Self-consistent extension of random-phase approximation enlarged beyond particle-hole configurations

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We present a new extension of the random-phase approximation method: the quasiboson approximation is avoided and correlations are included in the ground state without resorting to renormalized operators or renormalized matrix elements; the configuration space is enlarged by considering also elementary excitations corresponding to the annihilation of a particle (hole) and the creation of another particle (hole) on the correlated ground state, together with the particle-hole ones. Two new and relevant advantages of this method with respect to the existing extensions of random-phase approximation are highlighted: (i) the energy weighted sum rules are completely satisfied; (ii) the problem of the existence of nonphysical states in the spectrum, related to the inclusion of particle-particle and hole-hole configurations, is solved: a way to unambiguously disentangle physical from nonphysical states in the excitation spectrum is presented. The method is applied here to a three-level Lipkin model where its quality can be judged by comparing with the exact results. Both advantages (i) and (ii) shall lead to feasible future applications of this extended RPA to several realistic cases.

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I. INTRODUCTION

One of the common features characterizing finite-size many-body systems is the existence of collective modes of excitations. Low-lying and giant resonances in nuclei [1], dipole plasmons in clusters [2], and breathing modes in atomic gases [3] are some examples of many-particle coherent motion generating a collective excitation mode. The microscopic approach that is most currently used to analyze collective excitations in many-body systems is the random-phase approximation (RPA) [4], where the modes are described as superpositions of particle-hole and hole-particle individual configurations. This method is extensively used in different domains and, in general, successfully describes the energies and transition probabilities associated to the excitations.

In its standard version, it is a single-reference approach [5], where the reference state is the uncorrelated Hartree-Fock (HF) Slater determinant characterized by 1 and 0 occupation numbers in the single-particle states that construct it (quasiboson approximation (QBA) [4] based approach). The replacement of the correlated ground state with the HF one is justified only if ground state correlations are not too strong, otherwise a better and, if possible self-consistent, treatment of these correlations become important. For example, it has been shown in the context of metal clusters that correlations in the ground state can be quite strong and induce important deviations from 1 and 0 in the occupation numbers of hole and particle states, respectively [6]. On the other side, in nuclear physics, new measurements on weakly bound nuclei start revealing relevant changes in shell structure far from stability [7] and novel properties such as, for example, the existence of pygmy excited modes [8]. The behavior of exotic nuclei presents a more complex scenario with respect to the physics of stable nuclei and the introduction of correlations beyond standard mean field approaches may be expected to provide an important contribution in this case.

One possible direction is the use of a multireference approach [5]: the generator coordinate method (GCM) [9], for instance, is a multireference approach where the reference state is no longer a single state but a superposition of wave functions associated to some collective coordinates. Another possible way to explicitly introduce correlations in the ground state consists in constructing a RPA-like formalism where the violations of the Pauli principle related to the use of the QBA are cured. Two main lines have been developed in the past decades using either boson expansion methods [10] or extensions in the fermionic space [11]. Many of these approaches are based on the so-called renormalized RPA (RRPA) method starting from the early works of Hara and Rowe in the 1960s [12]. In all these RRPA models: (i) the ground state is explicitly correlated with occupation numbers different from 1 and 0 appearing in some renormalization factors; (ii) the Pauli principle is satisfied because the QBA is not adopted.

A very important feature of standard RPA is that it preserves energy weighted sum rules (EWSR) [9]. This property guarantees that spurious excitations corresponding to broken symmetries (as, for example, the translational invariance) separate out and are orthogonal to the physical states. On the contrary, the common limitation of all the approaches aimed to overcome QBA is that EWSR are not satisfied [13] and can be strongly violated (up to 20–30% in some cases [14]). Such violations are related to the fact that only particle-hole excitations are considered in such approaches.

An attempt to overcome this problem has been discussed in Ref. [15]. A RRPA approach has been introduced where particle-particle and hole-hole configurations have been added to the standard particle-hole configurations. The method has

been applied to a three-level Lipkin model [16] with one hole and two particle levels and it has been shown that the EWSR result exactly satisfied.

However, a new problem arises related to the existence of a so called 'spurious' mode that does not correspond to any level in the exact excitation spectrum of the model. In this work, we will call this mode more properly 'additional' or 'nonphysical' instead of 'spurious' because it does not actually correspond to any broken symmetry. Since the transition probability associated to this mode becomes non-negligible at some value of the interaction strength in the Hamiltonian [15], an application of this method to a realistic case would result as unfeasible: various additional modes would be generated and it would be impossible to isolate them from the physical spectrum by simply looking at their transition probabilities.

The appearance of these nonphysical states can be traced back to the use of the enlarged configuration space that, from the other hand, allows to exactly preserve EWSR (see also [17]). Although, in some realistic calculations carried out in extended RPA approaches, it has been discussed that the transition probabilities associated to these modes are quite small (see for example [18]), an approach that allows to identify *a priori* these modes should be very useful.

In this work, we present a new and more general method with respect to that of Ref. [15], where the renormalization scheme is not adopted and, in the range of the existence of RPA and, to some extent, beyond the RPA collapse, the EWSR are satisfied and the separation of the additional modes is unambiguously done.

The proposed approach is a generalization of the self-consistent RPA introduced in Ref. [19] in the context of metal clusters. The generalization with respect to [19] consists in enlarging the space including also particle-particle and hole-hole configurations following the line suggested in Ref. [15]. Similarly to what is done in Ref. [19], the operators Q_ν are non-renormalized and the one-body density matrix (OBDM) is not assumed diagonal.

The paper is organized as follows. In Sec. II we discuss the problem of EWSR and the origin of the violation of EWSR when only particle-hole excitations are considered in the phonon operators within generalized RPA approaches aimed to overcome QBA. In Sec. III we discuss how this problem is solved within the present approach. In Sec. IV we apply it to a solvable three-level Lipkin model and we compare the results obtained within this approach with the RPA and exact ones. Finally, in Sec. V, the main conclusions are drawn.

II. GENERALIZED RPA APPROACHES AND EWSR

Let us briefly recall how the problem of the EWSR raises when generalizations of RPA are considered, still remaining in the space of particle-hole elementary excitations. Within such space, the excited states of the system are written as

$$|\nu\rangle = Q_\nu^\dagger|0\rangle, \tag{1}$$

where

$$Q_\nu^\dagger = \sum_{ph}\left(X_{ph}^\nu a_p^\dagger a_h - Y_{ph}^\nu a_h^\dagger a_p\right) \tag{2}$$

with $a^\dagger$ and a denoting creation and annihilation operators and $|0\rangle$ is the correlated ground state, defined as the vacuum of the Q_ν operators

$$Q_\nu|0\rangle = 0. \tag{3}$$

.

The equations of motion method [9] leads to

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}\begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix} = \omega_\nu \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix}\begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix} \tag{4}$$

with

$$A_{ph,p'h'} = \langle 0|[a_h^\dagger a_p, H, a_{p'}^\dagger a_{h'}]|0\rangle, \tag{5}$$

$$B_{ph,p'h'} = -\langle 0|[a_h^\dagger a_p, H, a_{h'}^\dagger a_{p'}]|0\rangle, \tag{6}$$

$$G_{ph,p'h'} = \langle 0|[a_h^\dagger a_p, a_{p'}^\dagger a_{h'}]|0\rangle, \tag{7}$$

where the symmetrized double commutators are defined as

$$[A, B, C] = \tfrac{1}{2}\{[A, [B, C]] + [[A, B], C]\}. \tag{8}$$

The X and Y amplitudes satisfy the orthonormality conditions

$$\sum_{ph,p'h'}\left(X_{ph}^\nu X_{p'h'}^{\nu'} - Y_{ph}^\nu Y_{p'h'}^{\nu'}\right)G_{ph,p'h'} = \delta_{\nu\nu'}. \tag{9}$$

The standard RPA equations can be obtained by replacing, in the evaluation of the matrices (5)–(7), the state $|0\rangle$ with the (uncorrelated) HF one. In particular the norm matrix G acquires the simpler form

$$G_{ph,p'h'}^{(\mathrm{HF})} = \langle \mathrm{HF}|[a_h^\dagger a_p, a_{p'}^\dagger a_{h'}]|\mathrm{HF}\rangle = \delta_{hh'}\delta_{pp'}, \tag{10}$$

the matrices A and B are easily evaluated since

$$a_h^\dagger|\mathrm{HF}\rangle = a_p|\mathrm{HF}\rangle = 0 \tag{11}$$

and the RPA X and Y amplitudes satisfy the orthonormality conditions

$$\sum_{ph}\left(X_{ph}^\nu X_{ph}^{\nu'} - Y_{ph}^\nu Y_{ph}^{\nu'}\right) = \delta_{\nu\nu'}. \tag{12}$$

A very important feature of standard RPA is that it preserves EWSR [9,20]. It is well known that, if $|0\rangle$ and $|\nu\rangle$ are a complete set of exact eigenstates of the Hamiltonian, with eigenvalues E_0 and E_ν, the following identity holds:

$$\sum_\nu \omega_\nu|\langle\nu|F|0\rangle|^2 = \frac{1}{2}\langle 0|[F, [H, F]]|0\rangle, \tag{13}$$

where $\omega_\nu = E_\nu - E_0$ and F is any Hermitian single particle operator. The above equality is in general violated to some extent when $|0\rangle$, $|\nu\rangle$ and ω_ν are calculated with some approximation. To which extent it is satisfied is a measure of the adequacy of the approximation.

Transition amplitudes $\langle\nu|F|0\rangle$ induced by a one-body operator

$$F = \sum_{\alpha,\beta}\langle\alpha|F|\beta\rangle a_\alpha^\dagger a_\beta \tag{14}$$

between the ground state $|0\rangle$ and excited states $|\nu\rangle$ when these are described within RPA and RPA-like approaches

$$\langle\nu|F|0\rangle = \langle 0|[Q_\nu, F]|0\rangle, \tag{15}$$

where the definition (1) and the vacuum property (3) have been used. The above expression is general and it is valid independently of the explicit form of the Q operators. Within standard RPA, where the latter have the form (2), only the p-h components of the transition operator F are selected, obtaining

$$\langle \nu|F|0\rangle = \sum_{ph} \left\{ X^{\nu *}_{ph} \langle p|F|h\rangle + Y^{\nu *}_{ph} \langle h|F|p\rangle \right\}. \tag{16}$$

A very important feature of RPA, known as Thouless theorem [20], can be described as follows. When the left hand side of Eq. (13) is evaluated by using the energies and the X and Y amplitudes of RPA, one finds the same result as when the right hand side of the same equation is calculated by replacing the exact ground $|0\rangle$ with the $|\mathrm{HF}\rangle$ state. This result is very important also because it guarantees that spurious excitations corresponding to broken symmetries (as, for example, the translational invariance) separate out and are orthogonal to the physical states. We remark that, when the r.h.s. is evaluated in the HF state, only the p-h components of the transition operator F appear in it. The same happens in the l.h.s. but it is essentially related to the p-h nature of the Q operators. In fact, when the correlated $|0\rangle$ is maintained, it is still true that only the p-h components of the transition operator F appear in the l.h.s. and one has

$$\langle \nu|F|0\rangle = \sum_{php'h'} \left\{ X^{\nu *}_{ph} \langle p'|F|h'\rangle + Y^{\nu *}_{ph} \langle h'|F|p'\rangle \right\} G_{ph,p'h'}, \tag{17}$$

while this is no more the case in the r.h.s., where the whole structure of F appears. This is the reason why all extension of RPA, with only p-h excitations, violate Eq. (13). On the other hand, the use of the HF state in place of the correlated one, and thus the use of the QBA, is a well known limit of RPA and many efforts have been done in order to overcome this limit by taking into account ground state correlations neglected in standard RPA.

III. FORMULATION OF THE APPROACH

In the following we present an extension of RPA aimed to overcome the QBA, by taking into account ground state correlations and, at the same time, to obtain a scheme in which EWSR are preserved. As discussed above, the violations of the EWSR in extended RPA approaches are mainly due to the use of phonon operators containing only p-h excitations. In our approach the excited states $|\nu\rangle$ of the system are generated by more general operators

$$Q^{\dagger}_{\nu} = \sum_{\alpha>\beta} \left(X^{\nu}_{\alpha\beta} a^{\dagger}_{\alpha} a_{\beta} - Y^{\nu}_{\alpha\beta} a^{\dagger}_{\beta} a_{\alpha} \right), \tag{18}$$

where α and β are single-particle states and $\alpha > \beta$ means that $n_\alpha < n_\beta$ (n being the occupation number of the state). The equations to solve are similar to RPA equations with a different norm matrix:

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix} = \omega_\nu \begin{pmatrix} G & \bar{G} \\ -\bar{G}^* & -G^* \end{pmatrix} \begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix}. \tag{19}$$

The norm matrix is defined by the matrices G which read

$$\begin{aligned} G_{\alpha\beta,\alpha'\beta'} &= \langle 0|[a^{\dagger}_{\beta} a_{\alpha}, a^{\dagger}_{\alpha'} a_{\beta'}]|0\rangle \\ &= \delta_{\alpha\alpha'} \rho(\beta,\beta') - \delta_{\beta\beta'} \rho(\alpha',\alpha), \end{aligned} \tag{20}$$

where ρ is the OBDM

$$\rho(\alpha,\beta) \equiv \langle 0|a^{\dagger}_{\alpha} a_{\beta}|0\rangle \tag{21}$$

and

$$\bar{G}_{\alpha\beta,\alpha'\beta'} = G_{\alpha\beta,\beta'\alpha'}. \tag{22}$$

The new A and B matrices have the same expressions defined in Eqs. (5) and (6) but the $p's$ and $h's$ indices are replaced now by the $\alpha's$ and $\beta's$ ones, respectively.

All the matrix elements appearing in Eqs. (19) are calculated in the correlated ground state, which is defined as the vacuum of the operators Q. The explicit expressions of the A and B matrices is calculated by using the method of linearization of equations of motion [15,19]. It amounts to contract with respect to the correlated ground state $|0\rangle$ the two-body terms coming out from the inner commutators appearing in Eqs. (5) and (6). In a loose notations, this means

$$[H, a^{\dagger}a] \to a^{\dagger}a + a^{\dagger}a^{\dagger}aa \sim a^{\dagger}a + \langle 0|a^{\dagger}a|0\rangle a^{\dagger}a. \tag{23}$$

In such a way also, the A and B matrices are expressed in terms of the OBDM.

By using the number operator method [19,21] the ρ matrix is expressed in terms of the X and Y amplitudes:

$$\rho(p,p') = \sum_{\nu\nu'} \sum_{h} \left(\delta_{\nu\nu'} - \frac{1}{2} \sum_{p_1 h_1} R^{\nu'}_{h_1 p_1} R^{\nu *}_{p_1 h_1} \right) R^{\nu}_{ph} R^{\nu' *}_{hp'}, \tag{24}$$

$$\rho(h,h') = \delta_{hh'} - \sum_{\nu\nu'} \sum_{p} \left(\delta_{\nu\nu'} - \frac{1}{2} \sum_{p_1 h_1} R^{\nu'}_{h_1 p_1} R^{\nu *}_{p_1 h_1} \right) \times R^{\nu}_{ph} R^{\nu' *}_{h'p}, \tag{25}$$

$$\rho(p,h) = \sum_{\nu\nu'} \sum_{h_2} \left(\delta_{\nu\nu'} - \frac{1}{2} \sum_{p_1 h_1} R^{\nu'}_{h_1 p_1} R^{\nu *}_{p_1 h_1} \right) R^{\nu}_{ph_2} R^{\nu' *}_{h_2 h}, \tag{26}$$

where

$$R^{\nu}_{ij} = \sum_{\alpha>\beta} \left(X^{\nu}_{\alpha\beta} G_{ij,\alpha\beta} - Y^{\nu}_{\alpha\beta} G_{ij,\beta\alpha} \right). \tag{27}$$

Therefore Eqs. (19) are nonlinear and in order to solve them we use an iterative procedure. At the nth iterative step we compute the ρ matrix, and thus the A, B, and G matrices, by using the X and Y amplitudes of the $(n-1)$th step. As starting point, we use the HF OBDM, i.e.,

$$\rho^{(0)}(h,h') = \delta_{h,h'}; \quad \rho^{(0)}(p,p') = 0; \quad \rho^{(0)}(p,h) = 0, \tag{28}$$

obtaining thus the standard RPA solutions, which are used in the next step. The procedure is carried out until convergence is reached, namely, until the maximum relative difference in the excitation energies between two successive iterations is less than a chosen limit (10^{-7} in the following calculations).

Equations (19) are a generalized eigenvalue problem and can be solved directly by using, for example, QR or QZ algorithms [22]. Alternatively, one can extract a set of linear independent states by diagonalizing the norm matrix

$$\begin{pmatrix} G & \bar{G} \\ -\bar{G}^* & -G^* \end{pmatrix}, \tag{29}$$

and make thus a unitary transformation which allows to transform the generalized eigenvalue problem into a standard one [23]. We have used this second way for the following reason. Since, in the enlarged phonon operator (18) the α and β indices denote generic single particle states, both occupied and unoccupied in the HF ground state, particle-particle, particle-hole, and hole-hole configurations are included. This implies that nonphysical additional states come into play. The strong advantage of this method with respect to that of Ref. [15] is that by diagonalizing the norm matrix and by looking at the overlaps of the excited states with the ground state the nonphysical states can be isolated and eliminated from the spectrum. The implications of the above approach on the description of the collective states in realistic system will be studied in forthcoming works. In the next section we show the advantages of the method within a three-level Lipkin model.

IV. AN APPLICATION TO THE LIPKIN MODEL

The three-level Lipkin model consists of three energy levels, each of them is 2Ω-fold degenerate, in which the total number of fermions $N = 2\Omega$ is distributed. The Hamiltonian of the model is

$$H = \sum_{i\neq 0} \epsilon_i K_{ii} + V_0 \sum_{i,j\neq 0} K_{i0}K_{0j} + V_1 \sum_{i,j\neq 0} (K_{i0}K_{j0} + h.c.) + V_2 \sum_{i,j,k\neq 0} (K_{i0}K_{jk} + h.c.) + V_3 \sum_{i,j,k,l,\neq 0} K_{ij}K_{kl} \tag{30}$$

where ϵ are the energies of the levels and the coefficients V represent the strengths of the various interaction terms. The operators K have the following form:

$$K_{ij} = \sum_m a^\dagger_{im} a_{jm} \tag{31}$$

and satisfy the commutation relations

$$[K_{ij}, K_{kl}] = \delta_{jk}K_{il} - \delta_{il}K_{kj}. \tag{32}$$

The indices i and j indicate the three levels and m denotes the 2Ω degenerate states of each level. The operators K define a SU(3) algebra. The exact excitation spectrum of the model can be calculated and compared with the obtained results.

We present the results for two choices of the parameters ϵ and V in the Hamiltonian.

As a first set we have

$$\epsilon_0 = 0, \quad \epsilon_1 = \epsilon, \quad \epsilon_2 = 2.5\epsilon, \quad V_0 = -\chi, \quad V_1 = \chi, \quad V_2 = -\chi/2, \quad V_3 = \chi/10. \tag{33}$$

In Fig. 1 the excitation spectrum (a) and the eigenvalues of the norm matrix (b) are displayed as a function of the strength parameter $\tau = \chi/\epsilon$ for the first set of parameters. The present calculation is denoted in the figure with the acronym EERPA (extended and enlarged RPA). EERPA results (solid lines) are compared with the exact corresponding values (dashed lines) and with the standard RPA energies (dotted lines). The first two RPA excited states are indicated with (1) and (3). A much better agreement with the exact results is found for EERPA with respect to RPA. This is mostly evident for the first RPA excited state already before the RPA collapse point, $\tau = 0.026$. With EERPA the collapse point is located much further and the good agreement with the exact values is kept in the whole region where the EERPA results exist. The state (2), whose energy is equal to $\epsilon_2 - \epsilon_1$ when τ approaches 0, does not correspond to any exact solution. Thus it is nonphysical. This straightforward identification would not be possible when the EERPA is applied to the study of realistic systems since exact results are not available in that case. Quite in general, however, one can proceed as follows.

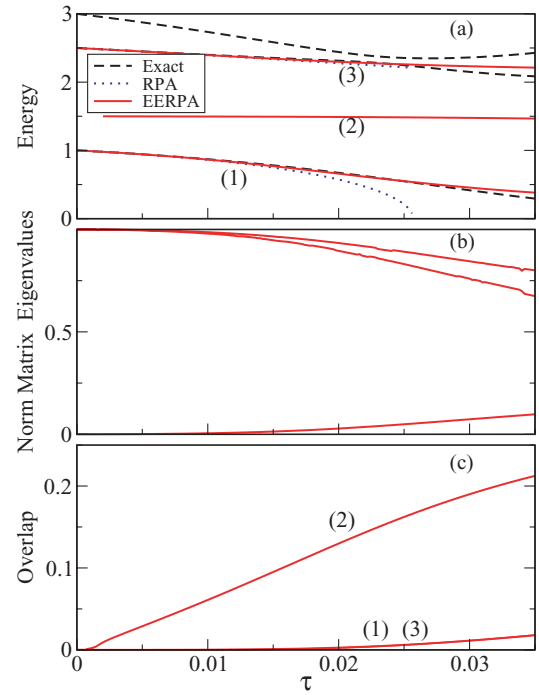

FIG. 1. (Color online) First set of parameters (33). (a): Excitation energies of the states (1), (2), and (3) and the eigenvalues of the norm matrix (b) as a function of the strength parameter $\tau = \chi/\epsilon$. The energies in the Y axis are expressed in units of ϵ. In (c) the corresponding overlaps [Eq. (34)] of the three states with the ground state are shown. See the text for more details.

The eigenvalues of the norm matrix Eq. (29), are plotted in panel (b) of the figure. One can observe that even beyond the RPA collapse point one eigenvalue is much smaller than the others, its value being equal to 0.03 at the RPA collapse. At $\tau = 0.035$, that is far beyond the collapse, the three eigenvalues are still well separated: 0.09, 0.69, and 0.84, respectively. This tells us that one of the three solutions is nonphysical. One condition that the three excited states should satisfy in order to be acceptable as physical states is their orthogonality with

the ground state. In panel (c) of Fig. 1 we show the overlaps

$$\langle 0|\nu\rangle = \langle 0|Q_\nu^\dagger|0\rangle = \sum_{\alpha>\beta} X^\nu_{\alpha\beta}\rho(\alpha,\beta) - Y^\nu_{\alpha\beta}\rho(\beta,\alpha) \qquad (34)$$

between the ground state and the states of panel (a). As it is very clearly visible, such overlap for the state labeled as (2) is strongly different from zero, while for the others it is much smaller. Thus we can conclude that indeed such state is the nonphysical one. Of course, in the case of a schematic, exactly solvable model such identification can be done directly by looking at the exact energy spectrum while this is not possible in realistic cases. On the other hand, the appearance of a (almost) zero eigenvalue of the norm matrix does not allow to identify the nonphysical state. On the contrary, looking at the values of the overlaps Eq. (34) is an unambiguous criterion to single out the states to be eliminated and it is viable also in realistic systems. The fact that the overlaps of the physical states with the ground state are not exactly zero, especially for large values of the strength, is related to the approximations present in the approach. For example, the OBDM is evaluated by using the number operator method truncated at a certain order, which probably is not enough for values of the strength well beyond the RPA collapse point. However, in physical cases we expect to be in situations corresponding to smaller values of the strength or, in worst cases, around the RPA collapse point. We remark that the above discussed criterion, to disentangle physical from nonphysical states, could not be applied in the previous approach [15] where, since the OBDM is assumed diagonal, the overlaps (34) are zero by construction for all the states $|\nu\rangle$ and thus the fulfillment of the orthogonality condition is not achievable.

It is worthwhile to underline that even when only particle-hole excitations are included in the phonon operators (18), the self-consistent treatment of the ground state correlations used in our approach leads to a better agreement with the exact results with respect to RPA, as we have checked. However, in this case, violations of EWSR are found. For example, near the RPA collapse point, deviations of the order of 30% are present. When instead the enlarged configuration space is used, EWSR are exactly satisfied, as shown in [15]. This has been numerically verified in the present calculations.

The first set of parameters allows to modelize the case where, in a realistic system, two particle (or hole) states which construct an elementary configuration are quite well separated in energy: the unperturbed energy of the corresponding configuration is then finite and greater than zero. It can happen, however, that a realistic single-particle spectrum presents some very close single-particle states. If a particle-particle or hole-hole configuration can be formed with two of these states, the associated unperturbed energy would be very close to zero. It has been shown in Ref. [18] for a realistic nuclear system that these additional states (that would contribute in the low-energy part of the excitation spectrum) have actually very low transition probabilities and do not contribute in practice to the strength distribution of the excitation mode. The second set of parameters in our Lipkin model has been chosen to mimic such a situation by using the same strength parameters V of (33)

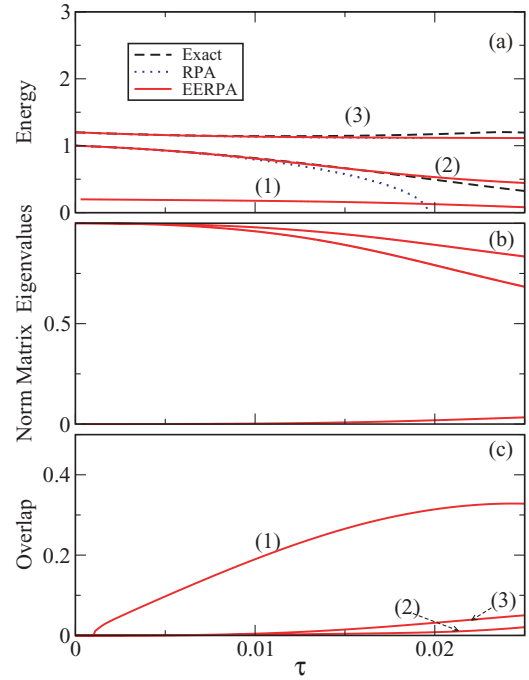

FIG. 2. (Color online) Same parameters of Eq. (33) but with $\epsilon_1 = \epsilon, \epsilon_2 = 1.2\epsilon$.

and as single particle energies:

$$\epsilon_0 = 0, \quad \epsilon_1 = \epsilon, \quad \epsilon_2 = 1.2\epsilon. \qquad (35)$$

In Fig. 2, where we show the same quantities of Fig. 1 for this second choice, we can see that EERPA results are again in better agreement with the exact ones with respect to the RPA, whose collapse point is now at $\tau = 0.019$. The energy of the nonphysical state, labeled as (1) in the figure, starts from a value equal to $\epsilon_2 - \epsilon_1$ when τ is near to 0 and then decreases by increasing τ. Indeed, the collapse of EERPA ($\tau = 0.036$), that we did not observe in Fig. 1, in this second case happens just because the energy of the nonphysical state approaches 0. However, the EERPA collapse is far beyond the RPA one. In panels (b) and (c) of Fig. 2, we plot the eigenvalues of the norm matrix (29) and the overlaps (34), respectively, which, as in the previous case, allow to disentangle physical from nonphysical states in a very clear way.

V. CONCLUSIONS

In conclusion, we have presented an extension of RPA in which ground state correlations are treated in a self-consistent way and no use of the quasiboson approximation is made. The method has been applied to a three-level Lipkin model and a better agreement with the exact results than in RPA has been found. By using an enlarged configuration space, with respect to that commonly used (particle-hole excitations), EWSR are exactly preserved. This means that the problem of spurious excitations corresponding to broken symmetries which arises in many, if not all, extensions of RPA is not present in this method. In a previous paper [15], an extension

of RPA with an enlarged basis of elementary excitations within the renormalized RPA framework was introduced and studied in the same three level Lipkin model. A serious problem of that approach is the appearance of a nonphysical state, having no counterpart in the exact spectrum. The strong and innovative advantage of the here presented approach is that by diagonalizing the norm matrix and by looking at the overlaps of the excited states with the ground state, nonphysical states can be isolated and eliminated from the spectrum. The obtained results strongly encourage the application of the present approach to more realistic systems. Work in this direction is in progress.

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