton in the deuteron is absorbed by the target nucleus and the neutron passes it. In particular, treatment of deuteron breakup processes is important for predicting the neutron yield accurately because the deuteron itself is a very loosely bound system. Therefore, the main subject of the present work is to analyze deuteron breakup reactions in $d + \text{Li}$ interactions. Recently, Pereslavtsev *et al.* have performed the nuclear data evaluation of $d + {}^6,7\text{Li}$ reactions for deuteron energies up to 50 MeV, which is dedicated to the IFMIF [4], and have shown their evaluated double differential neutron emission spectra with experimental data [2,3]. In their model calculation, the Serber model [5] is applied to describe a prominent bump observed at approximately half the incident energy in neutron

production in the forward direction. The deuteron breakup processes, namely both elastic breakup and proton stripping, are involved with formation of the bump. Since the Serber model describes the stripping process alone, the contribution of elastic breakup is neglected in their calculation. Accordingly, it will be necessary to estimate how the elastic breakup contributes to neutron production from $d + \text{Li}$ interactions to make a more accurate prediction of the neutron production cross sections.

In our early work [6], we have applied successfully the continuum discretized coupled channels (CDCC) method to the analysis of deuteron elastic scattering from ${}^6,7\text{Li}$ and deuteron reaction cross sections in the energy range up to 50 MeV. The CDCC method [7–9] deals with the deuteron breakup processes explicitly by using a three-body Hamiltonian in which the nucleon-nucleus interaction is represented by the optical model potential (OMP) at half the deuteron incident energy and an effective nucleon-nucleon potential is used for the p - n interaction. The CDCC method has so far been applied successfully to analysis of the elastic breakup process in coincidence (d, pn) measurements [8]. It is expected, therefore, that the CDCC method is reliable to estimate the elastic breakup component in the inclusive $\text{Li}(d, xn)$ reaction. The only input parameter in the CDCC calculation is the nucleon OMP. The most suitable nucleon OMP for ${}^6,7\text{Li}$, called the extended Chiba OMP, was found for energies up to 50 MeV in our preceding work [6]. Its use enables CDCC calculations without any adjustable parameters.

The proton stripping process should be included to provide a full description of neutron emission from deuteron breakup processes. Since the present version of the CDCC code [10] cannot deal with the proton stripping process, we apply the Glauber model [11] to the calculation of proton stripping, instead of the Serber model used in Ref. [4]. Up to now, the Glauber model has been widely used in analyses of projectile breakup of exotic and halo nuclei at intermediate energies [12–14]. Also there are some examples of deuteron-induced reactions at intermediate energies above 40 MeV/nucleon in Refs. [15,16]. Therefore, it is of great interest to investigate

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the applicability of the Glauber model to deuteron breakup reactions at incident energies below 40 MeV/nucleon. Since the eikonal phase shift in the Glauber model can be calculated by using the nucleon OMP for ${}^6,{}^7\text{Li}$ [6], there is an advantage that no adjustable parameter is included in our Glauber model calculation. Moreover, with the Glauber model one can calculate both total reaction and elastic breakup cross sections in a straightforward way. It is expected, therefore, that their comparison with the aforementioned CDCC calculation may be a useful way to verify the applicability of the Glauber model to deuteron breakup reactions at relatively low energies.

In the present work, the model calculation using the CDCC method for the elastic breakup process and the Glauber model for the nucleon stripping process is applied to analysis of the inclusive (d, xn) reaction on ${}^7\text{Li}$ at 40 MeV. In addition, experimental data of the inclusive (d, xp) reaction on ${}^9\text{Be}$ at 100 MeV [17] are compared with a model calculation of the ${}^7\text{Li}(d, xp)$ reaction. The contributions from pre-equilibrium and evaporation emission should also be taken into account to analyze the experimental inclusive spectra over the whole emission energy and angular ranges, although they are expected to be small in the forward neutron emission of our interest. Since we suppose that the statistical models such as the pre-equilibrium exciton model are not appropriate for a nine-nucleon system consisting of $d + {}^7\text{Li}$, the contributions are estimated phenomenologically in terms of the moving source (MS) model [18] using the experimental data for backward emission.

In Sec. II, the formalism of the CDCC, the Glauber model, and the moving source model is outlined. In Sec. III, both CDCC and Glauber model calculations are compared for total reaction and elastic breakup cross sections. Results of the ${}^7\text{Li}(d, xn)$ reaction at 40 MeV and the ${}^7\text{Li}(d, xp)$ reaction at 100 MeV are presented. The applicability of the Glauber model to the $d + {}^7\text{Li}$ reaction at the relatively low energy of 40 MeV is discussed. Finally, conclusions are given in Sec. IV.

II. THEORETICAL MODEL

Inclusive nucleon emission spectra from deuteron-induced reactions contain contributions from various reaction processes, namely, the direct processes such as elastic breakup and stripping processes and the statistical processes such as pre-equilibrium and evaporation processes. To analyze the experimental data of deuteron-induced reactions on Li, we use the following model approach. For the direct processes, the CDCC method is applied to calculation of the elastic breakup (EB) process and the Glauber model is used for the proton stripping processes (STR) in the continuum. Also the MS model is used to estimate the evaporation and pre-equilibrium components (EP). Thus, the double differential cross section (DDX) of (d, xn) reactions is expressed by the incoherent summation of three components:

$$\begin{aligned} \frac{d^2\sigma^{(d, xn)}}{dE_n^L d\Omega_n^L} &= \left. \frac{d^2\sigma_{\text{EB}}}{dE_n^L d\Omega_n^L} \right|_{\text{CDCC}} + \left. \frac{d^2\sigma_{\text{STR}}^p}{dE_n^L d\Omega_n^L} \right|_{\text{Glauber}} \\ &+ \left. \frac{d^2\sigma_{\text{EP}}}{dE_n^L d\Omega_n^L} \right|_{\text{MS}}, \end{aligned} \quad (1)$$

where the superscript L stands for physical quantities in the laboratory system. The DDX of (d, xp) reactions can also be calculated by replacing the subscript n by p and the superscript p by n on the right-hand side of Eq. (1). Each model is described briefly in the following sections.

Here we note that, in the preceding study [19], an attempt to separate the complete and incomplete components of the fusion cross section was carried out using the CDCC method, and importance of the breakup channels of the projectile was clarified. The incomplete fusion cross section in Ref. [19] corresponds to the stripping cross section in the present work. Unfortunately, however, formulation of the DDX of the stripping process, $d^2\sigma_{\text{STR}}^p/(dE_n^L d\Omega_n^L)$, with CDCC has not yet been performed. Since the DDX rather than the integrated cross section σ_{STR}^p is essentially important in the present work at the IFMIF, we adopt the Glauber model to calculate $d^2\sigma_{\text{STR}}^p/(dE_n^L d\Omega_n^L)$ as shown in the following. Another remark on Ref. [19] is that a possible contribution of the capture of all projectile fragments to the complete fusion process was naively neglected. Further study on the separation of the complete and incomplete fusion processes with CDCC will be necessary.

A. CDCC method applied to the elastic breakup process

The basic idea of the CDCC method is to discretize the continuum states of projectile with respect to its fragmentation and relative momentum (or excitation energy) and to introduce them to the coupled channel (CC) equations. Then, the S -matrix can be derived according to the asymptotic boundary condition and applied to analyses of nuclear reactions such as elastic scattering and breakup reactions, in which projectile breakup plays an important role.

Here let us consider the case where a neutron is detected via the elastic breakup process, $d + {}^7\text{Li} \rightarrow n + p + {}^7\text{Li}$. According to Ref. [8], the triple differential cross section in the laboratory system is expressed as

$$\frac{d^3\sigma_{\text{EB}}}{d\Omega_p^L d\Omega_n^L dE_n^L} = \frac{2\pi}{\hbar} \frac{\mu_{dA}}{P_d} |T_{fi}|^2 \rho(E_n^L), \quad (2)$$

in which Ω_p^L and Ω_n^L represent the emission direction of p and n , respectively, E_n^L is the neutron emission energy, μ_{dA} is the reduced mass of the deuteron and the target, P_d is the momentum of the incident deuteron, and $\rho(E_n^L)$ is the three-body phase space factor [20]. The transition matrix element, T_{fi} , is given as

$$\begin{aligned} T_{fi} &= i \frac{(2\pi\hbar)^3}{\sqrt{2\mu_{pn}\mu_{dA}} k \sqrt{P} P_d} \sum_{IJM} \sqrt{2L+1} \\ &\times [Y_I(\hat{\mathbf{k}}) \otimes Y_L(\hat{\mathbf{P}})]_{JM} e^{i\delta_{Ik}} S_{IL}^J(k), \end{aligned} \quad (3)$$

using the breakup S -matrix element, $S_{IL}^J(k)$, obtained from the discretized CC equations. Here μ_{pn} is the reduced mass of the p and n subsystem, k is the relative momentum between p and n , P is the momentum of the center of mass of the p - n pair relative to the target, and δ_{Ik} is the nuclear phase shift of the scattering between p and n . In the summation in Eq. (3), J and M are the total angular momentum of the $d + {}^7\text{Li}$ system and its z component, L is the orbital angular momentum between

the center of mass of the p - n system and the target, and l is the relative angular momentum of the p - n system.

Finally, the double differential cross section with respect to the neutron emission energy and angle is obtained by integrating Eq. (2) over Ω_p^L :

$$\left. \frac{d^2\sigma_{\text{EB}}}{d\Omega_n^L dE_n^L} \right|_{\text{CDCC}} = \int d\Omega_p^L \frac{d^3\sigma_{\text{EB}}}{d\Omega_p^L d\Omega_n^L dE_n^L}. \quad (4)$$

It is noted that the DDX for proton detection can be expressed by exchanging the subscripts n and p in Eqs. (2) to (4).

According to Ref. [17], the Coulomb breakup of the deuteron contributes very little to the inclusive (d, xp) reaction on ${}^9\text{Be}$ at 100 MeV. Therefore, it is not considered in the present CDCC calculation as in Ref. [6].

B. Glauber model

The Glauber model [11], as a semiclassical approach, gives a rather good prediction of the reaction cross sections involving the loosely bound projectiles, by assuming the eikonal and adiabatic approximations.

According to the Glauber model, the S -matrix to describe the deuteron interaction with the target nucleus A is given by

$$S(b) = e^{i\chi(b)}, \quad \text{with } \chi(b) = -\frac{1}{\hbar v} \int_{-\infty}^{+\infty} dz V_{dA}(R), \quad (5)$$

where $\chi(b)$ is the eikonal phase shift, b is the impact parameter perpendicular to the deuteron incident direction along the z axis, v is the relative velocity between the deuteron and target nucleus, $V_{dA}(R)$ is the OMP between the deuteron and target nucleus, and $R = \sqrt{b^2 + z^2}$. If V_{dA} is expressed by the sum of the proton-target OMP, V_{pA} , and the neutron-target OMP, V_{nA} , the phase shift $\chi(b)$ is simply the sum of the phase shift for the proton, $\chi_{pA}(b_p)$, and that for the neutron, $\chi_{nA}(b_n)$:

$$\chi(b) = \chi_{pA}(b_p) + \chi_{nA}(b_n), \quad (6)$$

where b_p and b_n are the impact parameters of the proton and the neutron perpendicular to the z axis (i.e., $\vec{b}_p = \vec{b} + \frac{1}{2}\vec{r}_\perp$ and $\vec{b}_n = \vec{b} - \frac{1}{2}\vec{r}_\perp$, where \vec{r}_\perp corresponds to the projection of \vec{r} , which is the relative coordinate between the proton and neutron in the deuteron, perpendicular to the z axis). Thus, the S -matrices for the nucleon-target interaction, S_ν ($\nu = p, n$), are defined by

$$\begin{aligned} S_\nu(b_\nu) &= e^{i\chi_{\nu A}(b_\nu)} \\ &= \exp\left[-\frac{i}{\hbar v} \int_{-\infty}^{+\infty} dz V_{\nu A}(\sqrt{b_\nu^2 + z^2})\right], \end{aligned} \quad (7)$$

where the phenomenological OMP [6] is used for $V_{\nu A}$ in the present work. Since the integral in Eq. (7) for the Coulomb part of the proton OMP diverges, we use the same prescription as in Ref. [21], in which the Coulomb eikonal phase shift is added to the $\chi_{pA}(b_p)$ calculated using V_{pA} without the Coulomb potential.

Using $S_p(b_p)$ and $S_n(b_n)$, we can formulate the total reaction cross section of the d -target collision, σ_R , the proton stripping cross section, σ_{STR}^p , the neutron stripping cross section, σ_{STR}^n ,

and the elastic breakup cross section, σ_{EB}^p , as follows:

$$\sigma_R = \int d^2\vec{b} \left[1 - \left| \int d^3\vec{r} |\psi_{00}(\vec{r})|^2 S_p(b_p) S_n(b_n) \right|^2 \right], \quad (8)$$

$$\sigma_{\text{STR}}^p = \int d^2\vec{b} \int d^3\vec{r} |\psi_{00}(\vec{r})|^2 |S_n(b_n)|^2 (1 - |S_p(b_p)|^2), \quad (9)$$

$$\sigma_{\text{STR}}^n = \int d^2\vec{b} \int d^3\vec{r} |\psi_{00}(\vec{r})|^2 |S_p(b_p)|^2 (1 - |S_n(b_n)|^2), \quad (10)$$

$$\begin{aligned} \sigma_{\text{EB}} &= \int d^2\vec{b} \left[\int d^3\vec{r} |\psi_{00}(\vec{r})|^2 |S_p(b_p) S_n(b_n)|^2 \right. \\ &\quad \left. - \left| \int d^3\vec{r} |\psi_{00}(\vec{r})|^2 S_p(b_p) S_n(b_n) \right|^2 \right], \end{aligned} \quad (11)$$

where $\psi_{00}(\vec{r})$ is the wave function of the deuteron ground state.

The differential cross section for the proton stripping process is given by the following expression [13,22–24]:

$$\begin{aligned} \frac{d^3\sigma_{\text{STR}}^p}{d^3k_n^C} &= \frac{1}{(2\pi)^3} \int d^2\vec{b}_p \left\{ [1 - |S_p(b_p)|^2] \right. \\ &\quad \left. \times \left| \int d^3\vec{r} e^{-i\vec{k}_n^C \cdot \vec{r}} S_n(b_n) \psi_{00}(\vec{r}) \right|^2 \right\}, \end{aligned} \quad (12)$$

in the center of mass of p - n system, where k_n^C is the neutron wave number. The double differential cross section can be given by transforming Eq. (12) from the center of mass system to the laboratory system:

$$\left. \frac{d^2\sigma_{\text{STR}}^p}{dE_n^L d\Omega_n^L} \right|_{\text{Glauber}} = \frac{m_n k_n^L}{\hbar^2} \frac{d^3\sigma_{\text{STR}}^p}{d^3k_n^C}, \quad (13)$$

where E_n^L , k_n^L , and Ω_n^L are the energy, the wave number, and the solid angle of the neutron in the laboratory system, respectively. The neutron stripping cross section can also be calculated by exchanging p and n in the superscripts and subscripts in Eqs. (12) and (13).

C. Moving source model

Nucleon emission via evaporation and pre-equilibrium processes is included in the inclusive nucleon emission spectra from deuteron-induced reactions. An empirical method based on experimental data, called the moving source (MS) model, is applied to estimate these components. Thus, the following MS formula is used to calculate the DDX of these components in Eq. (1):

$$\begin{aligned} \left. \frac{d^2\sigma_{\text{EP}}}{dE_n^L d\Omega_n^L} \right|_{\text{MS}} &= \sum_{i=1,2} N_{0,i} \sqrt{E_n^L} \exp\{-[E_n^L + E_{1,i} \\ &\quad - 2\sqrt{E_n^L E_{1,i} \cos\theta_n^L}]/T_i\}, \end{aligned} \quad (14)$$

where the parameters $N_{0,i}$, $E_{1,i}$, and T_i are determined by fitting the experimental DDXs at backward angles because the contribution from the direct process is expected to be very small. The suffixes $i = 1$ and 2 represent the evaporation and pre-equilibrium processes, respectively.

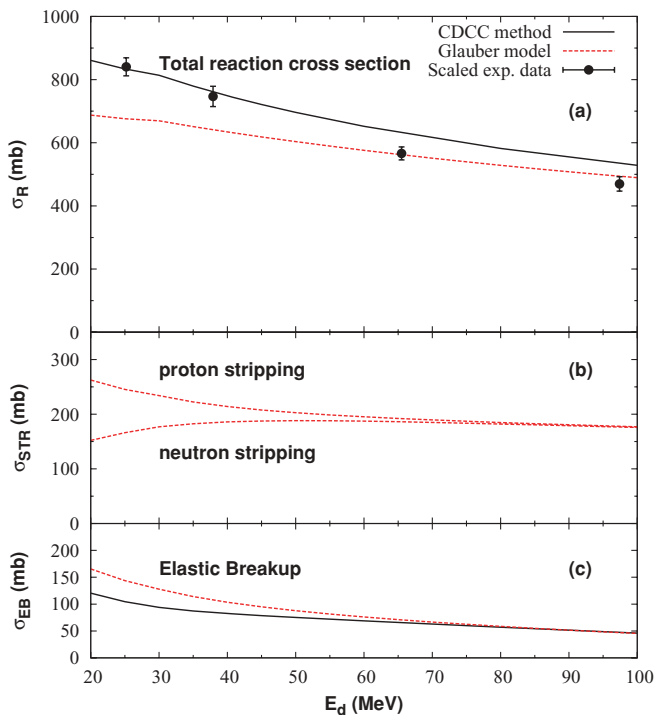


FIG. 1. (Color online) (a) Total reaction cross section, (b) proton and neutron stripping cross sections, and (c) elastic breakup cross section for the $d + {}^7\text{Li}$ reaction. The solid and dashed curves represent the calculations using the CDCC method and the Glauber model, respectively. The closed circles are the “scaled” experimental data of ${}^9\text{Be}$ explained in Ref. [6].

III. RESULTS AND DISCUSSION

The models outlined in Sec. II are applied to inclusive (d, xn) and (d, xp) reactions on ${}^7\text{Li}$ at energies less than 100 MeV to investigate the deuteron breakup processes. The CDCC calculations are performed using the codes [8,10] with the same input data as in our preceding work on deuteron elastic scattering from ${}^6,7\text{Li}$ [6]. The major input data necessary in the Glauber model calculation are the nucleon OMP of ${}^7\text{Li}$ and the deuteron ground-state wave function, which are the same as in the CDCC calculation. Both the CDCC and Glauber model calculations use the extended Chiba OMP of ${}^7\text{Li}$ [6] at half the incident deuteron energy. For the OMP, there are two parameter sets, called Set 1 and Set 2. Our preliminary calculation has confirmed that both calculations with the Set 1 and Set 2 parameters agree within several percent. Thus, the results with Set 1 are presented in the following.

The calculation result of integrated cross sections is illustrated in Fig. 1, which shows the total reaction cross section,

the proton stripping cross section, the neutron stripping cross section, and the elastic breakup cross section given by the CDCC method (solid curves) and the Glauber model (dashed curves) as a function of deuteron incident energy. The total reaction cross section calculated by the CDCC method is the same as that in Fig. 9(b) in Ref. [6], and the explanation on the “scaled” experimental data of ${}^9\text{Be}$ is given there. The Glauber model calculation provides a smaller total reaction cross section over the whole energy range than the CDCC calculation, whereas the elastic breakup cross section calculated by the Glauber model is larger than the CDCC calculation at energies below 80 MeV. The differences between the two calculations are at most 20% at 40 MeV, where the experimental data of the (d, xn) reaction is available and the model analysis will be shown later. The differences decrease with increasing incident energy and both calculated elastic breakup cross sections coincide at 80 MeV or higher. The stripping cross sections calculated with the Glauber model are also depicted in Fig. 1. They are not compared with the CDCC calculation because our present CDCC cannot calculate the quantity. The Glauber model prediction shows that the stripping process is more important than the elastic breakup process in the deuteron breakup reactions on ${}^7\text{Li}$. However, the contribution from the elastic breakup process is not negligible. According to Fig. 1, the elastic breakup cross section amounts to about 50% of the proton stripping cross section at 40 MeV, and still about 25% of that even at 100 MeV. This comparison shows that the elastic breakup process makes a considerable contribution to neutron (or proton) production from the $d + \text{Li}$ reaction.

The CDCC and Glauber model calculations are then applied to analyses of the continuum energy spectra and angular distribution of neutrons produced from the ${}^7\text{Li}(d, xn)$ reaction at 40 MeV. Figure 2 shows a comparison between the calculated energy spectra and the experimental data [3] for emission angles smaller than 20° . The result of angular distribution is also presented in Fig. 3. It should be noted that the experimental angular distribution is given by integrating the double differential cross section data over emission energy up to 40 MeV. In Figs. 2 and 3, the dotted curves are the elastic breakup component calculated by the CDCC method and the dashed curves are the proton stripping component calculated by the Glauber model. The evaporation and pre-equilibrium components plotted by the dot-dashed curves are estimated using the MS model calculation. The parameters used in Eq. (14) are determined by fitting the experimental data at backward angles, where the direct breakup contribution is expected to be negligible. Figure 4(a) shows the fitting result at 110° . The obtained parameters are listed in Table I.

The calculation reproduces the prominent bump observed around 20 MeV in the experimental spectra fairly well in

TABLE I. Parameters of the moving source model.

	Processes	N_0 (mb/MeV $^{3/2}$ sr)	E_1 (MeV)	T (MeV)
${}^7\text{Li}(d, xn)$ at 40 MeV	Evaporation	9.91	2.0×10^{-4}	1.31
	Pre-equilibrium	3.01	0.79	4.41
${}^9\text{Be}(d, xp)$ at 100 MeV	Pre-equilibrium	1.06	2.0	6.98

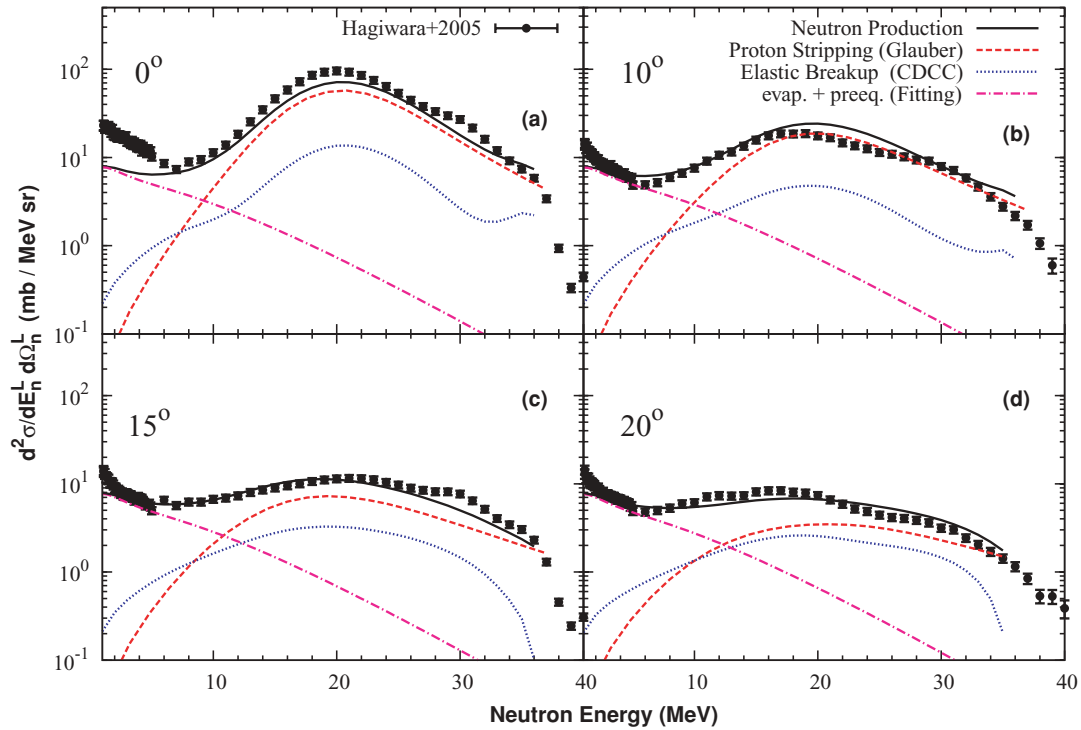


FIG. 2. (Color online) Comparison of the experimental data and calculated DDXs of ${}^7\text{Li}(d, xn)$ at 40 MeV for different neutron emission angles. The proton stripping and elastic breakup components are plotted as the dashed and the dotted curves, respectively. The sum of the evaporation and pre-equilibrium components is denoted by the dot-dashed curve. The solid curve represents the sum of these three components.

Fig. 2. The deuteron breakup processes are strongly involved with the formation of the bump as expected. The proton stripping process is more predominant than the elastic breakup process. Here it is worth noting that the strong contribution from the stripping process is also indicated by the distorted wave Born approximation (DWBA) calculation for (d, px) reactions on medium-heavy nuclei at 25.5 MeV [25]. The relative contribution from the stripping process is reduced as the angle increases. From the comparison between the calculated and experimental angular distributions in Fig. 3, it is found that the deuteron breakup processes dominate neutron

production at forward angles and are negligible at backward angles, whereas the evaporation and pre-equilibrium processes have a major contribution at backward angles. Consequently, the present analysis suggests that an accurate description of the deuteron breakup reactions including both the elastic breakup and proton stripping processes is essential for reliable design of the neutron sources using the ${}^7\text{Li}(d, xn)$ reaction.

In Fig. 2, a discrepancy between the MS model calculation and the experimental data is seen only in the low emission energy range at 0° . The MS model gives almost isotropic angular distribution for the evaporation component, whereas the experimental data in the energy range show a sudden increase at 0° . A proper treatment of neutron decay from compound and residual nuclei might be necessary for better understanding of low-energy neutron production. However, this is beyond our scope of the present work.

Next we examine the applicability of the present model calculation to $d + {}^7\text{Li}$ reactions at incident energies higher than 40 MeV. Although there are no available data for ${}^7\text{Li}$, a measurement of double differential ${}^9\text{Be}(d, xp)$ cross sections has been reported for an incident energy of 100 MeV [17]. The calculation for the ${}^7\text{Li}(d, xp)$ reaction is compared with the experimental data of ${}^9\text{Be}$, because the deuteron reaction cross sections calculated using the empirical formula [26] have a modest 12% difference between ${}^7\text{Li}$ and ${}^9\text{Be}$ and both the (d, xp) measurements are expected to provide similar cross sections of deuteron breakup reactions.

Figure 5 shows a comparison between the calculation and the experimental data. The parameters of the MS model are determined by fitting the data at 100° and are listed in Table I.

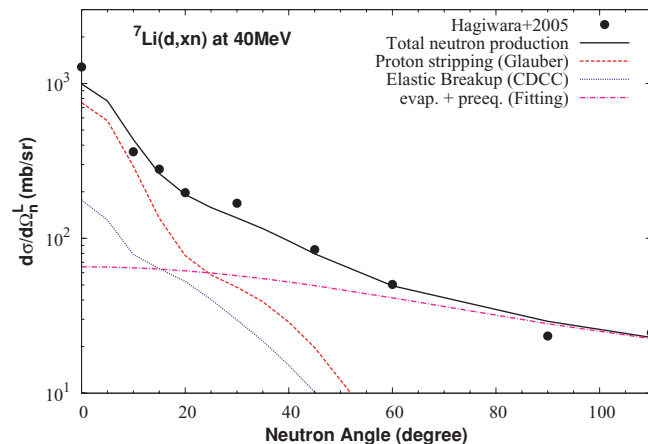


FIG. 3. (Color online) The same as in Fig. 2 but for the angular distribution.

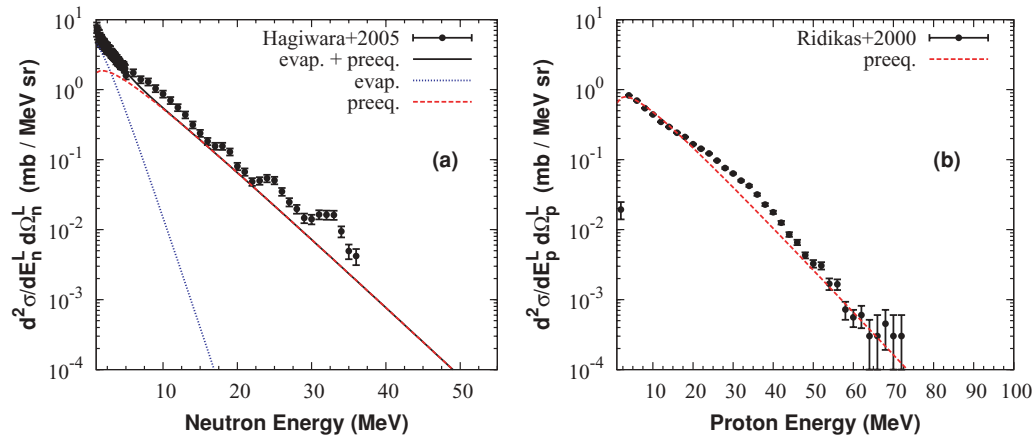


FIG. 4. (Color online) Fitting of the experimental data of (a) ${}^7\text{Li}(d, xn)$ at 40 MeV and 110° and (b) ${}^9\text{Be}(d, xp)$ at 100 MeV and 100° using the moving source model. The dotted and dashed curves represent the components of evaporation and pre-equilibrium processes, respectively. The solid curve gives their sum.

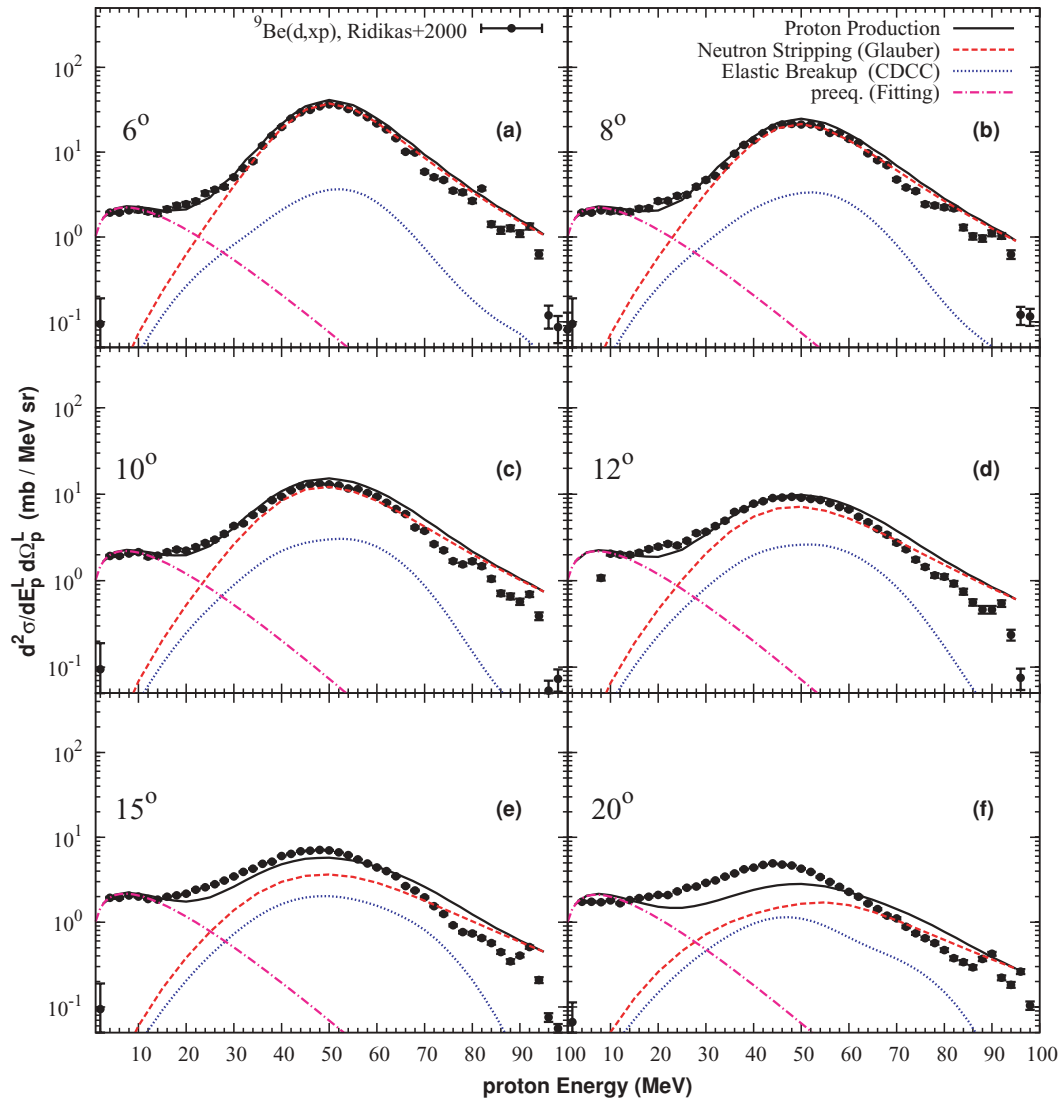


FIG. 5. (Color online) The same as in Fig. 2 but for comparison of the calculation of ${}^7\text{Li}(d, xp)$ at 100 MeV with the experimental data of ${}^9\text{Be}(d, xp)$ at 100 MeV for different proton emission angles.

The result is shown in Fig. 4(b). Since it was found that only the pre-equilibrium component is enough to reproduce the energy spectrum over the wide emission energy in this case, the evaporation component is omitted. As shown in Fig. 5, the model calculations are in excellent agreement with the experimental data for small angles up to 20° . The prominent bump seen around 50 MeV is reproduced fairly well by the calculation for each angle except at 20° . As can be seen at 20° , an upward shift in the peak position appears in the Glauber model stripping component with increasing emission angle. This shift makes the agreement with the experimental data worse at large angles. The neutron stripping process dominates over the elastic breakup process at small angles, and the relative fraction is reduced with increasing emission angle. This is the same trend as seen in Fig. 2 for the ${}^7\text{Li}(d, xn)$ reaction at 40 MeV. It should be noted that the Coulomb breakup of the deuteron is not included in the present calculation, because Ridikas *et al.* [17] show a negligibly small contribution from the Coulomb breakup for ${}^9\text{Be}$, although it plays a crucial role at forward proton emission for the heavy nuclei, such as Pb.

Finally, let us discuss why the Glauber model works well even for relatively low incident energies less than 100 MeV. The eikonal approximation requires a condition in which the wavelength of the projectile is short compared to the effective range of the potential between the projectile and the target. This condition can be expressed in terms of the relative wave number between the projectile and the target, k , and the interaction range, a , as follows:

$$ka \gg 1. \quad (15)$$

Then, the eikonal approximation also needs a relatively high incident energy, E_{in} , compared with the potential depth between the deuteron and the target nucleus, V_0 , namely,

$$E_{\text{in}} \gg V_0. \quad (16)$$

In the case of the $d + {}^7\text{Li}$ reaction at 40 MeV, the first condition given by Eq. (15) is satisfied reasonably well, because

$$\begin{aligned} ka &= k(R_d + R_{\text{Li}}) \\ &\approx 6.3 \gg 1, \end{aligned} \quad (17)$$

where R_d and R_{Li} are the radii of d and Li, respectively. The second condition given by Eq. (16) is satisfied if the relative distance between the center of mass of the deuteron and the target, R , is large enough because the potential depth decreases with R as shown in Fig. 6, where the potential is the real part of the deuteron OMP given by the sum of both proton and neutron OMPs for ${}^7\text{Li}$. In the figure, the differential proton stripping cross section with respect to the impact parameter b is also plotted to see where the proton stripping process occurs mainly at 40 MeV. It is noted that a similar dependence on b in the case of inclusive projectile fragmentation was shown schematically by Hussein and McVoy [22]. It is obvious that the major contribution comes from the peripheral region. The potential depth at the distance R , where the contribution is the largest, is about 10 MeV, sufficiently smaller than

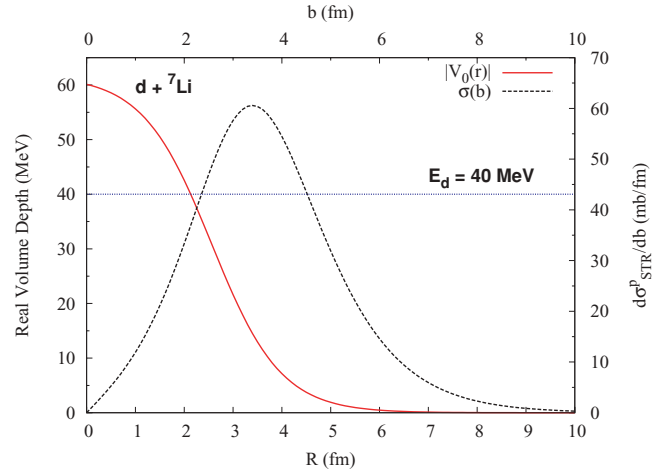


FIG. 6. (Color online) Real volume depth (solid curve) of the deuteron OMP on ${}^7\text{Li}$ at 40 MeV as a function of d -Li relative distance, R , and differential proton stripping cross section (dashed curve) with respect to the impact parameter, b .

40 MeV. Thus, the second condition given by Eq. (16) holds modestly at the deuteron energy of 40 MeV. From this discussion, it can be understood why the Glauber model can be successfully applied to the analysis of the stripping reaction on Li even at the relatively low incident energy of 40 MeV.

IV. SUMMARY AND CONCLUSIONS

Deuteron breakup reactions on ${}^7\text{Li}$ at incident energies of 40 and 100 MeV were analyzed using the model calculation with the CDCC theory for the elastic breakup process, the Glauber model for the stripping process, and the moving source model for evaporation and pre-equilibrium processes. It should be noted that the use of the most suitable nucleon optical potential to describe nucleon elastic scattering from ${}^7\text{Li}$ results in a model calculation with no free parameters for either of the direct breakup processes. The calculations reproduce the experimental energy spectra in the continuum at small angles of less than 20° quantitatively well, even at a relatively low incident energy of 40 MeV. The analysis clarified that the stripping process is more dominant than the elastic breakup process in the $d + {}^7\text{Li}$ reaction. The dependence of the stripping reaction on the impact parameter was investigated for the ${}^7\text{Li}(d, xn)$ reaction at 40 MeV, and the stripping reaction was found to occur predominantly in the peripheral region of the target nucleus. As a result, it was confirmed that the condition that the incident energy E_{in} is much larger than the depth V of the optical potential between the deuteron and ${}^7\text{Li}$ in the Glauber approximation is satisfied even at 40 MeV, because V becomes shallow around the nuclear surface and is much lower than E_{in} .

Although our model calculation reproduces successfully well the experimental data at small angles, the Glauber model calculation shows that the peak position in the emission spectra shifts to high energy as the emission angle

increases, and it fails to reproduce the experimental spectra at large angles. This may suggest a limitation of applying the Glauber model to large momentum transfer. Therefore, it is desirable to extend the CDCC method to deal with the stripping process and to predict nucleon production from deuteron breakup reactions in a unified way in the future.

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