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Pion pair condensed nuclear matter without a σ meson

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The nonlinear σ model that includes vector meson contributions is utilized for the study of nuclear matter. Here, the σ meson degree of freedom is eliminated by the condition that $\sigma^2 + \boldsymbol{\pi}^2$ is invariant under both chiral transformation and isospin rotation. The nuclear matter consists of nucleons, pions, and vector (ω) mesons. In the model, there are only three free parameters, and the minimum energy and desired incompressibility at the normal nuclear density for the pion pair condensed nuclear matter can be obtained by adjusting these parameters.

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I. INTRODUCTION

The bulk properties of nuclear matter with mesons were first studied by Walecka [1]. In the original σ-ω model, the minimum energy at the normal nuclear density for nuclear matter was successfully reproduced, but the resulting value of incompressibility was very large. To obtain the desired incompressibility for nuclear matter, higher-order σ meson terms [2] or derivative σ-nucleon coupling [3] was introduced into the standard σ-ω model. In both models, the higher-power σ terms were taken into consideration to obtain the desired value of incompressibility.

Another approach is that based on the linear σ model [4]. In addition to vector meson contributions, additional scalar meson terms [5], scalar-vector interaction terms [6], or higher-order terms [7,8] of the chiral loop ($\sigma^2 + \boldsymbol{\pi}^2$) were introduced to reproduce the bulk properties of nuclear matter.

The density dependence of the classical σ meson field plays an essential role in these calculations. However, the σ meson has not yet been experimentally confirmed. The nonlinear σ model has been proposed to eliminate the degree of freedom associated with the uncertain σ meson [9]. The σ meson σ is replaced by pions under the condition that the chiral loop ($\sigma^2 + \boldsymbol{\pi}^2$) remains invariant under both isospin rotation and chiral transformation. In this model, a σ meson field is expressed as the square root of the square of the pion field and is expanded to have an infinite number of terms representing pion fields.

In this study, we examine this idea for pion pair condensed nuclear matter and discuss the properties of nuclear matter, which consists of nucleons, pions, and ω mesons. It is shown that in the model, there are only three parameters for adjusting the minimum energy, the incompressibility, and the normal nuclear density according to the type of nuclear matter; we can set the three parameters to reproduce nuclear matter properties well.

II. MODEL

The Lagrangian density of the linear σ model, which comprises vector meson contributions, the nucleon mass term, and the pion mass term, can be expressed as

$$\begin{aligned}\mathcal{L} = {}& \overline{\psi} i\gamma^\mu \partial_\mu \psi - \Delta M \overline{\psi}\psi + g_\pi \overline{\psi}(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi \\ & - g_\omega \overline{\psi}\gamma^\mu \omega_\mu \psi + \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - \tfrac{1}{2}A_\pi f_\pi^2 \boldsymbol{\pi}^2 + \tfrac{1}{2}A_\omega f_\pi^2 \omega_\mu\omega^\mu - B f_\pi^3 \sigma,\end{aligned} \tag{2.1}$$

with

$$F^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu, \tag{2.2}$$

where ψ, $\boldsymbol{\pi}$, and ω_μ denote the nucleon field, the pion, and the ω meson, respectively.

The nucleon mass, pion mass, and linear σ terms, represented by $\Delta M\overline{\psi}\psi$, $\frac{1}{2}A_\pi f_\pi^2\boldsymbol{\pi}^2$, and $Bf_\pi^3\sigma$, respectively, are not chiral invariant. The nucleon mass term renders the π-N coupling constant g_π a free parameter that controls attraction. The quantities, g_π, g_ω, A_π, A_ω, ΔM, and B, are constants to be determined to obtain the desired properties of the nuclear matter.

The apparent σ meson field σ is expressed in terms of $\boldsymbol{\pi}$ according to the condition

$$\sigma^2 + \boldsymbol{\pi}^2 = A^2, \tag{2.3}$$

where A is a constant corresponding to the radius of the chiral loop. The Lagrangian density thus obtained has the same form as that of the extended linear σ model [8,10] in which the Condition (2.3) holds.

The Dirac equation for a nucleon is given by

$$i\gamma^\mu\partial_\mu\psi - \Delta M\psi + g_\pi(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi - g_\omega\gamma^\mu\omega_\mu\psi = 0. \tag{2.4}$$

The Klein-Gordon equation for a pion is given by

$$\begin{aligned}&\left(\frac{\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}}{\sigma^2}\right)^2\boldsymbol{\pi} + \frac{(\partial_\mu\boldsymbol{\pi})^2}{\sigma^2}\boldsymbol{\pi} + \frac{\boldsymbol{\pi}\cdot\partial_\mu\partial^\mu\boldsymbol{\pi}}{\sigma^2}\boldsymbol{\pi} + \partial_\mu\partial^\mu\boldsymbol{\pi} \\ &\quad = g_\pi\overline{\psi}\left(-\frac{\boldsymbol{\pi}}{\sigma} + i\gamma_5\boldsymbol{\tau}\right)\psi - A_\pi f_\pi^2\boldsymbol{\pi} + Bf_\pi^3\frac{\boldsymbol{\pi}}{\sigma},\end{aligned} \tag{2.5}$$

and the Proca equation for an ω meson is given by

$$\partial_\mu F^{\mu\nu} = g_\omega\overline{\psi}\gamma^\nu\psi - A_\omega f_\pi^2\omega^\nu. \tag{2.6}$$

Further, the Hamiltonian density is given by

$$\begin{aligned}\mathcal{H} = {}& \psi^\dagger\boldsymbol{\alpha}\cdot\boldsymbol{p}\psi + \Delta M\overline{\psi}\psi - g_\pi\bar{\psi}(\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)\psi \\ & + g_\omega\overline{\psi}\gamma^\mu\omega_\mu\psi + \tfrac{1}{2}(\partial_0\sigma)^2 + \tfrac{1}{2}(\nabla\sigma)^2 + \tfrac{1}{2}(\partial_0\boldsymbol{\pi})^2\end{aligned}$$

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$$+\tfrac{1}{2}(\nabla\boldsymbol{\pi})^2+\tfrac{1}{2}A_\pi f_\pi^2\boldsymbol{\pi}^2+\tfrac{1}{2}(\partial_0\omega_\mu)^2+\tfrac{1}{2}(\nabla\omega_\mu)^2$$
$$-\tfrac{1}{2}A_\omega f_\pi^2\omega_\mu\omega^\mu+Bf_\pi^3\sigma. \quad (2.7)$$

Because the vector current is expressed as

$$\mathcal{J}^\mu = \bar\psi\gamma^\mu\frac{\boldsymbol{\tau}}{2}\psi+\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}, \quad (2.8)$$

it can easily be ensured that the condition

$$\partial_\mu\mathcal{J}^\mu = 0 \quad (2.9)$$

is satisfied and that the Lagrangian density (2.1) is invariant under isospin transformation. The axial vector current is given by

$$\mathcal{J}_5^\mu = \bar\psi\gamma^\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi+\sigma\partial^\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial^\mu\sigma, \quad (2.10)$$

and the dispersion of the axial vector current results in the following condition:

$$\partial_\mu\mathcal{J}_5^\mu = \Delta M\bar\psi i\gamma_5\boldsymbol{\tau}\psi-A_\pi f_\pi^2\sigma\boldsymbol{\pi}+Bf_\pi^3\boldsymbol{\pi}. \quad (2.11)$$

This implies that the nucleon mass, pion mass, and σ linear terms are not chiral invariant.

From the weak decay of the charged pions $\pi^+\to\mu^++\nu_\mu$ and $\pi^-\to\mu^-+\overline{\nu}_\mu$, the following relation can be obtained

$$\langle 0|\partial_\mu\mathcal{J}_5^\mu(0)|\boldsymbol{\pi}(k)\rangle = k_\mu k^\mu f_\pi = m_\pi^2 f_\pi, \quad (2.12)$$

where m_π and f_π are the pion mass at vacuum (zero nuclear density) and the decay constant of a charged pion, respectively. From a comparison of Eqs. (2.11) and (2.12), the following equation can be obtained

$$m_\pi^2 = Bf_\pi^2-A_\pi f_\pi\langle\sigma\rangle_0, \quad (2.13)$$

where $\langle\sigma\rangle_0$ denotes an apparent classical σ meson field in vacuum.

III. PION PAIR CONDENSED NUCLEAR MATTER

We use spatially and temporally uniform mean field approximations for studying the nuclear matter. Because of parity conservation of the nuclear ground state, the classical field of a single pion $\langle\boldsymbol{\pi}\rangle$ does not exist. However, the scalar part of a combination of pions can exist as a classical field [10]. Thus, we assume

$$\boldsymbol{\pi}\longrightarrow\langle\boldsymbol{\pi}\rangle=0 \quad \boldsymbol{\pi}^2\longrightarrow\langle\boldsymbol{\pi}^2\rangle, \quad (3.1)$$
$$\omega_\mu\longrightarrow\langle\omega\rangle\delta_{\mu0}, \quad (3.2)$$

and

$$\sigma = \pm\sqrt{A^2-\boldsymbol{\pi}^2}\longrightarrow\langle\sigma\rangle = \pm\sqrt{A^2-\langle\boldsymbol{\pi}^2\rangle}. \quad (3.3)$$

The sign of σ is determined later.

The mean field Hamiltonian density is given by

$$\langle\mathcal{H}\rangle = \langle\psi^\dagger\boldsymbol{\alpha}\cdot\boldsymbol{p}\psi\rangle+(\Delta M-g_\pi\langle\sigma\rangle)\langle\overline{\psi}\psi\rangle$$
$$+g_\omega\langle\omega\rangle\langle\overline{\psi}\gamma^0\psi\rangle+\tfrac{1}{2}A_\pi f_\pi^2\langle\boldsymbol{\pi}^2\rangle-\tfrac{1}{2}A_\omega f_\pi^2\langle\omega\rangle^2$$
$$+Bf_\pi^3\langle\sigma\rangle. \quad (3.4)$$

The condition that the mean field Hamiltonian density $\langle\mathcal{H}\rangle$ has the minimum value for the pion pair classical field $\langle\boldsymbol{\pi}^2\rangle$ leads the equation

$$(A_\pi\langle\sigma\rangle-Bf_\pi)f_\pi^2 = -g_\pi\langle\overline{\psi}\psi\rangle. \quad (3.5)$$

Using this equation, $\langle\sigma\rangle$ and then $\langle\boldsymbol{\pi}^2\rangle$ can be expressed as functions of nuclear density. Nuclear matter in which the pion pair classical field $\langle\boldsymbol{\pi}^2\rangle$ exists is called pion pair condensed nuclear matter.

The ω meson classical field $\langle\omega\rangle$ should obey the equation of motion of the ω meson field ω_μ (2.6). Thus,

$$g_\omega\langle\overline{\psi}\gamma^0\psi\rangle = A_\omega f_\pi^2\langle\omega\rangle. \quad (3.6)$$

Here, the classical ω meson field $\langle\omega\rangle$ is expressed as a function of nuclear density.

A. Mass

The meson fields can be expressed in terms of their classical fields and their fluctuations as follows:

$$\boldsymbol{\pi}\longrightarrow\tilde{\boldsymbol{\pi}} \quad \boldsymbol{\pi}^2\longrightarrow\langle\boldsymbol{\pi}^2\rangle+\tilde{\boldsymbol{\pi}}^2, \quad (3.7)$$
$$\omega_\mu\longrightarrow\langle\omega\rangle\delta_{\mu0}+\tilde\omega_\mu, \quad (3.8)$$

and

$$\sigma = \pm\sqrt{A^2-\boldsymbol{\pi}^2}\longrightarrow\langle\sigma\rangle+\tilde\sigma = \pm\sqrt{A^2-\langle\boldsymbol{\pi}^2\rangle-\tilde{\boldsymbol{\pi}}^2}$$
$$= \pm\sqrt{A^2-\langle\boldsymbol{\pi}^2\rangle}\left\{1-\frac{\tilde{\boldsymbol{\pi}}^2}{2(A^2-\langle\boldsymbol{\pi}^2\rangle)}-\frac{\tilde{\boldsymbol{\pi}}^4}{8(A^2-\langle\boldsymbol{\pi}^2\rangle)^2}-\cdots\cdots\right\}$$
$$= \langle\sigma\rangle-\frac{\tilde{\boldsymbol{\pi}}^2}{2\langle\sigma\rangle}-\frac{\tilde{\boldsymbol{\pi}}^4}{8\langle\sigma\rangle^3}-\cdots\cdots. \quad (3.9)$$

By substituting these equations in the Lagrangian density (2.1), the effective nucleon mass can be defined by

$$M^* = \Delta M-g_\pi\langle\sigma\rangle \quad (3.10)$$

as the coefficient of the term $\overline{\psi}\psi$. The square of the effective pion mass is given by

$$m_\pi^{*2} = A_\pi f_\pi^2-\frac{Bf_\pi^3}{\langle\sigma\rangle} \quad (3.11)$$

as the coefficient of $\tilde{\boldsymbol{\pi}}^2/2$. That of the effective ω meson mass is also given by

$$m_\omega^{*2} = A_\omega f_\pi^2. \quad (3.12)$$

The effective ω meson mass does not depend on the nuclear density, and thus, it must be equal to the ω meson mass at zero nuclear density. Thus, the coefficient A_ω can be determined as

$$A_\omega = \frac{m_\omega^2}{f_\pi^2}. \quad (3.13)$$

By comparing Eqs. (2.13) and (3.11) under the condition of zero nuclear density, the following relation was derived:

$$\langle\sigma\rangle_0 = -f_\pi. \quad (3.14)$$

Thus,

$$A = f_\pi, \quad (3.15)$$

and

$$\sigma = -\sqrt{f_\pi^2 - \boldsymbol{\pi}^2}. \tag{3.16}$$

Further

$$A_\pi + B = \frac{m_\pi^2}{f_\pi^2}, \tag{3.17}$$

and

$$\Delta M = M_N - g_\pi f_\pi, \tag{3.18}$$

where M_N is the nucleon mass at zero nuclear density.

B. Energy

For pion condensed nuclear matter, other distributions that are different from the Fermi gas distribution have been suggested [11]. However, here we assume the existence of the classical field of the scalar part of combined pions, and we do not aim to discuss the structures of pion condensed nuclear matter. Thus, we assume that the nuclear distribution for pion pair condensed nuclear matter is the simplest Fermi gas distribution. The energy per nucleon of the nuclear matter with the Fermi momentum p_F is given by

$$\mathcal{E} = \mathcal{E}_N - M_N + \mathcal{E}_\omega + \mathcal{E}_\pi, \tag{3.19}$$

where

$$\begin{aligned}\mathcal{E}_N &= \frac{2}{\pi^2}\int_0^{p_F} p^2\sqrt{p^2+M^{*2}}dp \,/\, \rho_B \\ &= \frac{M^{*4}}{4\pi^2}\left[\frac{p_F}{M^*}\left\{2\left(\frac{p_F}{M^*}\right)^2+1\right\}\sqrt{1+\left(\frac{p_F}{M^*}\right)^2} - \log\left|\frac{p_F}{M^*}+\sqrt{1+\left(\frac{p_F}{M^*}\right)^2}\right|\right]\Big/\rho_B,\end{aligned} \tag{3.20}$$

$$\mathcal{E}_\omega = \tfrac{1}{2}g_\omega\langle\omega\rangle, \tag{3.21}$$

and

$$\mathcal{E}_\pi = \left\{\tfrac{1}{2}A_\pi f_\pi^2\langle\boldsymbol{\pi}^2\rangle + Bf_\pi^3(\langle\sigma\rangle + f_\pi)\right\}\big/\rho_B. \tag{3.22}$$

The baryon density (nuclear density) ρ_B of the Fermi gas nuclear matter is given by

$$\rho_B = \langle\overline{\psi}\gamma^0\psi\rangle = \sum_{s,I}\int_0^{\vec{p}_F}\frac{d^3p}{(2\pi)^3} = \frac{2p_F^3}{3\pi^2}, \tag{3.23}$$

where s and I are the spin and isospin degrees of freedom, respectively.

IV. NUMERICAL CALCULATION

The Lagrangian density (2.1) has six unknown coefficients: g_π, g_ω, A_π, A_ω, B, and ΔM. These coefficients are related to each other as given below:

$$A_\omega = \frac{m_\omega^2}{f_\pi^2}, \tag{4.1}$$

$$A_\pi + B = \frac{m_\pi^2}{f_\pi^2}, \tag{4.2}$$

and

$$\Delta M = M_N - g_\pi f_\pi. \tag{4.3}$$

Thus, the number of free parameters to be determined is three: g_π, g_ω, and A_π. The classical pion pair field $\langle\boldsymbol{\pi}^2\rangle$ is determined by using Eq. (3.5), where $\langle\overline{\psi}\psi\rangle = \rho_s$ is the scalar density. For the Fermi gas nuclear matter ρ_s is given by

$$\rho_s = \frac{M^{*3}}{\pi^2}\left\{\frac{p_F}{M^*}\sqrt{1+\left(\frac{p_F}{M^*}\right)^2} - \log\left|\frac{p_F}{M^*}+\sqrt{1+\left(\frac{p_F}{M^*}\right)^2}\right|\right\}. \tag{4.4}$$

Condition (3.5) indicates that the square of the effective pion mass (3.10) is zero at zero nuclear density. This means that Condition (3.5) cannot be applied in the case of nuclear density values near zero. In fact, Eq. (3.5) does not have a positive $\langle\boldsymbol{\pi}^2\rangle$ solution below a certain critical nuclear density ρ_c. Thus, in the case of low nuclear density, the following conditions must be satisfied:

$$\langle\boldsymbol{\pi}^2\rangle = 0, \tag{4.5}$$

$$\langle\sigma\rangle = -f_\pi, \tag{4.6}$$

$$M^* = \Delta M + g_\pi f_\pi = M_N, \tag{4.7}$$

and

$$m_\pi^{*2} = A_\pi f_\pi^2 + Bf_\pi^2 = m_\pi^2. \tag{4.8}$$

The classical ω meson field $\langle\omega\rangle$ is proportional to the nuclear density, as shown in Eq. (3.6).

The parameters g_π, g_ω, and A_π are determined such that the pion pair condensed nuclear matter has the minimum energy per nucleon $\mathcal{E} = -16.3$ MeV [12] and such that the incompressibility is $K \sim 300$ MeV [13] at normal nuclear density $\rho_0 = 0.153$ fm^{-3} ($p_F = 259.15$ MeV/c). For numerical calculations, the values of the pion decay constant f_π, nucleon mass M_N, pion mass m_π, and ω meson mass m_ω at zero nucleon density were 93, 938.9, 139.6, and 781.94 MeV, respectively.

The results of the numerical calculations are given in Table I.

TABLE I. Solutions with $\mathcal{E} = -16.3$ MeV at $\rho_0 = 0.153$ fm^{-3}.

g_π	g_ω	A_π	K (MeV)
25.09	15.81	−30.0	316.09
25.27	15.81	−30.5	308.93
25.45	15.80	−31.0	302.44
25.62	15.78	−31.5	293.40
25.79	15.77	−32.0	285.03

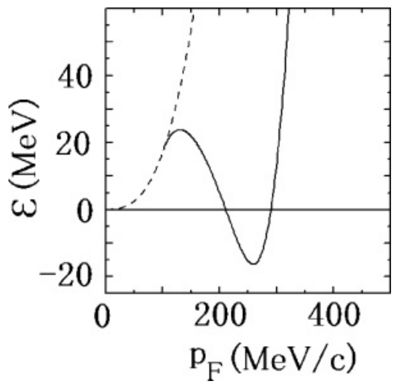

FIG. 1. Energy per nucleon $\mathcal{E}$. The solid and dashed lines denote results pertaining to pion pair condensed nuclear matter and nuclear matter without $\langle \boldsymbol{\pi}^2 \rangle$, respectively.

The most preferable set of parameters is $g_\pi = 25.45$, $g_\omega = 15.80$, and $A_\pi = -31.0$. Equation (3.5) has positive $\langle \boldsymbol{\pi}^2 \rangle$ solutions when $p_F \geqslant 102$ MeV/c. In this region, pion pair condensed nuclear matter has a lower energy than nuclear matter without $\langle \boldsymbol{\pi}^2 \rangle$, as shown in Fig. 1. The effective masses of the nucleon, pion, and ω meson obtained from the same parameter set are given in Fig. 2. Classical fields of the pion pair and the ω meson divided by f_π, i.e., $\sqrt{\langle \boldsymbol{\pi}^2 \rangle}/f_\pi$ and $\langle \omega \rangle / f_\pi$, respectively, are also shown in Fig. 3. The solid and dashed lines in all figures show the numerical results pertaining to pion pair condensed nuclear matter and nuclear matter without $\langle \boldsymbol{\pi}^2 \rangle$, respectively.

V. CONCLUDING REMARKS

Fermi gas nuclear matter consisting of nucleons, pions, and ω mesons can be used to reproduce the desired properties of nuclear matter in pion pair condensation. The apparent σ meson degree of freedom is replaced by pions satisfying the condition $\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2$. In the Lagrangian density (2.1), the free parameters are g_π, g_ω, and A_π. When $g_\pi = 25.45$, $g_\omega = 15.80$, and $A_\pi = -31.0$, the pion pair condensed nuclear matter has a minimum energy of $\mathcal{E} = -16.3$ MeV at a normal nuclear density of $\rho_0 = 0.153$ fm^{-3} and the desired incompressibility $K = 302.44$ MeV. The critical density of the pion pair condensed nuclear matter is $p_c = 102$ MeV/c. In other words, solutions of the pion pair classical field $\langle \boldsymbol{\pi}^2 \rangle > 0$ exist for nuclear matter with a Fermi momentum greater than $p_F = 102$ MeV/c. Here, pion pair condensed nuclear matter certainly has an energy lower than that of nuclear matter in which the pion classical field is absent, and the effective pion mass m_π^* increases with nuclear density.

As the pion pair classical field $\langle \boldsymbol{\pi}^2 \rangle$ increases with nuclear density, the effective nucleon mass M^* decreases rapidly; this effect results in a decrease in the energy $\mathcal{E}_N$. The contribution from the ω meson $\mathcal{E}_\omega$ is a repulsive contribution, as shown in Fig. 4. The contribution from the pion $\mathcal{E}_\pi$ is a repulsive contribution at low energies, while that from the nucleon $\mathcal{E}_N$ is an attractive contribution. The attraction is mainly caused by the reduction in the effective nucleon mass M^*. In fact, $M^* = 360$ MeV at the normal nuclear density is lower than the suggested nucleon mass in a nucleus by a fair amount. The attraction is controlled by the coupling constant g_π via variations in M^*, and the repulsion is controlled by g_ω via the vector meson contributions. By varying the parameter A_π, the strength of the higher-power terms of $\boldsymbol{\pi}^2$ can be controlled and the incompressibility can be adjusted.

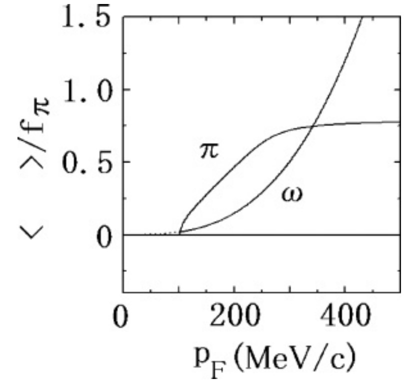

FIG. 3. Classical meson fields divided by f_π. $\langle \quad \rangle / f_\pi$ represents $\sqrt{\langle \boldsymbol{\pi}^2 \rangle}/f_\pi$ and $\langle \omega \rangle / f_\pi$. The solid lines denote the results pertaining to pion pair condensed nuclear matter, and the dashed line denotes the results pertaining to $\langle \omega \rangle / f_\pi$ without $\langle \boldsymbol{\pi}^2 \rangle$, respectively.

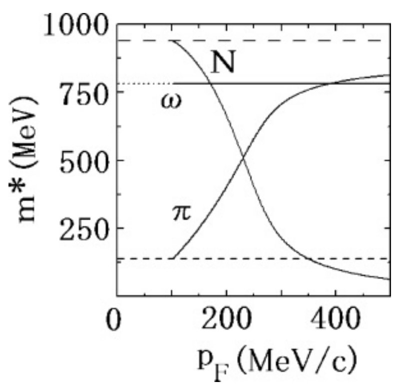

FIG. 2. Effective masses. The solid and dashed lines denote results pertaining to pion pair condensed nuclear matter and nuclear matter without $\langle \boldsymbol{\pi}^2 \rangle$, respectively.

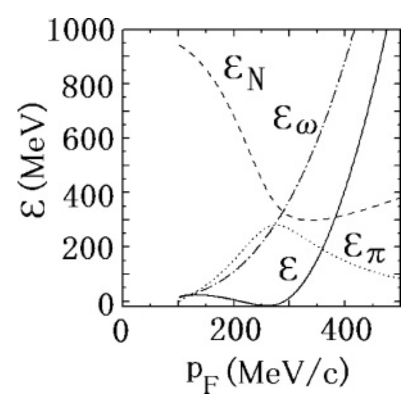

FIG. 4. Contributions of nucleon $\mathcal{E}_N$, pion $\mathcal{E}_\pi$, and $\mathcal{E}_\pi$ to the energy $\mathcal{E}$.

To obtain the desired incompressibility for nuclear matter, higher-powered σ meson terms were introduced in both the σ-ω model and linear σ model. In our model, the pion degrees of freedom are used instead of the σ meson degrees of freedom, and the condition $\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2$ results in higher-power $\boldsymbol{\pi}^2$ terms. From the higher-power $\boldsymbol{\pi}^2$ terms, the desired incompressibility can be obtained for the pion pair condensed nuclear matter.

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