

# Single particle states in neutron-rich $^{101}\text{Zr}$ , $^{103,105,107}\text{Mo}$ , and $^{109,111}\text{Ru}$

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The multipole mixing ratios of  $\Delta I = 1$  transitions between levels in rotational bands built on single-particle states in odd neutron nuclei are dependent on the configurations of the states. In particular, the mixing ratio can be used to distinguish between several possible single-particle configurations if interpreted with the particle-axial-rotor model. This work features the first determination of the ground-state configurations of  $^{109,111}\text{Ru}$ . The single-particle structures of the ground states of  $^{101}\text{Zr}$  and  $^{103,105,107}\text{Mo}$  as well as excited states in  $^{103,107}\text{Mo}$  are also investigated, with a new result found in  $^{107}\text{Mo}$ .

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## I. INTRODUCTION

Neutron-rich nuclei such as  $^{101}\text{Zr}$ ,  $^{103,105,107}\text{Mo}$ , and  $^{109,111}\text{Ru}$  with  $A \geq 100$  are known to be highly deformed [1], with the  $h_{11/2}$  neutron orbital playing a role in driving the deformation although the proton-neutron interaction may be also important. It has been shown by Orlandi *et al.* [2] that the determination of the  $g$  factors of states and multipole mixing ratios from angular correlations between prompt  $\gamma$  transitions can be used to deduce the Nilsson orbitals for single-particle states of nuclei in this region. These nuclei are produced in spontaneous fission of  $^{252}\text{Cf}$ . Orlandi *et al.* were able to assign neutron single-particle configurations in  $^{101}\text{Zr}$  ( $Z = 40$ ) and  $^{103,105}\text{Mo}$  ( $Z = 42$ ). A similar method was used by Urban *et al.* [3] to determine the single-particle structure of  $^{107}\text{Mo}$ . The ruthenium isotopes ( $Z = 44$ ) have been investigated recently by Wu *et al.* [4], who found that the levels in  $^{109,111}\text{Ru}$  were most likely based on the  $5/2[413]$  or  $5/2[402]$  neutron orbitals, although they were not able to distinguish between the two. Therefore, it is of interest to distinguish between these configurations if possible.

The neutron single-particle configurations of  $^{101}\text{Zr}$ ,  $^{103,105,107}\text{Mo}$ , and  $^{109,111}\text{Ru}$  are determined in this work by measuring the mixing ratios of  $\Delta I = 1$  transitions in rotational bands. The mixing ratios are interpreted in terms of the particle-axial-rotor model.

## II. EXPERIMENT AND METHOD

The data for this analysis were taken by using the Gammasphere detector array, which was located at Lawrence Berkeley National Laboratory. A  $^{252}\text{Cf}$  spontaneous fission source with an  $\alpha$  activity of  $62 \mu\text{Ci}$  was placed between two iron foils. The foils were thick enough to stop the fission fragments, eliminating the need for a Doppler correction. Approximately

$5.7 \times 10^{11}$  three- and higher-fold  $\gamma$  coincidence events were recorded. More details about this experiment can be found in Luo *et al.* [5].

The method for measurement of angular correlations with Gammasphere triple-coincidence data used in this work is described in detail by Daniel *et al.* [6]. In this method, the coincidence intensity between a cascade of two  $\gamma$  rays as a function of angle is fitted to Eq. (1).

$$W(\theta) = 1 + G_2 A_2(\delta) P_2(\cos\theta) + G_4 A_4(\delta) P_4(\cos\theta) \quad (1)$$

In this equation the  $A_k$  are calculable coefficients depending on the spin sequence of the levels involved and the multipolarities and mixing ratios of the transitions between them. The  $G_k$  coefficients relate to attenuations of the correlation and are equal to unity unless some interaction is present during the lifetime of the intermediate state of the cascade that can cause precession of the nuclear spin. In this work the  $^{252}\text{Cf}$  source was mounted between unmagnetized iron foils so the the fission fragments were implanted into the Fe lattice. The recoil energies of the fragments (50–100 keV) lead to high occupation of substitutional lattice sites at which the nuclei experience strong magnetic hyperfine field  $B_{\text{HF}}$ . The interaction between the hyperfine field and the nuclear magnetic moment of the fragments produces Larmor precession of the nuclear spin about the field through a mean precession angle  $\phi = \omega_L \tau$ , where  $\omega_L$  is the the Larmor precession frequency. For randomly oriented field directions as in unmagnetised iron, this interaction leads to  $G_k$  given by

$$G_k = \frac{1}{2k+1} \left( 1 + 2 \sum_{q>0}^k \frac{1}{1+q^2\phi^2} \right), \quad (2)$$

where

$$\phi = -\frac{g\mu_N B_{\text{HF}}\tau}{\hbar}. \quad (3)$$

In these equations  $g$  is the nuclear  $g$  factor,  $B_{\text{HF}}$  is the nuclear hyperfine field in iron,  $\mu_N$  is the nuclear magneton, and  $\tau$  is the mean lifetime of the intermediate state. For the elements involved in this work  $B_{\text{HF}}$  is less than about 50 T giving  $\omega_L \sim 10^9$  rad s $^{-1}$  for a  $g$  factor  $\sim 0.4$ . Unless the lifetime of the intermediate state is greater than  $\sim 0.7$  ns,  $\phi$  is  $< 0.1$  rad and the  $G_k$  are close to unity. Thus, for intermediate states of short lifetime and/or small  $g$  factor, attenuation affects may be neglected and measured angular correlations used to extract multipole mixing ratios, in this work specifically  $\delta(E2/M1)$  ratios in  $\Delta I = 1$  transitions, provided the spins of the levels and the mixing ratio of the other transitions involved are known. For correlations involving intermediate states of longer lifetimes, provided the multipolarities of both  $\gamma$  transitions are known, attenuation factors, deduced by comparison of the measured with the calculated correlation coefficients, can be used to extract the  $g$  factor of the state, depending on knowledge of its lifetime and the hyperfine field acting.

The multipole mixing ratios measured in this work are interpreted in terms of the particle-plus-axial rotor model. Following the procedure outlined in Ref. [2], mixing ratios of  $\Delta I = 1$  transitions within a rotational band are given by

$$\delta(E2/M1) = \sqrt{\frac{5}{12}} \frac{1}{K} \frac{E_\gamma}{1.2 \times 10^3} \frac{Q_0}{(g_K - g_R)} \frac{\langle I_f K 20 | I_i K \rangle}{\langle I_f K 10 | I_i K \rangle}, \quad (4)$$

where  $Q_0$  is the quadrupole moment;  $g_K$  and  $g_R$  are the intrinsic and rotational  $g$  factors, respectively; and  $E_\gamma$  is the  $M1$  transition energy in keV. For these calculations,  $g_R$  is taken to be equal to  $\frac{1}{2}(Z/A)$  [7] and the quadrupole moment  $Q_0$  is calculated from the deformation parameter,  $\beta$ , using the relation

$$\beta = 91.7 \frac{Q_0}{ZA^{2/3}} \quad (5)$$

with  $\beta$  assumed to take a common value,  $\beta = 0.3$ . The single-particle  $g_K$  factor depends on the Nilsson configuration of the particle and can be calculated by using the  $M1$  matrix elements tabulated in Ref. [8] for various single-particle configurations by

$$g_K = g_l + \frac{g_s - g_l}{2K} \sum (\alpha_{l,K-1/2}^2 + \alpha_{l,K+1/2}^2) \quad (6)$$

where the  $\alpha$ 's are Nilsson coefficients [9] tabulated in Ref. [8] as a function of the deformation parameter  $\beta$ , and  $g_l$ ,  $g_s$  are the orbital and spin  $g$  factors. In these calculations, which involve neutron states, the values  $g_l = 0$  and  $g_s^{\text{eff}} = 0.6g_s^{\text{free}} = -2.296$  are used.

Fitting the parameter  $A_2$  to angular correlation data involving one pure multipole transition and a second transition of mixed multipolarity with mixing ratio  $\delta$  usually produces two values of  $\delta$ . In this work both  $\delta$  values are given in the last column in Table I. As an example, Fig. 1 shows the variation of the  $A_k$  parameters for the correlation of the pure  $E2$ ,  $13/2^+ - 9/2^+$  540.7-keV transition and the mixed  $E2/M1$   $9/2^+ - 7/2^+$  222.7-keV transition in the decay of  $^{109}\text{Ru}$ , as a function of  $\delta^2/(1+\delta^2)$ , where  $\delta$  is the  $E2/M1$

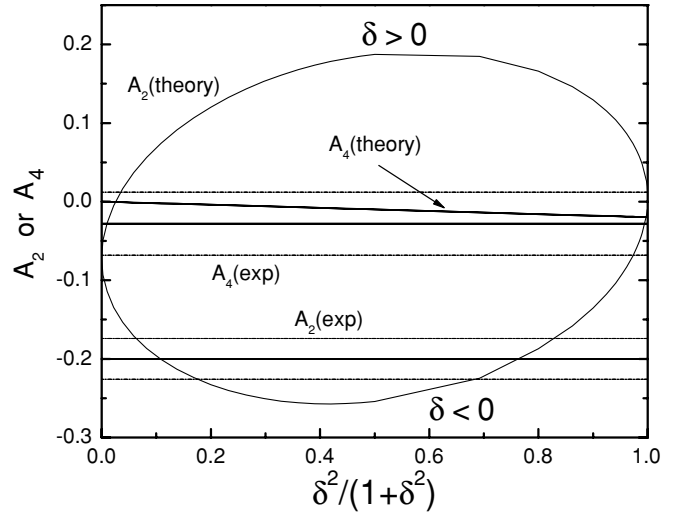


FIG. 1.  $A_k$  coefficient analysis to extract  $\delta(E2/M1)$  for the cascade  $13/2^+ - 9/2^+ - 7/2^+$  in  $^{109}\text{Ru}$  (see text). The experimental  $A_k$  values are shown as heavy horizontal lines with error limits above and below. Intersection of the  $A_2$  results with the  $A_2$  (theory) ellipse yields two values for the sign and magnitude of  $\delta$  given in Table I.

mixing ratio of the 222.7-keV transition. In principle a choice between the two values from  $A_2$  can be made on the basis of the extracted  $A_4$ , which, however, is often small and, as in this case, not accurate enough for the purpose. In this work, as has been done by others, the more  $M1$  dominated  $\delta$  value is adopted as the mixing ratio for the  $\Delta I = 1$  transition of the rotational bands given in the last column in Table II. In certain cases, either because the measured  $A_2$  value is close to either the lower or upper limit of its range or because of large errors in the data, the fit results in a single range for  $\delta$  and for these the lower limit value of  $\delta$  has been adopted in Table II.

Analysis of angular correlations gives not only the magnitude of the mixing ratio  $\delta$  but also its sign (see Fig. 1). Although the magnitude of  $\delta$  can also be extracted from, for example, measurements of electron conversion coefficients, the sign of  $\delta$  is valuable in identifying the nuclear configurations involved.

### III. RESULTS

Results are presented on a series of cascades in Zr, Mo, and Ru isotopes. The analysis of states in Zr and Mo largely confirms results reported by Orlandi *et al.* [2] using a related technique and establishes the method. The results on Ru are new and clearly identify the configurations present in these isotopes.

#### A. $^{101}\text{Zr}$

The partial level scheme of  $^{101}\text{Zr}$  is shown in Fig. 2. The mixing ratio of the mixed  $E2/M1$ ,  $5/2^+ - 3/2^+$  98.2-keV transition from the first excited state is determined from its

TABLE I. Angular correlations measured in this work. In most cases, there were two values for the experimental mixing ratio ( $\delta(\text{exp})$ ) of the  $\gamma$  transition in parentheses.

Nucleus	Cascade	$A_2^{\text{exp}}, A_4^{\text{exp}}$	$\delta(\text{exp})$
$^{101}\text{Zr}$	$9/2^+ \rightarrow 5/2^+ \rightarrow 3/2^+$ 310.0-(98.2)	-0.12(2), -0.00(4)	-0.16(9) -2.1(4)
	$7/2^- \rightarrow 5/2^+ \rightarrow 3/2^+$ 222.9-(98.2)	0.08(2), 0.02(3)	-0.13(10) -2.2(6)
$^{103}\text{Mo}$	$7/2^- \rightarrow 5/2^+ \rightarrow 3/2^+$ 251.4-(102.8)	0.09(1), 0.01(2)	-0.19(5) -1.9(2)
	$11/2^- \rightarrow 7/2^- \rightarrow 5/2^+$ 144.5-251.4	-0.054(13), 0.007(20)	<i>g</i> factor No mixing
	$13/2^- \rightarrow 9/2^- \rightarrow 7/2^-$ 372.3-(124.9)	-0.23(3) - 0.04(5)	-0.29 to -1.94 Single range
	$9/2^- \rightarrow 7/2^- \rightarrow 5/2^+$ (124.9)-251.4	+0.25(4) - 0.07(6)	-0.49 $^{+0.14}_{-0.22}$ -2.66 $^{+0.92}_{-1.40}$
$^{105}\text{Mo}$	$11/2^- \rightarrow 7/2^- \rightarrow 5/2^-$ 283.2-(95.3)	-0.12(1), 0.02(2)	-0.12(3) -2.9(5)
	$13/2^- \rightarrow 9/2^- \rightarrow 7/2^-$ 390.6-(138.3)	-0.17(1), -0.01(2)	-0.25(4) -2.3(2)
$^{107}\text{Mo}$	$9/2^- \rightarrow 7/2^+ \rightarrow 5/2^+$ 306.4-(152.1)	0.17(1), -0.00(3)	-0.44 to -1.44 Single range
	$11/2^+ \rightarrow 7/2^+ \rightarrow 5/2^+$ 414.5-(152.1)	-0.21(3), -0.01(4)	-0.31 to -1.76 Single range
	$9/2^- \rightarrow 7/2^- \rightarrow 5/2^+$ (110.2)-348.3	0.14(3), 0.06(4)	-0.18(9) -0.0012 $^{+0.057}_{-0.0001}$
$^{109}\text{Ru}$	$11/2^+ \rightarrow 7/2^+ \rightarrow 5/2^+$ 472.8-(185.1)	-0.16(2), 0.01(3)	-0.25(6) -2.0(3)
	$13/2^+ \rightarrow 9/2^+ \rightarrow 7/2^+$ 540.7-(222.7)	-0.20(3), -0.03(4)	-0.35 $^{+0.09}_{-0.12}$ -1.8(4)
$^{111}\text{Ru}$	$9/2^- \rightarrow 7/2^+ \rightarrow 5/2^+$ 166.6-(150.2)	0.15(2), 0.03(2)	-1.29 $^{+0.94}_{-0.34}$ -0.48(5)
	$7/2^- \rightarrow 7/2^+ \rightarrow 5/2^+$ 103.8-(150.2)	-0.37(1), -0.00(2)	-0.32(2) -1.73(9)
	$11/2^+ \rightarrow 7/2^+ \rightarrow 5/2^+$ 431.3-(150.2)	-0.22(1), -0.03(4)	-1.23(16) -0.51(7)

correlation with the pure  $E2$ ,  $9/2^+ - 5/2^+$  310.0 keV transition and its correlation with the pure  $E1$ ,  $7/2^- - 5/2^+$  222.9-keV transition. The data for the 310.0- to 98.2-keV cascade are shown in Fig. 3. The correlations are unattenuated because the known lifetime of the 98.2-keV state, 0.6(2) ns, its measured  $g$  factor 0.05(3) [2] and the hyperfine field,  $B_{\text{HF}}(\text{Zr Fe})$  27.4 T [11] gives a value for the mean precession angle in this state  $\phi \sim 0.04$  rad.

The mixing ratio values of the 98.2-keV transition extracted from the data for the two cascades given in Table I, are in good agreement, with an average (lower) result  $\delta(E2/M1)_{98.2} =$

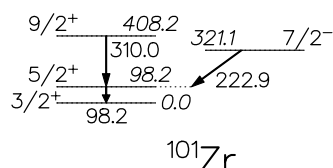


FIG. 2. Partial level scheme of  $^{101}\text{Zr}$  [10].

-0.15(6) that is fully consistent with the previous published result -0.11(4) [2].

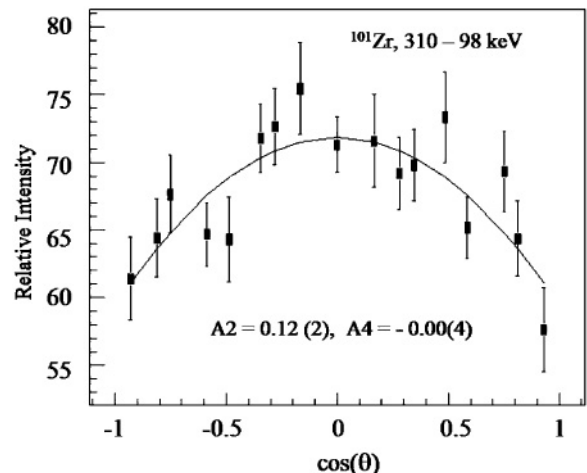


FIG. 3. The 310.0- to 98.2-keV angular correlation in  $^{101}\text{Zr}$ .

TABLE II. Selected experimental mixing ratios,  $\delta(\text{exp})^S$ , for  $\Delta I = 1$  transitions within rotational bands of odd- $A$  nuclei measured in this work. The experimental values are compared to the predictions of the particle-axial-rotor model for various single-particle states.

Nucleus	Transition (keV)	Orbital	$Q_0$	$\delta$ (calc)	$\delta$ (exp) <sup>S</sup>
<sup>101</sup> Zr	98.2	3/2[411]	2.84	-0.13	-0.15(6)
		3/2[422]	2.84	0.44	
<sup>103</sup> Mo	102.8	3/2[402]	2.84	0.22	
		3/2[411]	3.02	-0.15	-0.19(5)
		3/2[422]	3.02	0.50	
<sup>105</sup> Mo	124.9	5/2[532]	3.02	-0.15	-0.49 <sup>+0.14</sup> <sub>-0.22</sub>
		5/2[532]	3.06	-0.15	-0.12(3)
		5/2[532]	3.06	-0.17	-0.25(4)
<sup>107</sup> Mo	152.1	5/2[413]	3.09	0.79	
		5/2[402]	3.09	-0.20	-1.0(7)
<sup>109</sup> Ru	110.2	7/2[523]	3.09	-0.16	-0.18(9)
		5/2[413]	3.28	1.07	
<sup>109</sup> Ru	185.1	5/2[402]	3.28	-0.26	-0.25(6)
		5/2[413]	3.28	0.98	
		5/2[402]	3.28	-0.24	-0.35 <sup>+0.09</sup> <sub>-0.12</sub>
<sup>111</sup> Ru	150.2	5/2[413]	3.32	0.85	
		5/2[402]	3.32	-0.21	-0.32(2)
		5/2[402]	3.32	-0.21	-0.48(5)
					-0.51(7)

**B. <sup>103,105,107</sup>Mo**

The partial level schemes of <sup>103,105,107</sup>Mo are shown in Fig. 4.

**I. <sup>103</sup>Mo**

The ground-state band in <sup>103</sup>Mo has been assigned to the 3/2[411] configuration, and an excited band beginning at 354.2 keV has been assigned the configuration 5/2[532] [10]. Recent work verified both these assignments through  $g$  factor and mixing ratio measurements [2]. Here these assignments are further checked.

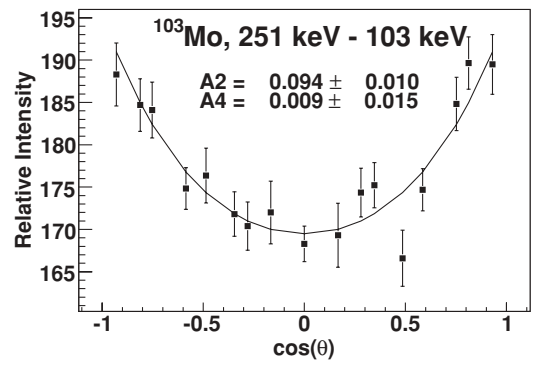


FIG. 5. The 251.4- to 102.8-keV angular correlation in <sup>103</sup>Mo.

The  $g$  factor of the 102.8 keV,  $5/2^+$  state,  $g = 0.057(13)$  [2], combined with the lifetime  $\tau = 0.63(2)$  ns and the hyperfine field  $B_{\text{HF}}(\text{MoFe}) = 25.6$  T [11], give a predicted small mean precession angle,  $\phi \sim 0.05$  rad. Thus the correlation of the pure  $E1$ ,  $7/2^- - 5/2^+$ , 251.4 keV transition with the mixed  $E2/M1$ ,  $5/2^+ - 3/2^+$  102.8-keV transition is unattenuated. The mixing ratio  $\delta(E2/M1)_{102.8}$  extracted from the data on this correlation, shown in Fig. 5, is  $-0.19(5)$ , in reasonable agreement with the value  $-0.28(9)$  reported by Ref. [2].

The cascade from the  $11/2^-$  state at 498.1 keV through the bandhead 354.2 keV  $7/2^-$  state to the 102.8-keV  $5/2^+$  state involves a pure  $E2$  transition followed by a pure  $E1$  transition and does show attenuation. The predicted correlation coefficients in this case are  $A_2^{\text{calc}} = -0.071$  and  $A_4^{\text{calc}} = 0.000$ .

The measured coefficients are  $A_2^{\text{exp}} = -0.054(13)$  and  $A_4^{\text{exp}} = 0.007(20)$ , yielding an attenuation coefficient  $G_2 = 0.76(18)$ . Taking the hyperfine field to be  $B_{\text{HF}}(\text{MoFe}) = 25.60(1)$  [11] and the lifetime of the 354.2-keV state  $\tau(7/2^-) = 1.7(1)$  ns, the  $g$  factor of the bandhead is found to be  $g(7/2^-) = -0.21(13)$ . The sign, undetermined by the present method, has been adopted from Ref. [2]. This result is in general agreement with the previously reported value,  $g = -0.094(31)$  [2].

Access to the  $E2/M1$  mixing ratio of the 124.9-keV  $9/2^- - 7/2^-$  transition is given by two cascades as listed in Table I. Both indicate negative sign and the correlation with the pure  $E1$   $7/2^- - 5/2^+$  254.1-keV transition yields the result  $\delta_{124.9} = 0.49^{+0.14}_{-0.22}$ .

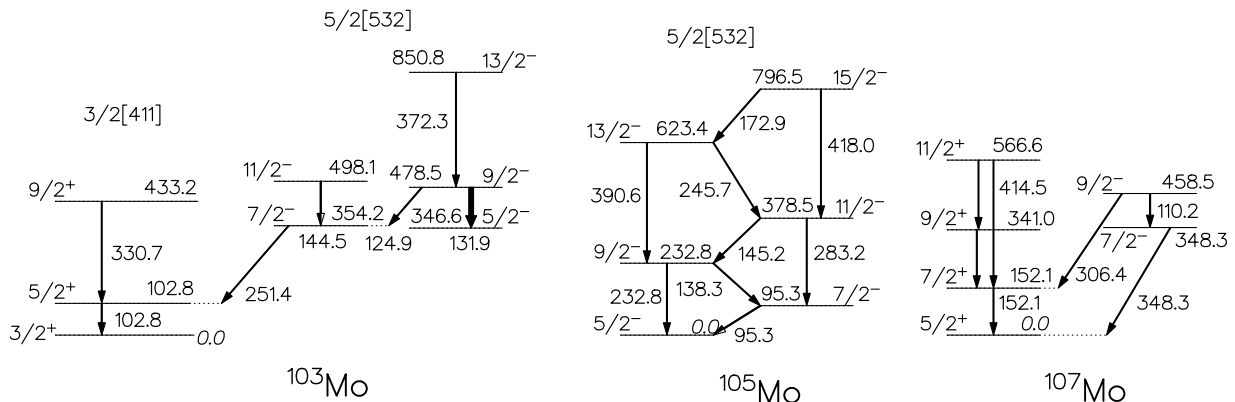


FIG. 4. Partial level schemes of <sup>103,105,107</sup>Mo [3,13].

2.  $^{105}\text{Mo}$ 

The two lowest rotational bands in  $^{105}\text{Mo}$  have been identified as the ground-state band built on the  $5/2[532]$  configuration (see Fig. 4) and a second band built on the  $3/2[411]$  configuration with bandhead at 246.3 keV [12]. Note that the sequence is reversed as compared to  $^{103}\text{Mo}$ , indicating evolution of the single-particle levels with changing shape.

Correlations in this work are used to check the assignment of the ground-state configuration. Two cascades have been analyzed as detailed in Table I and shown in Fig. 6.

The first, between the pure  $E2$ ,  $11/2^- - 7/2^-$ , 283.5-keV transition and the mixed  $E2/M1$ ,  $7/2^- - 5/2^-$ , 95.3-keV transition, yielded the mixing ratio of the latter,  $\delta_{95.3} = -0.12(3)$ . No attenuation is to be expected in the intermediate state at 95.3 keV because the lifetime,  $\tau = 0.63(3)$  ns,  $g$  factor  $g_{95.3} = -0.064(8)$  [2] and hyperfine field  $B_H F \text{ MoFe} = 25.6$  T combine to predict a precession  $\phi \approx 0.06$  rad. The second correlation, between the pure  $E2$ ,  $13/2^- - 9/2^-$ , 390.6-keV transition and the mixed  $E2/M1$ ,  $9/2^- - 7/2^-$ , 138.3-keV transition, yielded the mixing ratio,  $\delta_{138.5} = -0.25(4)$ . The lifetime of 0.16(2) ns and  $g$  factor of  $-0.03(13)$  [2] again predict negligible attenuation in the 232.8-keV intermediate state.

These two mixing ratio results may be combined to predict the correlation between the two mixed transitions 138.3–95.3 keV giving the values  $A_2 = +0.353$  and  $A_4 = +0.0002$ . These agree very closely with measured parameters for this cascade,  $A_2 = +0.33(1)$  and  $A_4 = +0.01(2)$ . The data for this strong correlation are shown in Fig. 7.

We note the similarity between this figure and Fig. 5 of Orlandi *et al.* [2]. It seems likely that the negative sign assigned by them to the  $A_2$  coefficient of this correlation is a typographical error because their values for the mixed transition ratios agree well with the present results.

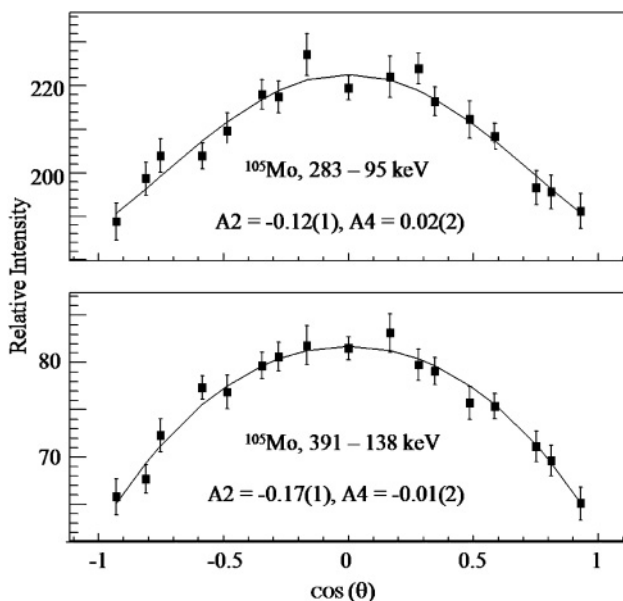


FIG. 6. The 283.2- to 95.3-keV and 390.6- to 138.3-keV angular correlations in  $^{105}\text{Mo}$ .

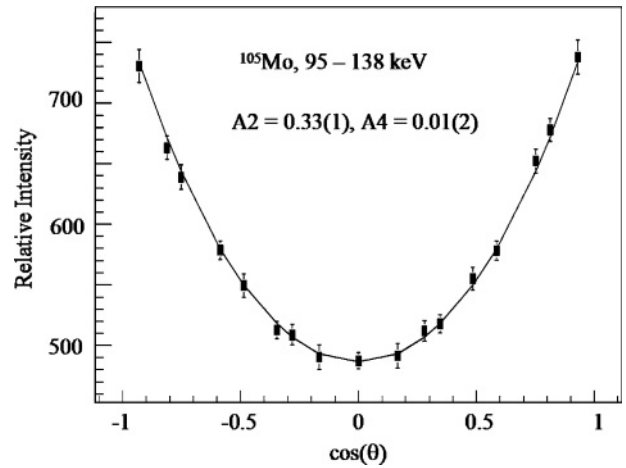


FIG. 7. The 95.3- to 138.3-keV angular correlation in  $^{105}\text{Mo}$ .

3.  $^{107}\text{Mo}$ 

For analysis of results in  $^{107}\text{Mo}$ , we adopt the spins and parities assigned by Urban *et al.* [3] rather than those of Hwang *et al.* [14] who did not have angular correlation information. Urban *et al.* identified the ground-state, positive-parity, band as built on the configuration  $5/2[402]$  based on measured properties of the band. They also made a somewhat more tentative assignment of the negative-parity band with bandhead at 348.3 keV as built on the  $7/2[523]$  configuration. With the aim of confirming these assignments three correlations in this isotope have been analyzed as detailed in Table I. Although in this nucleus there are no  $g$  factor data for the intermediate states involved, namely the  $7/2^+$  state at 152.1 keV and the  $7/2^-$  state at 348.3 keV, we assume negligible attenuation based on their general similarity to the levels in neighboring nuclei detailed above. The  $E2/M1$  mixing ratio of the 152.1-keV transition in the ground state band is extracted from two cascades involving the pure  $E1$ ,  $9/2^- - 7/2^+$ , 306.4-keV transition and the pure  $E2$ ,  $11/2^+ - 7/2^+$ , 414.5-keV transition, respectively. In both cases the values of  $A_2$  are considerably larger than those reported by [3] and yield a single range for the mixing ratio  $\delta$ . Data for the 414.5- to 152.1-keV cascade are shown in Fig. 8.

The mixing ratio in the  $9/2^- - 7/2^-$  110.2-keV transition in the negative parity band was evaluated from data on its cascade

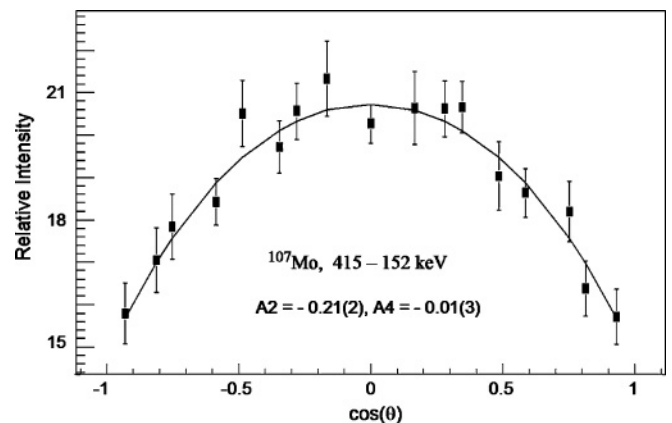


FIG. 8. The 414.5- to 152.1-keV angular correlation in  $^{107}\text{Mo}$ .

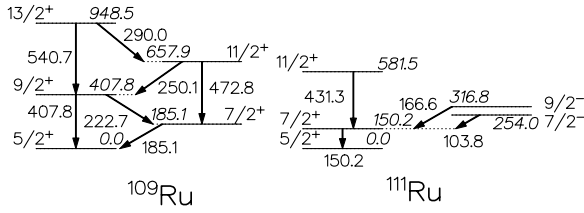


FIG. 9. Partial level schemes of  $^{109,111}\text{Ru}$  [13,16].

with the interband, pure  $E1$ ,  $7/2^- - 5/2^+$  348.3-keV transition, yielding the result  $\delta_{110.2} = -0.18(9)$ .

**C.  $^{109,111}\text{Ru}$**

The levels of interest in  $^{109,111}\text{Ru}$  are shown in Fig. 9. In both these isotopes Wu *et al.* [4] made tentative assignment of the single-particle configurations of the ground-state bands but were unable to distinguish between the  $5/2[413]$  and  $5/2[402]$  possibilities on the basis of their data. There is no direct evidence concerning possible attenuation of the correlations; however, the relatively large  $A_2$  coefficients extracted suggest that attenuation is insignificant.

**1.  $^{109}\text{Ru}$**

Two correlations in  $^{109}\text{Ru}$  have been analyzed involving pure  $E2$  transitions feeding the  $7/2^+$  and  $9/2^+$  members of the ground-state band, followed by the mixed transitions  $7/2^+ - 5/2^+$  at 185.1 keV and  $9/2^+ - 7/2^+$  at 222.7 keV, respectively. The two mixed transitions show closely similar mixing ratios,  $\delta_{185.1} = -0.25(6)$  and  $\delta_{222.7} = -0.35(11)$ . Data for the 472.8- to 222.7-keV cascade are shown in Fig. 10.

**2.  $^{111}\text{Ru}$**

Three correlations have been analyzed in  $^{111}\text{Ru}$ , each yielding a value for the  $E2/M1$  mixing ratio in the 150.2 keV,  $7/2^+ - 5/2^+$ , transition between the lowest levels of the ground-state band. The first cascade involves the pure  $E2$   $11/2^+ - 7/2^+$  431.3-keV transition in the ground-state band and the other two pure  $E1$  interband transitions at 103.8 and

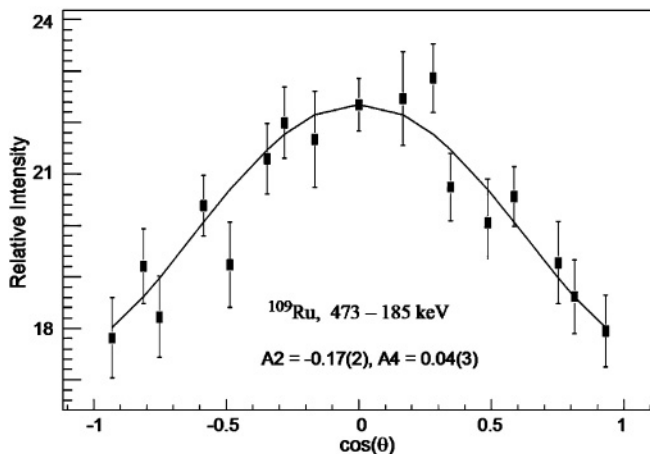


FIG. 10. The 472.8- to 185.1-keV angular correlation in  $^{109}\text{Ru}$ .

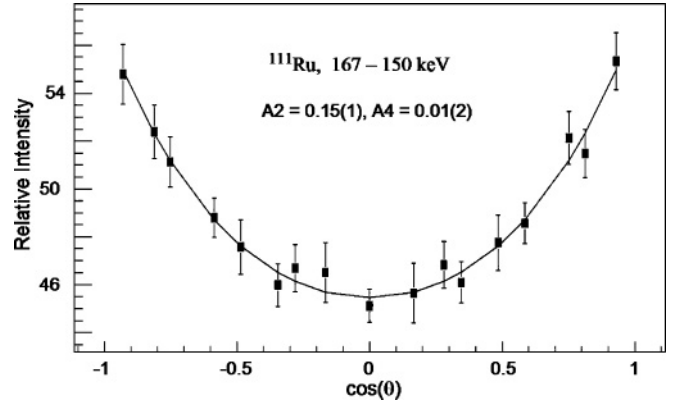


FIG. 11. The 166.6- to 150.2-keV angular correlation in  $^{111}\text{Ru}$ .

166.6 keV, respectively. Data for the 166.6- to 150.2-keV cascade are shown in Fig. 11. The extracted lower  $\delta$  values are generally consistent, being all negative between  $-0.3$  and  $-0.5$ .

**IV. DISCUSSION**

To interpret the mixing ratios of the  $\Delta I = 1$  transitions within rotational bands, the experimental values were compared to calculations made with a particle plus axial rotor model for various single-particle states as outlined in Sec. II. The results are shown in Table II. Although only an approximation for these nuclei, which are known to be triaxial [15], this model has been shown to reproduce their spectroscopic properties reasonably well [2].

Table II shows the mixing ratios calculated for several relevant single-particle states close to the Fermi surface in these isotopes. It is seen that the experimental mixing ratios, taken to be those having the stronger  $M1$  component of the two results in Table I, show good agreement with and can serve to identify, specific single-particle shell-model configurations in all instances. For example, the mixing ratio of the 102.8-keV transition in  $^{103}\text{Mo}$  is  $\delta_{102.8} = -0.19(5)$ , which is in close agreement with the calculated value  $-0.15$  for a  $3/2[411]$  level but disagrees strongly with model prediction for other nearby possibilities  $3/2[422]$  and  $3/2[402]$ . The Nilsson configurations of single-particle states determined in this work are shown in Fig. 12.

$5/2[532]$	$7/2[523]$	
$5/2^-$ 346.6	$7/2^-$ 348.3	
$3/2^+$ 0.0	$5/2^-$ 0.0	$5/2^+$ 0.0
$3/2[411]$	$5/2[532]$	$5/2[402]$
$^{103}\text{Mo}$	$^{105}\text{Mo}$	$^{107}\text{Mo}$
$3/2^+$ 0.0	$5/2^+$ 0.0	$5/2^+$ 0.0
$3/2[411]$	$5/2[402]$	$5/2[402]$
$^{101}\text{Zr}$	$^{109}\text{Ru}$	$^{111}\text{Ru}$

FIG. 12. The single-particle configurations determined in this work. Level energies can be obtained from Figs. 2, 4, and 9.

Orlandi *et al.* [2] established the  $3/2[411]$  assignment of the ground band of  $^{101}\text{Zr}$ , which is also supported in this work. They found the ground band in  $^{103}\text{Mo}$  to have the same configuration, which is also supported by this work. They further showed the negative parity  $5/2^-$  bandhead at 347 keV in  $^{103}\text{Mo}$  to have the  $5/2[532]$  configuration and identified this configuration as the ground-state band in  $^{105}\text{Mo}$ . The present results in  $^{105}\text{Mo}$  support this assignment.

In  $^{107}\text{Mo}$ , the configuration of the ground state was assigned by Urban *et al.* [3] to be  $5/2[413]$  based on the measurement of  $E2/M1$  branching ratios in the rotational band. Two independent values of the mixing ratio of the 152.1-keV transition between the lowest levels of this band in  $^{107}\text{Mo}$ , are reported in this work. They are both negative and, taking the stronger  $M1$  limit from the experimental fit ranges, give a  $\delta$  around  $-0.5$ , showing strong preference for the  $5/2[402]$  assignment over the  $5/2[413]$  for which the predicted  $\delta$  value is positive and close to unity. It would seem that Urban *et al.*, who obtained  $\delta$ , without sign, chose  $\delta$  positive contrary to the present direct sign measurement. For the negative-parity bandhead at 348.4 keV, the configuration was assigned  $7/2[523]$  [3], which is supported by the present result on mixing in the 110.2-keV transition.

In  $^{109,111}\text{Ru}$ , Wu *et al.* [4] attempted to determine the single-particle configurations of the ground-state bands based on  $E2/M1$  branching ratios but were not able to distinguish between the  $5/2[413]$  and  $5/2[402]$  configurations. In this work angular correlation data gives clear evidence that the ground states of  $^{109,111}\text{Ru}$  both have a  $5/2[402]$  configuration. Calculations for a  $5/2[402]$  state yield  $\delta = -0.26$  and  $\delta = -0.21$ , respectively, in excellent agreement with the measured

values of  $-0.25(6)$  and  $-0.32(2)$ , whereas calculations for the  $5/2[413]$  configuration yield large positive values.

## V. CONCLUSION

The data presented here further demonstrate the ability of  $E2/M1$  mixing ratios in intraband  $\Delta I = 1$  transitions to establish single-particle configurations of bandheads in nuclei in this region, with extension to Ru isotopes. Previous identifications of the ground-state, positive-parity, bandheads in  $^{101}\text{Zr}$  and  $^{103,107}\text{Mo}$  as based on the  $g_{7/2}$  subshell are confirmed. In addition, in  $^{103}\text{Mo}$  and  $^{107}\text{Mo}$  a negative-parity bandhead based on the  $h_{11/2}$  intruder orbital, assigned  $5/2[532]$  and  $7/2[523]$ , respectively, is seen at around 250–350 keV. However, in  $^{105}\text{Mo}$  the  $5/2[532]$  becomes the ground state. New results in  $^{109,111}\text{Ru}$  establish the ground-state band configurations as  $5/2[402]$ .

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