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Unified framework for understanding pair transfer between collective states in atomic nuclei

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A new interpretation of two-nucleon pair transfer in collective nuclei is presented. It differs from traditional models and unifies, within a consistent framework, the entire range of monopole pair-transfer phenomenology in collective nuclei. This includes the well-known examples of large cross sections to excited 0^+ states in phase transitional nuclei, and small ones in many other nuclei, but also predicts large cross sections elsewhere under particular circumstances. These predictions can be tested experimentally.

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The search for signatures that elucidate the evolution of structure in many-body systems with emergent collectivity is an active area of study in many fields. This is particularly the case when such systems undergo large structural changes as a function of some variable. Atomic nuclei are fascinating mesoscopic systems with the particular feature that the number of bodies (nucleons) can be varied in a controlled way. Thus reactions which transfer nucleons from an initial to a final nucleus are a sensitive probe of structure and of changes in structure. Particularly important are two-nucleon transfer reactions of which there are many early examples, including [1–12], along with a number of newer studies using ultra-high resolution spectrometry [13–16]. A striking example occurs where large observed jumps [1–6] in cross sections to the first excited 0^+ state in specific isotopes were interpreted long ago [1–5,17] as evidence for sudden shape changes and, recently [18,19], as evidence of a first order quantum phase transition (QPT) in the equilibrium shape. Understanding the signatures of QPT's in nuclei has broader relevance to understanding QPT's in other mesoscopic systems [20,21].

Aside from the large cross sections to excited states just cited, in virtually all cases the ground state cross sections dominate: excited states are populated with $<15\%$ of the ground state cross section [6,18,22,23]. (Notable exceptions include pairing vibrations [22] near closed shell nuclei and the occurrence of favored microscopic configurations such as those in the neutron midshell Yb isotopes [23]). The traditional interpretation [24] is that, for ground state to ground state transitions in collective nuclei, the amplitudes in the transfer matrix element add constructively because of the coherent nature of the ground states, whereas they add destructively for population of orthogonal excited states. The one dramatic exception to this behavior, referred to above, is in the Sm [1–3] and Gd [4,5] isotopes near $N = 90$, where excited 0^+ state(s) are populated with cross sections *comparable* to the ground state. The correlation with a rapid shape transition from spherical to deformed nuclei in this region has led to the standard interpretation [1–5,17–19] in terms of mixing of coexisting states of different shapes and their similarity with the target ground state. For example, the transitional nucleus ^{154}Gd, whose ground state is considered a mixture of spherical and deformed configurations, would have large (p,t) cross sections both to the spherical ground state of ^{152}Gd and to its coexisting deformed first excited 0^+ state (see Fig. 1).

It is the purpose of this Rapid Communication to propose that this interpretation is, in fact, not correct, and to propose a new interpretation of the transfer cross sections for two identical nucleons [e.g., (p,t)] to 0^+ states in which the cross sections depend only on the structural change between the ground states of the target and final nucleus, as measured by a simple observable, and not on whether such changes involve a region of shape/phase coexistence. This interpretation, which accounts for both the normally encountered small cross sections and the occasional large ones, also involves mixing, but at a more microscopic level rather than mixing of preformed coexisting collective states. It provides a unifying framework, grounded in simple nucleon-pair configurations. We also explain why previous interpretations of large cross sections nevertheless *appeared* to be reasonable. Finally we discuss a case where our interpretation is supported experimentally.

To do this, we exploit the IBA model [26] which spans a variety of collective structures and, with its s and d boson structure, is well-suited to pair transfer reactions to 0^+ states which proceed by the addition or removal of an s boson [19,26]. It includes well-known [26] selection rules for transitions at the symmetry limits. We use the simple Hamiltonian

$$H = \epsilon n_d + \kappa Q \cdot Q = c\left[(1-\zeta)n_d + \frac{\zeta}{4N_B} Q \cdot Q\right], \quad (1)$$

where N_B is the boson number and

$$Q = s^\dagger \tilde{d} + d^\dagger s + \chi (d^\dagger \tilde{d})^2. \quad (2)$$

In terms of the symmetry triangle (see inserts to Fig. 2), ζ (or ϵ/κ) reflects the competition between the spherical-driving ϵn_d term and the deformation-inducing $Q \cdot Q$ term. It ranges from $\zeta = 0$ for the U(5) vibrator limit to unity for deformed nuclei. χ specifies the degree of γ-softness, from axially symmetric [SU(3)] for $\chi = -\sqrt{7}/2 = -1.32$ to γ-soft [O(6)] for $\chi = 0$.

Figure 1 includes IBA calculations for the Gd isotopes in both (p,t) and (t,p). They describe the trends in the data well, especially considering the uncertainties in the specific orbits involved (cross sections are sensitive to the spatial overlap of the two transferred nucleons). The IBA also reproduces

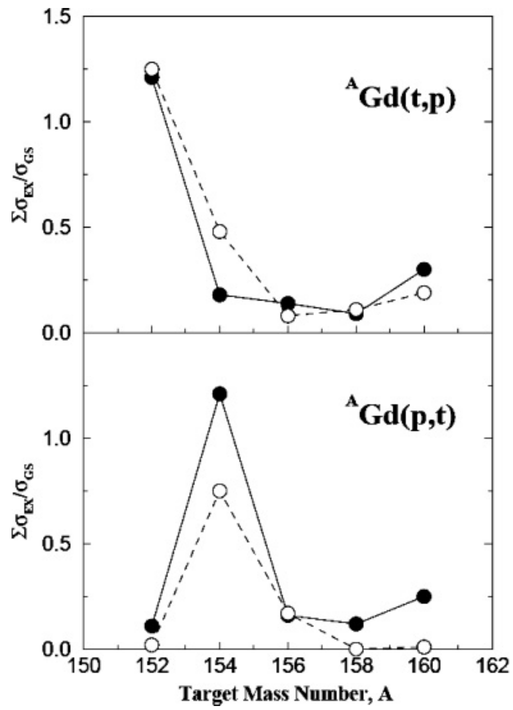

FIG. 1. Comparison of two nucleon transfer data (solid dots) for Gd [5,10–12] with IBA calculations (open symbols), using parameters from Ref. [25], showing the sum of cross sections to excited 0^+ states as a ratio to the ground state cross section.

a wealth of (p,t) and (t,p) data throughout the rare earth region [27], with the exception of a few well known, isolated, instances where particular single particle effects come into play [23]. Thus it is an apt model to explore these reactions. We will show that the mechanism for both large and small cross sections turns out to be completely different from the standard interpretations.

To understand the key point, we have carried out IBA calculations spanning the triangle. These calculations cover a fine mesh of locations of the target and final nuclei in the triangle along a variety of trajectories. Of course, calculations for a realistic case will utilize appropriate parameters for the specific initial and final nuclei. Here, our purpose is more general—to investigate how the cross sections behave as a function of the initial and final parameters. All the calculations point to a consistent result which we illustrate in Fig. 2 with several specific cuts through the triangle for different sets of target and final nucleus parameters. Panel a) shows results for the case where the initial and final nuclei lie along the bottom axis of the triangle. It gives the sum of the first nine excited 0^+ state cross sections in two-valence nucleon removal reactions [$N_B = 10 \rightarrow 9$, e.g., (p,t) below midshell], normalized to the ground state cross section, against the initial value of ζ for $\chi = -\sqrt{7}/2$ (bottom leg of the triangle). Perhaps more physically intuitive than ζ is $R_{4/2} = E(4_1^+)/E(2_1^+)$, ranging from 2.0 for a harmonic vibrator to 3.33 for a symmetric rotor. The $R_{4/2}^{\rm init}$ values (nonlinear in $\zeta_{\rm init}$) are shown along the top. The panel includes contours for different values of the change in $R_{4/2}$, $\delta R_{4/2}$, between initial and final nuclei.

The results are completely unexpected in the context of the traditional interpretation. Regardless of $R_{4/2}^{\rm init}$, successive contours of larger $\delta R_{4/2}$ lie systematically higher, independent of the structure of the initial or final nucleus, including whether or not either is in a region of shape coexistence. For these reactions there is nothing conceptually different about the phase transitional region. Figure 2(b) shows a complementary cut through Fig. 2(a) in which the cross sections are shown against $\delta R_{4/2}$ and for different $\zeta_{\rm init}$ values (i.e., $R_{4/2}^{\rm init}$ values). They vividly confirm the same idea: the cross sections increase monotonically with $\delta R_{4/2}$. The same result is also shown for a trajectory along the O(6) to SU(3) leg ($\zeta = 1$), in which χ is allowed to vary for fixed ζ, that is, target and final nuclei have different χ values (degrees of γ-softness). Along this leg it is known [28] that no phase transition occurs. One concludes that excited 0^+ state cross sections can indeed easily be a significant fraction of the ground state cross section or even orders of magnitude larger (for $\delta R_{4/2} \gtrsim 0.5$) and this can occur equally well in shape transitional or other regions.

These points are the essence of this Rapid Communication. They differ from the traditional interpretation which associates large cross sections to excited 0^+ states with a spherical-deformed transition region, as stated for example in Ref. [19] which suggests that large cross sections are a signature of such shape changes.

We note that each contour in Fig. 2(a) has a characteristic behavior. It is large when the target is at the vibrator limit ($R_{4/2}^{\rm init} = 2.0$), decreases thereafter, and, at some point, suddenly turns around and increases rapidly, peaking when the final nucleus has $\zeta = 1$ [SU(3) symmetry when $\chi = -\sqrt{7}/2$]. The situation in which either the target or final nucleus is near the phase transitional point is, ironically, actually near the minimum of each contour.

These results are not an accident of looking only along the bottom (first order phase transitional) axis of the triangle. As seen in Fig. 2(c), we find the same behavior along the U(5)-O(6), or vibrator to γ-soft leg (with $\chi = 0$), and for intermediate trajectories (e.g., for $\chi = -0.7$).

For large $\delta R_{4/2}$ values the cross section is spread over several excited 0^+ states. However, the same general behavior characterizes the cross sections to the first excited 0^+ state alone (Fig. 2(d)). Finally, two-valence nucleon adding reactions [e.g., (t,p) in the first half of a major neutron shell] and results for other boson numbers share the same characteristics.

We can now understand the origin of the large cross sections in the $N = 90$ region. Figure 3 gives empirical values of $\delta R_{4/2}$ showing that large $\delta R_{4/2}$ are very rare, occurring only in narrow regions a few neutrons from shell closures. That is, virtually all large $\delta R_{4/2}$ occur in regions of shape coexistence and so it was natural to associate this with the unusual cross sections. In our view, this is an accidental correlation, masking a more general phenomenon, of which phase transitional regions are the most striking example. This also explains why previous calculations (e.g., [19]) ascribed the large cross sections to excited 0^+ states to the occurrence of a phase transition: their parameters were chosen such that the only large $\delta R_{4/2}$ values occurred precisely where the initial and final nuclei spanned the phase transitional region [e.g., for the U(5) to SU(3) leg,

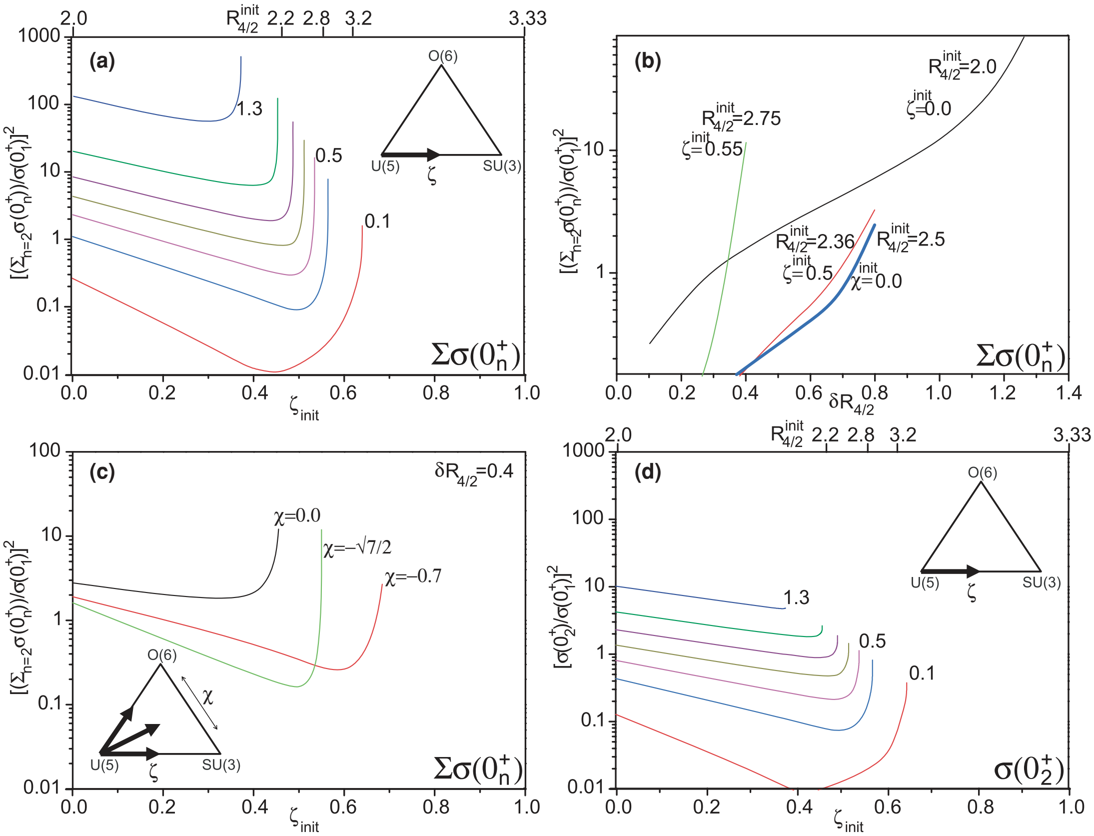

FIG. 2. (Color online) Calculated two nucleon transfer IBA cross sections to the first nine excited 0^+ states, normalized by the ground state cross section, for $N_B = 10 \to 9$. (a) Contours of $\delta R_{4/2}$ (from 0.1 to 1.3 in steps of 0.2) for the sum over these excited 0^+ states as a function of $\zeta_{\rm init}$ for $\chi = -\sqrt{7}/2 = -1.32$. An equivalent $R_{4/2}^{\rm init}$ scale is at the top. (b) Three vertical cuts through the top plot in (a) as well as calculations for $\zeta = 1$ and varying χ, all showing monotonic increases with $\delta R_{4/2}$. (c) Similar to (a) for three χ values and $\delta R_{4/2} = 0.4$. (d) Same as in (a) except for the first excited 0^+ state only. The triangle inserts show trajectories followed in the calculations, where relevant.

where the initial and final nuclei jumped the transitional $R_{4/2} \sim 3.0$ boundary]. While smaller increases in excited state cross sections were seen [19] along the O(6) to SU(3) leg, where no phase transition occurs, $\sigma(0_2^+)/\sigma(\text{g.s.})$ was always <1 and so it was natural to cite large ratios as a possible signature for phase

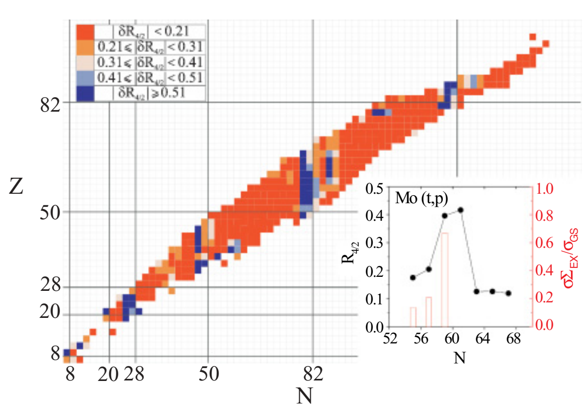

FIG. 3. (Color online) Nuclear chart showing experimental values of $\delta R_{4/2}$. Figure courtesy of Burcu Cakirli [29]. Lower right: inset of Mo (t,p) cross sections [7–9] to 0^+ states in comparison with empirical $\delta R_{4/2}$ values (plotted at the average of the initial and final neutron numbers.)

transitional behavior. Here, by carrying out calculations fully spanning all possible initial and final nuclei in the triangle, we see that the controlling element is largely $\delta R_{4/2}$ (or a similar observable that monitors structural change). Changes along the O(6) to SU(3) leg fit into the overall pattern as seen in Fig. 2(b). Of course, $\delta R_{4/2}$ is not the only ingredient, as again seen in Fig. 2(b) or Fig. 2(c). γ-softness also plays a role. However, the overriding physics conclusion is that it is the overall change in structure between initial and final nuclei that determines the cross sections. Finally, one element of our results is that we now see why no large excited state cross sections appear in well-deformed nuclei, since such nuclei must have $\delta R_{4/2} \lesssim 0.2$.

Our interpretation is supported by the data in one example of large $\delta R_{4/2}$ without a phase change, namely $^{100}\text{Mo}(t,p)^{102}\text{Mo}$, with $\delta R_{4/2} = 0.39$ (2.12 to 2.51). This is a region of spherical-deformed shape change where the transitional nuclei have $R_{4/2} \sim 3.0$. The observed cross sections are compared with empirical $\delta R_{4/2}$ values in Fig. 3. The correlation of large $\delta R_{4/2}$ values with large (t,p) cross sections is evident.

Lastly, it is worthwhile to analyze the origin of the behavior we have found. The essential point is evident from Eq. (1). The U(5) eigenstates (solutions to $H = \epsilon n_d$) have good s and d boson numbers. The $Q \cdot Q$ term mixes these (it has terms in $\delta n_d = 0, 1, 2$), and the mixing is proportional to

κ (or equivalently, ζ). Therefore, the cross section which would normally go to the ground state is shared among the excited states. The mechanism is indeed one of mixing but at the nucleon pair configuration level not at that of special collective modes, and is therefore general, describing the full range of two-neutron transfer phenomenology in a unified framework.

The two-nucleon transfer cross sections calculated in the IBA involve a sum of matrix elements of the two nucleon transfer operator (either s or $s^\dagger$) over all pairs of components of the initial and final wave functions. Thus these calculations probe the complete correlations in the wave functions. In general, with hundreds of terms contributing, either constructively or destructively, it is not easy to see the origin of a resulting cross section by inspection. However, there is one special case where this can be done and it is instructive to analyze it in order to develop an intuitive feeling for how the sensitivity to the *difference* in initial and final wave functions correlates with the predicted cross sections. This case is that in which the target has the exact U(5) symmetry. The IBA target ground state wave function then has $(n_s,n_d) = (N_B,0)$. Allowed (p,t) cross sections occur only to components $[N_B - 1, 0]$ in the final wave functions. Figure 4 shows distributions of the squares of these components (that is, the relative cross sections) in the successive 0^+ states of the final nucleus for three values of $\delta R_{4/2}$. For a final nucleus close to the vibrator (small $\delta R_{4/2}$), such amplitudes are small and occur only for the first excited 0^+ state. For, larger $\delta R_{4/2}$ values, two changes occur—the amplitudes grow, and they are spread over more 0^+ states, thus giving larger individual excited 0^+ state cross sections and a greater distribution of significant strength. For non-U(5) initial nuclei the mechanism is similar, except that many target ground state components are connected by an s-boson operator to components in the final state.

In summary, we have presented a new interpretation of two (identical) nucleon transfer cross sections to excited 0^+ states that is different from the traditional one that large cross sections only occur in phase transitional regions due to mixing of coexisting phases. Our IBA calculations show that the cross sections depend on the *overall change* in structure, as measured by the change in $R_{4/2}$ (with a smaller dependence on γ-softness). Our mechanism does not require that a transfer reaction passes through a critical point of a phase transition. Large cross sections in such regions emerge as just one manifestation (a dramatic and well documented one) of a general behavior. Moreover, these results also encompass the case of small cross sections and the explanation of Ref. [24] developed with small $\delta R_{4/2}$ in mind. If large $\delta R_{4/2}$ are discovered in exotic nuclei (e.g., ^{188,190}W with $\delta R_{4/2} = 0.35$ [30]), inverse kinematics pair transfer reactions could test these ideas. Of course, such tests are needed since these ideas are in the context of a particular model.

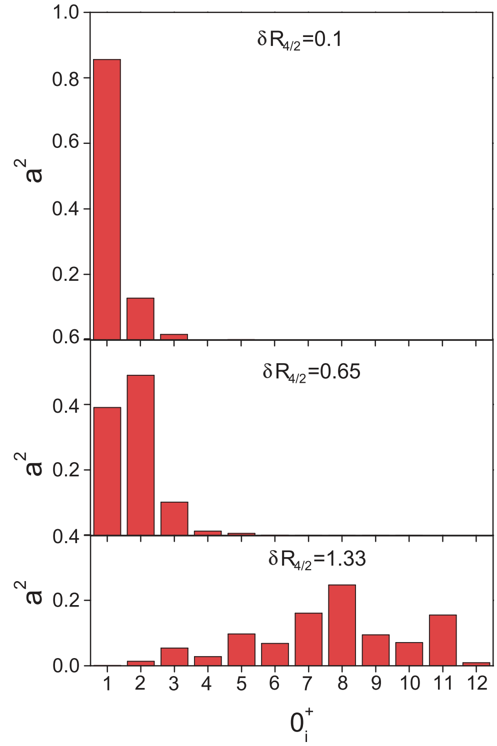

FIG. 4. (Color online) Squared amplitudes for pair transfers to successive excited 0^+ states for a U(5) target ($R_{4/2} = 2.0$) with $N_B = 10$. Calculations for $\chi = -\sqrt{7}/2$.

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