Temperature dependence of nuclear matter generalized isovector symmetry energy with Skyrme-type interactions

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The temperature dependence of the nuclear matter isovector symmetry energy coefficient ($\mathcal{A}_{0,1}$) is investigated in the framework of the generalized nuclear polarizability with Skyrme interactions, as worked out in previous articles [F. L. Braghin, Nucl. Phys. **A665**, 13 (2000); Phys Rev. C **71**, 064303 (2005); Int J. Mod. Phys. E **12**, 755 (2003)]. The variation of $\mathcal{A}_{0,1}(T)$ is very small (of the order of 1 MeV) for temperatures (T) in the range of 0 and 18 MeV. Different behaviors with temperature are found to strongly depend on the Skyrme parametrization, in particular at densities lower than the saturation density ρ_0.

I. INTRODUCTION

The neutron-proton nuclear symmetry energy is currently under intense theoretical and experimental investigations not only because of its relevance to different aspects of nuclear structure and dynamics but also because of its relevance to the description of dense stars' structure and of the Supernovae mechanism [1–9]. Actually, the possibility of extracting better experimental values for it at different densities and temperatures is under constant improvement. In this respect, (multi)fragmentation processes in heavy ion collisions provide experimental knowledge about the liquid-gas phase transition and therefore about the nuclear forces at low nuclear densities and excitation energies. For example, the isoscaling found in yields of multifragmentation experiments [8] depends directly on the isovector symmetry energy (coefficient), which was found to decrease considerably with the excitation energy according to experimental data [10]. In fact, under experimental conditions it is very difficult to extract unambiguous behavior with each of the thermodynamic observables involved (T, ρ) [10] and in earlier descriptions of experimental results some groups have considered a seemingly too strong variation of the symmetry energy with temperature [11]. In different (more recent) analysis it has been found that the strong decrease of the symmetry energy under experimental conditions should also be due to the excitation energy dependence of surface effects [12] and to the expansion of the system, which implies the lowering the total density [13,14]. In addition, recently De and Samaddar [15] have argued that the symmetry free energy [16] is the parameter that appears in the scaling of multifragmentation. However, their arguments stand for the specific analysis of multifragmentation processes and we intend to discuss rather the behavior of the symmetry energy coefficient. Concerning the specific dependence on the temperature (up to $T \simeq 20$ MeV) there are several theoretical indications of very weak decrease [16–20]; although in some works, for finite nuclei and nuclear matter, a small (or very small) increase was found [1,18,21]. In some of these works no unique tendency was found for different finite nuclei, for example, in Ref. [18]. In Ref. [1] few preliminary numerical results showed a very small variation of the nuclear matter symmetry energy coefficient with temperature using Skyrme forces. In the present work we perform a quite extensive investigation of this subject with the generalized polarizability as proposed in Refs. [1] and [13].

In usual mass formulas the binding energy per nucleon depends on the n-p asymmetry with the following form:

$$\frac{E}{A} = \frac{H_0(A, Z)}{A} + a_\tau \frac{(N - Z)^2}{A^2} + \cdots, \quad (1)$$

where N, Z, and A are the neutron, proton, and mass numbers and the isovector symmetry energy coefficient (s.e.c.) is a_τ. This coefficient is a measure of the energy needed to increase n-p asymmetry. Different powers of the n-p asymmetry $(N - Z)^j/A$ ($j \neq 2$) are much smaller, although the n-p asymmetry modifies many other terms [22]. The value of the n-p s.e.c. in fits of the mass formulas for stable nuclei is quite well known [2]. For an infinite medium, roughly speaking, nucleon numbers (N, Z) might be replaced by the corresponding densities ρ_n, ρ_p for many purposes depending on the volume occupied by each nucleon species (see, for example, Ref. [23]).

A particularly interesting way of obtaining the symmetry energy coefficient, eventually under different conditions, was found by using the nuclear isovector polarizability $\mathcal{A}_{0,1}$ [24,25]. Generalized nuclear polarizabilities and their dependencies on several parameters simultaneously (such as total density ρ, exchanged momentum and energy q, ω, and neutron-proton asymmetry $b = \frac{\rho_n}{\rho_p} - 1$) were investigated quite extensively with Skyrme-type forces [13,25–27]. Deviations from the quadratic form of expression (1) might appear depending on the particular density fluctuations considered for a given set of nucleon densities for these calculations with Skyrme density functionals. Recent investigations have revealed further relations of these (nonrelativistic) effective interactions with relativistic models [28,29].

This work exhibits the temperature dependence of the isovector generalized s.e.c., as proposed in Refs. [1] and [13] using Skyrme type forces. For that, the generalized isovector screening function with different parametrizations of Skyrme

effective interactions are shown at different densities and n-p asymmetries.

II. GENERALIZED POLARIZABILITIES

For an asymmetric medium in n-p densities, the generalized screening function using Skyrme forces, for zero energy and momentum exchange ($\omega = q = 0$), can be written in a compact notation as [1,13]

$$\mathcal{A}_{s,t} = \frac{\rho}{2N} \Big\{ 1 + 2\overline{V_0^{(s,t)}} N_c + 6V_1^{(s,t)} M_p^* (\rho_c + \rho_d)$$
$$+ 12 M_p^* V_1^{(s,t)} \overline{V_0^{(s,t)}} (N_c \rho_d - \rho_c N_d)$$
$$+ \big(V_1^{(s,t)}\big)^2 (36(M_p^*)^2 \rho_c \rho_d - 16 M_p^* M_c N_d) \Big\}, \quad (2)$$

where $\overline{V_0^{(s,t)}}$ and $V_1^{(s,t)}$ are functions of the Skyrme forces parameters in a given channel of the effective nuclear interaction of (spin, isospin) denoted by superscripts (s,t). Therefore they carry the main contributions of the effective NN interaction, distinguishing each of the channels of the particle-hole interaction. In particular, the function $V_1^{(0,1)} = (t_2(1 + 2x_2) - t_1(1 + 2x_1))/16$ is a combination of (momentum dependent) Skyrme parameters that contributes to the usual nucleon effective masses in the framework of the Skyrme calculation, $m_{n,p}^*$. The effective masses of neutron and protons are functions of the total density and of the neutron and proton densities (see, for example, Ref. [24]). However, the function $V_0^{(0,1)}$ depends mainly on t_0 and t_3. While $V_0^{(0,1)}$ of each of the Skyrme parametrizations used in this work do not have meaningful different values, the function $V_1^{(0,1)}$ has considerably different values because it might be zero, i.e., for some forces $V_1^{(0,1)} = 0$. Although relevant, these differences will be shown to not be enough to provide very different behaviors for $\mathcal{A}_{s,t}(T)$. The mixed functions ρ_v, M_v, and N_v reduce to densities of the Skyrme-Hartree-Fock approach at zero temperature, being respectively the nucleon and kinetic energy densities and the densities of states. They are the zero frequency and zero momentum generalized Lindhard functions calculated in Ref. [1]. The total densities ρ, N are written without any index. These functions (ρ_v, M_v, N_v) are given, respectively, by

$$\rho_v = v\rho_n + (1-v)\rho_p,$$
$$M_v = vM_n + (1-v)M_p, \quad (3)$$
$$N_v = vN_n + (1-v)N_p.$$

In these expressions v stands for two different n-p asymmetry coefficients (c, d), being that we made use of the following asymmetry coefficients:

$$a = \frac{m_p^*}{m_n^*} - 1, \quad b = \frac{\rho_n}{\rho_p} - 1, \quad c = \frac{1+b}{2+b},$$
$$d = \frac{1}{1 + (1+b)^{\frac{2}{3}}}.$$

Therefore, by fixing the parameter of density asymmetry b (for a given total nuclear density) the neutron and proton densities

are found as well as the other asymmetry parameters (a, c, d) for a given Skyrme force [1,13].

There is a further mass parameter in Eq. (2), M_p^*, which is in fact a kind of reduced mass. It is given by

$$M_p^* \equiv \frac{m_p^*}{(1 + \frac{a}{2})} = \frac{2m_p^* m_n^*}{m_p^* + m_n^*}.$$

Although the calculation has been carried out in such a way to provide final expressions for each of the channels of the particle-hole interaction (isovector, spin, spin-isovector, and scalar), only the n-p one is investigated in this article.

A. Varying temperature

The densities N_α, ρ_α, and M_α (for $\alpha = n$, p, neutrons and protons) are the basic input for the temperature dependence of the polarizabilities. At finite temperature these functions are given, respectively, by integrals written as

$$(N_\alpha, \ 3\rho_\alpha, \ 4M_\alpha) = -\frac{1}{\pi^2} \int df_\alpha(k) \big(k.m_\alpha^*; \ k^3; \ k^5\big). \quad (4)$$

In these expressions, $df_\alpha(k)$ is the measure of integration in terms of the usual free fermion occupation numbers $f_\alpha(k)$ for neutrons and protons ($\alpha = n$, p). At $T = 0$ for the usual Fermi occupation number we have

$$df_\alpha(k) = -\delta\big(k - k_F^{(\alpha)}\big)dk, \quad (5)$$

where $k_F^{(\alpha)}$ is the Fermi momentum for each of the nucleon species. In this case the integration is trivial. Therefore all the temperature dependence of the density-like quantities ρ, N, M is encapsulated in the integrals above (4), which can, at most, yield smoother results for the integral. Furthermore $N_v(T)$ and $M_v(T)$ are the only parameters that vary with temperature because ρ_α are kept constant. The zero temperature limit was considered previously [1,13] and it shows more explicitly, as mentioned above, the effect of each of the Skyrme force parameters through the functions $V_0^{(s,t)}$ and $V_1^{(s,t)}$ as well as the effective masses. This issue is extremely relevant for the resulting $\mathcal{A}_{0,1}(T)$.

From the general expression in Eq. (2) a useful (simplified) limit is recovered in which the behavior with temperature can be understood in detail. For instance, consider the limit in which the function $V_1^{(0,1)}$ appears only in the leading order of the symmetric n-p function. This is achieved with $b = a = 0$ and $d = c = 1/2$, yielding $m^* = m_n^* = m_p^* = M_p^*$ and $\rho_p = \rho_n = \rho/2$. We obtain an expression of the following form, for $(s,t) = (0,1)$, in the n-p symmetric limit:

$$\mathcal{A}_{0,1} \to a_\tau = \frac{\rho}{2} \left(\frac{1}{N} + 2V_0^{(0,1)} + 6m^* V_1^{(0,1)} \frac{\rho}{N} \right) + \text{h.o.}, \quad (6)$$

where h.o. stands for the higher order terms in $V_1^{(0,1)}$. This expression reproduces exactly a_τ, which is the usual symmetry energy coefficient in the Skyrme-Hartree-Fock approximation [24]. The first and third terms of a_τ depend on N and therefore they show a very small variation with temperature only due to $N(T)$ according to Expression (4). The behavior of the function $N_\alpha(T)$ is monotonic and it decreases with T. This

is the main feature for understanding the numerical results of Expression (2).

The higher order terms, in the limit of n-p symmetric matter, are given by

$$\text{h.o.} \rightarrow \left(V_1^{(0,1)}\right)^2 (9(m^*)^2\rho^2 - 8m^*MN)\frac{\rho}{2N}. \qquad (7)$$

In these terms, and mainly for n-p asymmetric matter [in the complete expression (2)], the imbalance between the T dependence of $M_v(T)$ and $N_v(T)$ determines whether the increasing behavior of $N(T)$ with temperature is the leading one or not. Basically this is seen from the (overall) denominator of Expression (2) by reminding that $N(T)$ is a decreasing function of the temperature in the Skyrme-Hartree-Fock level and $M_v(T)$ is an increasing function of the temperature. The different resulting behaviors might appear because of the relative values of their coefficients, i.e., $V_0^{(0,1)}$ and $V_1^{(0,1)}$. This is noticed in the results exhibited in the next section. Although it might be expected that for asymmetric n-p matter the variation of the polarizability is larger because there are more terms at work, this will be shown to be not really sizable for the forces considered in this work. The complete expression is quite complicated such that it might not exhibit a simple and unique behavior in more general situations.

It is worth mentioning some recent results claiming that the symmetry free energy is the quantity that really rules multifragmentation [15,16,30]. Basically this corresponds to considering the entropic contribution, which amounts basically to extra additive terms. This can be qualitatively seen as follows. The calculation of the polarizability in such a case should depart from a free energy in the presence of a n-p asymmetry and of an infinitesimal external source (ε) that induces fluctuations of the densities of neutrons from protons; i.e., $\mathcal{F}(\rho + \delta\rho_{np}) = \mathcal{E}(\rho + \delta\rho_{np}) - TS(\rho + \delta\rho_{np}) + \epsilon\delta\rho_{np}$, where $\delta\rho_{np} = \delta\rho_n - \delta\rho_p$. The entropy can be expanded in terms of $\rho_n - \rho_p$ to make explicit its contribution to the symmetry (free) energy: $\mathcal{F}(T) = \mathcal{E}_0(T) + a_\tau(\delta\rho_{np})^2 + S^{(1)}(T)\delta\rho_{np} - S^{(2)}(T)(\delta\rho_{np})^2 + \epsilon\delta\rho_{np} + \cdots$, where $S^{(i)}(T)$ are the leading contributions of the entropy for the symmetry free energy. As it was shown in Ref. [13], the linear term in $\delta\rho_{np}$ might be incorporated into the usual calculation (considering only the quadratic terms) and it is not considered explicitly below. The polarizability is then given by

$$\Pi \equiv \frac{\delta\rho_{np}}{\epsilon} = -\frac{\rho}{2(a_\tau - S^{(2)})}. \qquad (8)$$

The final symmetry free energy coefficient can be written as $a_\tau^f(T) = a_\tau(T) - S^{(2)}(T)$. Therefore we can expect that the entropic contribution would appear mainly as additive terms for the screening function. This is seen in the results of Refs. [15,16,30]. However, a microscopic investigation of this quantity, with its eventual relevance for the multifragmentation processes, is outside the scope of the present work.

B. Results

The temperature dependence of the isovector polarizability $\mathcal{A}_{0,1}(T)$ is shown in Figs. 1 to 4 for the following Skyrme forces: SGII from Ref. [31], SLyb from Ref. [32] (which
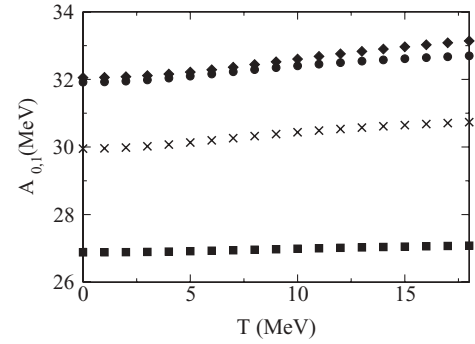

FIG. 1. The inverse of isovector polarizability, $\mathcal{A}_{0,1}$, as a function of the temperature for different Skyrme forces: SkSC 4 [33] (circles), SGII [31] (squares), SLyb [32] (diamonds), SkSC 6 [33] ($\times$ or $+$). For: ($\rho/\rho_0 = 1$ and $b = 0$).

is sometimes referred to as SLy4 in the literature), and two parametrizations SkCS4 and SkSC6 from Ref. [33]. These last two Skyrme parametrizations have a slightly more intricate density dependence, although the resulting functions $V_1^{(0,1)}$ are zero. The zero function $V_1^{(0,1)} = 0$ (which carries the main part of the momentum dependent Skyrme forces) brings a lot of simplification to the dependence on the temperature as discussed in the last section and it is noticed in the figures below. In these cases the behavior of $\mathcal{A}_{s,t}(T)$ is always monotonic depending on $N(T)$ and slightly less on $V_0^{(0,1)}$. Different total densities and n-p density asymmetries are also considered.

In Fig. 1 the function $\mathcal{A}_{0,1}(T)$ is shown at the saturation density, $\rho = \rho_0$, and zero n-p density asymmetry ($b = 0$) with the following Skyrme parametrizations: SkSC4 (circles) [33], SkSC6 ($\times$) [33], SGII (squares) [31], and SLyb (diamonds) [32]. The variation with temperature (up to $T \simeq 18$ MeV) is quite small, reaching $\Delta\mathcal{A}_{0,1} \simeq 0.5$–$1.0$ MeV depending on the interaction and even nearly zero for the SGII force. The slope is always positive, although smaller at high temperatures. As noticed after Expression (6), the function $N(T)$ decreases with temperature within the Skyrme-Hartree-Fock approach. The force SLyb is the one with larger variation in $\mathcal{A}_{0,1}(T)$. This trend of small variation was found before [1], although
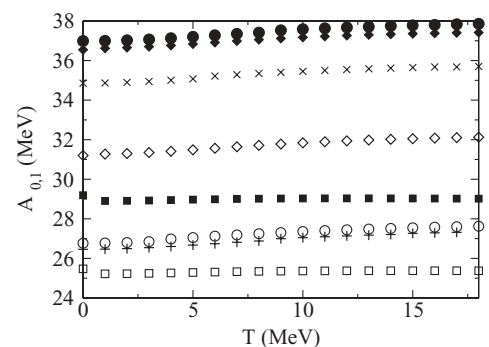

FIG. 2. The inverse of isovector polarizability, $\mathcal{A}_{0,1}$, as a function of the temperature for the same Skyrme forces as in figure 1. Considering: full symbols and symbol ($\times$) for ($\rho/\rho_0 = 0.75$ and $b = 0.25$) and empty symbols and $+$ for ($\rho/\rho_0 = 0.75$ and $b = 0$.).
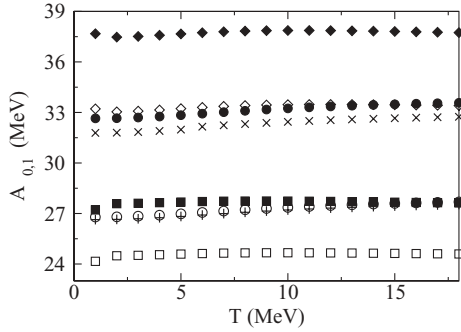
FIG. 3. The inverse of isovector polarizability, $\mathcal{A}_{0,1}$, as a function of the temperature for the same Skyrme forces as figure 1. Considering: full symbols and symbol × for ($\rho/\rho_0 = 0.5$ and $b = 0.5$) and empty symbols and + for ($\rho/\rho_0 = 0.5$ and $b = 0.25$).

it disagrees with the small decrease of the s.e.c. with T found in different works for lower densities [16–21]. However, it is also worth pointing out that in these references the s.e.c. was investigated in the regime of very low total density. The variation found for the range of $T = 0$ up to $T \simeq 15$ MeV is not large in all these works, and it can be of the order of 1 up to 3 MeV (though negative), a little bit larger than the present results.

Some further remarks to understand the behavior with temperature are in order. The chemical potential fixes the nucleonic density, which is kept constant for all temperatures. In the calculations with Skyrme interactions, the kinetic part of a_τ and the terms with $N(T)$, $M(T)$ in Expression (2) are temperature-dependent. In particular whereas the functions N_α slightly decrease with the temperature, the densities M_α increase slightly. Having this in mind and analyzing Expression (2) we can expect that n-p asymmetry (b, d, $c \neq 0$) favors different behaviors of $\mathcal{A}_{0,1}(T)$. The relative variation of the potential energy part of the symmetry energy at the Hartree Fock level is nearly zero because it depends mostly on ρ at not very low densities.

The same Skyrme parametrizations (and symbols) are used in Fig. 2. In this figure, $\mathcal{A}_{0,1}(T)$ is exhibited for $\rho = 0.75\rho_0$ with $b = 0.25$ (solid symbols and ×) and $b = 0$ (open symbols and +). For these cases (of lower nuclear matter
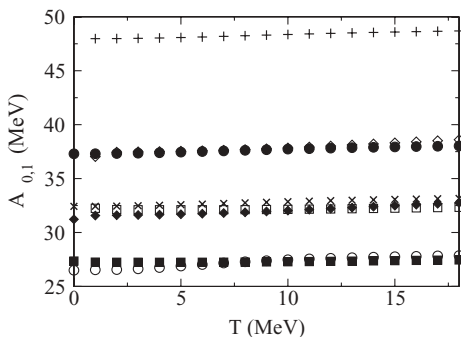

FIG. 4. The inverse of isovector polarizability, $\mathcal{A}_{0,1}$, as a function of the temperature for the same Skyrme forces as figure 1. Considering: full symbols and symbol × for ($\rho/\rho_0 = 1.33$ and $b = 0$.) and empty symbols and + for ($\rho/\rho_0 = 1.33$ and $b = 0.25$).

density), the behavior of the polarizability with temperature is nonmonotonic in the case of force SGII (squares). For Skyrme forces SkSC4, SkSC6, and SLy, the polarizability $\mathcal{A}_{0,1}(T)$ is nearly constant at very low temperatures and it starts increasing smoothly around $T \simeq 5$ MeV until $T \simeq 10$–15 MeV. For the force SGII, $\mathcal{A}_{0,1}$ decreases for low temperatures and smoothly increases for temperatures higher than nearly 3–5 MeV. By comparing the relative variation of the polarizability for n-p symmetric and for $b = 0.25$ n-p asymmetric matter, we find no further meaningful difference. It is worth emphasizing that the lower total density makes possible this nonmonotonic behavior. This is produced by the different behaviors with temperature of the functions $N_\alpha(T)$ and $M_\alpha(T)$ for given $V_0^{(0,1)}$ and $V_1^{(0,1)}$. This becomes clearer in Fig. 3, where $\rho = 0.5\rho_0$. As noticed above, the n-p asymmetry might amplify this nonmonotonic behavior although the difference is very small in the cases we show.

The same Skyrme parametrizations (and symbols) are used in Fig. 3, where $\mathcal{A}_{0,1}(T)$ is exhibited for the still lower density $\rho = 0.5\rho_0$ with $b = 0.5$ (solid symbols and ×) and $b = 0.25$ (open symbols and +). Different from all the results shown above the only Skyrme force that exhibits the nonmonotonic behavior is SLyb for both $b = 0.5$ and $b = 0.25$. Furthermore we notice that the stronger variation for all the forces (even if they are very small of the order of .5 MeV) occurs below $T \simeq 5$ MeV or $T \simeq 10$ MeV. Above these temperatures, the isovector polarizability variation is smaller. The reason why the SGII and SLyb forces have the nonmonotonic behavior at different densities is explained by the different values of the functions $V_0^{(0,1)}$, in terms of the t_0, t_3 Skyrme parameters [1,13], and also $V_1^{(0,1)}$ (nonzero).

For the range of lower nuclear densities some further conclusions can be extracted by comparing the Figs. 2 and 3, in particular for $b = .25$, (which means $\rho_n = 1.25\rho_p$). We notice that the behavior of $\mathcal{A}_{0,1}(T)$ is different depending on the effective force. While the parametrizations SkSC 4 and SkSC 6 do not provide any different behavior [apart from an eventual overall total variation of $\mathcal{A}_{0,1}(T)$], the forces SGII and SLyb present different trends for $\rho = .5\rho_0$ (Fig. 3: open squares, SGII; open diamonds, SLyb) and $\rho = .75\rho_0$ (Fig. 2: solid squares and solid diamonds, respectively). The nonmonotonic behavior appears for the SGII parametrization at $\rho = .75\rho_0$ whereas for the SLyb one it appears when $\rho = .5\rho_0$. As discussed in the beginning of the last section, because of the complicated form of Expression (2) and of the behavior of the functions $N(T)$ and $M(T)$, the results from the polarizabilities with Skyrme forces are not always monotonic. A suitable quantity for comparing the results from different forces, and even different methods, is the total variation $\Delta A_{0,1} = A_{0,1}(T = 20) - A_{0,1}(T = 0)$. This quantity is (quite) small in all the works of the field, with small differences also due to the particular nuclear density under consideration being also seen also in other works using different approaches [14,17,19,20] as analyzed in Ref. [26]. One of the main outcomes of these comparisons goes along with the above remarks: the behavior of $\mathcal{A}_{0,1}(T)$ might be different at ρ_0, $0.75\rho_0$, and $0.5\rho_0$, eventually for still lower densities analyzed in other works.

In Fig. 4, $\mathcal{A}_{0,1}(T)$ is exhibited for a density higher than ρ_0, i.e., $\rho = 1.33\rho_0$ with $b = 0$ (solid symbols and $\times$) and $b = 0.25$ (open symbols and $+$). The same kind of behavior found for $\rho = \rho_0$ in Fig. 1 is present in Fig. 4. The isovector polarizability very smoothly increases with temperature, although the variation is considerably smaller at high temperatures.

This analysis suggests that the T dependence of the s.e.c. is strongly dependent on the nuclear matter density ρ, and it is also suitable for shedding light on the nuclear effective interactions expected to be reliable with good predictive power. Nevertheless we emphasize that experimental data with temperature are very difficult to extract unambiguously [10].

III. SUMMARY

To summarize we conclude that the bulk isovector symmetry energy does not vary considerably in a quite wide range of temperatures within the isovector polarizability with Skyrme forces. As noticed in other works the dependence of $\mathcal{A}_{0,1}$ on the n-p density asymmetry is probably too strong. This issue, however, does not modify the variation with the temperature meaningfully. In the framework of the Skyrme-Hartree-Fock parametrization, temperature effects arise from the functions M and N given by Expression (4). They depend on the general properties of the Skyrme-Hartree-Fock approach. The variation of $\mathcal{A}_{0,1}(T)$ depends strongly on the particular Skyrme force, being always very smooth and small. The different contributions of the potential and kinetic parts of the symmetry energy for fixed densities, by means of the functions $N(T)$ and $M(T)$, as well as the relative values of $V_0^{(0,1)}$ and $V_1^{(0,1)}$, are responsible for these different results of each Skyrme interaction. The variation (decrease) of $N_\alpha(T)$ (which reduces to the n, p densities of states at zero temperature) with temperature is, however, the most relevant contribution for the results. The larger variation of $A_{0,1}(T)$ occurs for $\rho < \rho_0$, depending strongly on the effective force parametrization (for which $V_1^{(0,1)} \neq 0$, i.e., SGII and SLyb). The final behavior is not always monotonic with temperature. The differences in the overall variation of $\mathcal{A}_{0,1}(T)$ for different densities below ρ_0 are seen also in other works using different approaches [14,17,19,20] as pointed out and compared in Ref. [26]. The trends exhibited by the Skyrme parametrizations suggest that the eventual experimental knowledge of the behavior of the symmetry energy with the temperature will also contribute to better fine-tuning of the effective interaction as well as to improving its predictive power. This is clearer in the comparison between Figs. 2 and 3 (mainly for Skyrme forces SGII and SLyb).

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