Recoil Effects in Single-Nucleon-Transfer Heavy-Ion Reactions

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The effect of recoil in single-nucleon-transfer heavy-ion reactions is found to be twofold: Extra transfer angular momenta are introduced and the radial integrals are changed. Comparison of an exact recoil distorted-wave Born-approximation program with experimental data from heavy-ion reactions induced on ^{11}B and ^{12}C gives excellent quantitative agreement. Recoil effects on other targets and energies are considered.

1. INTRODUCTION

Reactions induced by heavy ions (here $5 \lesssim z_{\text{projectile}} \lesssim 10$) appear to be a promising new tool to study nuclear structure provided that, first of all, the reaction dynamics are well understood. The charge product (zZ) and small velocities keep the ions well separated at sufficiently low energies ($zZe^2/E \gtrsim 10$) so that the motion of the ions may be described in terms of semiclassical orbits. The actual transfer process is, however, quantum mechanical and, therefore, perhaps not amenable to semiclassical descriptions. At higher energies, as the distance of closest approach between the two ions is reduced, we might expect semiclassical descriptions to be of even less value.

In this paper the transfer of a single nucleon between two heavy ions is treated quantum mechanically with the distorted-wave Born approximation (DWBA). Recoil is found to be extremely important in these reactions, particularly as the energy is increased. Quantitative agreement is obtained with experimental data of heavy-ion reactions induced on ^{11}B and ^{12}C targets. The effects of recoil as a function of energy and target mass are studied.

2. RECOIL. WHAT IS IT?

Figure 1(a) shows the relevant vector diagram for the transfer reaction $A(a, b)B$ ($B = A + x$, $a = b + x$). As will be seen later, DWBA methods require integration over the vectors $\vec{r}_a$ and $\vec{r}_b$ (i.e., a six-dimensional integral). Therefore simplifications are usually sought. For heavy-ion reactions, the cores A and b are expected (at sufficiently low energies) to stay relatively far apart as shown in Fig. 1(b). We may write

$$\vec{r}_a = \vec{R} - (x/a)\vec{r}_{bx}$$

and

$$\vec{r}_b = (A/B)\vec{R} + (x/B)\vec{r}_{bx}, \tag{1}$$

where $x/a \equiv m_x/m_a$, etc. Ignoring the $\vec{r}_{bx}$ terms is called (quaintly) "neglecting recoil effects" and is explicitly done by Buttle and Goldfarb[1] and by Schmittroth, Tobocman, and Golestaneh.[2] The treatment in the latter paper is slightly different: They use the coordinates $\vec{r}_a$ and $\vec{r}_b = (A/B)\vec{r}_a$. Neglecting recoil, therefore, has two consequences:
(a) $\hat{r}_a = \hat{r}_b$; in effect the particle x is restricted to a line between A and b.
(b) r_b and r_a are simply related by the constant A/B and the distances $(x/a)r_{bx}$ and $(x/B)r_{bx}$ are neglected.

We might expect that (a) will break down as the two cores come closer together, i.e., as the inci-

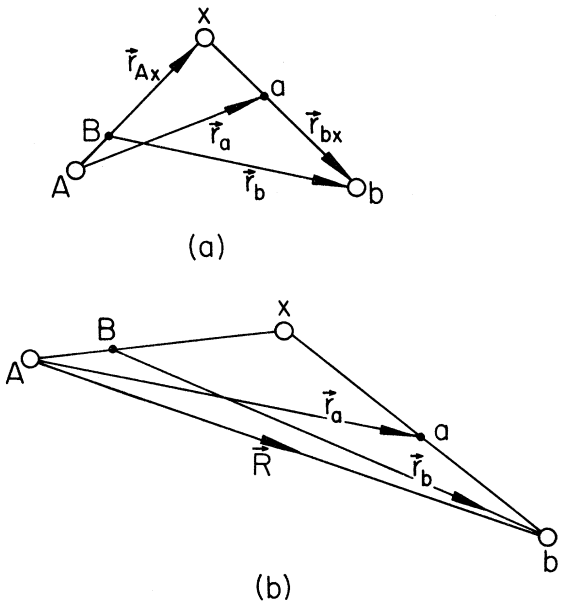

FIG. 1. Coordinate systems for the reaction $A(a, b)B$ where $a = b + x$, $B = A + x$. (a) DWBA position vectors exactly including recoil; (b) situation thought to occur in heavy-ion reactions. As explained in the text "neglecting recoil" corresponds to setting $\vec{r}_a = \vec{R}$ and $\vec{r}_b = (A/B)\vec{R}$.

dent energy is increased[3] and Fig. 1(b) goes to Fig. 1(a). Consequence (b) means that recoil effects will be more important for heavier targets as pointed out by Buttle and Goldfarb.[4] This is because, for a given projectile, the distance $(x/a)r_{bx}$ is constant. However, in order to keep the reaction near or above the Coulomb barrier the incident energy must be increased. Thus the wavelength decreases and the reaction will be more sensitive to the neglected distance.

3. WHAT RECOIL DOES TO THE DWBA

A DWBA amplitude looks like:

$$T_{\mathrm{DWBA}} \propto \int d\vec{r}_a \int d\vec{r}_b\, \chi_b^{(-)*}(\vec{r}_b) \times \psi_B^*(\vec{r}_{Ax})\, V_a(\vec{r}_{bx})\, \psi_a(\vec{r}_{bx})\, \chi_a^{(+)}(\vec{r}_a)\,, \tag{2}$$

where the vectors are defined in Fig. 1 and χ represents a distorted wave; ψ a bound state with an interaction potential V_a (using the post approximation for example). A great simplification occurs if recoil is neglected, for example, in the formalism of Buttle and Goldfarb:

$$T_{\mathrm{DWBA}}^{\mathrm{nonrecoil}} \propto \int d\vec{R}\, \chi_a^{(+)}(\vec{R})\chi_b^{(-)*}(B/A\vec{R}) \times \int d\vec{r}_{Ax}\, \psi_B(\vec{r}_{Ax})\, V_a(\vec{R}-\vec{r}_{ax})\psi_a(\vec{R}-\vec{r}_{ax})\,. \tag{3}$$

A very similar equation (replace $\vec{R}$ by $\vec{r}_a$) is derived by Schmittroth, Tobocman, and Golestaneh,[2] and is evaluated by them, and by Kubo, Duhm, and Ueta[5] with Woods-Saxon forms to obtain the bound-state wave functions. Buttle and Goldfarb use Hankel or Yukawa functions in the integral. The same basic procedure has been extended to the transfer of two nucleons by Baltz and Kahana.[6] Bonche and Giraud[7] make the approximation that the transferred particle lies at the center-of-mass position between the cores A and b, i.e., an extension of the condition (a) above.

For comparison, the zero-range approximation replaces

$$V_a(\vec{r}_{bx})\, \psi_a(\vec{r}_{bx}) = D_0 \delta(\vec{r}_{bx})$$

which physically is justifiable when the projectile is small and in an internal S state. This gives a DWBA amplitude[8]

$$T_{\mathrm{DWBA}}^{Z\text{-}R} \propto D_0 \int d\vec{r}_a\, \chi_a^{(+)}(\vec{r}_a)\chi_b^{(-)*}(A/B\,\vec{r}_a)\psi_B(\vec{r}_a),$$

which is rather similar but has no radial dependence for the bound state of b.

4. WHAT RECOIL DOES TO THE REACTION SELECTION RULES

If the particle x is bound to the core A (b) with orbital angular momentum l_2 (l_1) then the selection rules for the reaction are[9]:

$$\vec{j} = \vec{J}_A - \vec{J}_B\,, \quad \vec{l} = \vec{l}_1 - \vec{l}_2\,,$$
$$\vec{s} = \vec{s}_a - \vec{s}_b\,, \quad \vec{j} = \vec{l} + \vec{s}\,.$$

If we assume that the directions of $\vec{J}_A$ and $\vec{s}_b$ do not change in the reaction (inert core), then[10]:

$$s = j_1\,, \quad \text{where } \vec{j}_1 = \vec{l}_1 + \vec{s}_x\,,$$

and

$$j = j_2\,, \quad \text{where } \vec{j}_2 = \vec{l}_2 + \vec{s}_x\,.$$

Here j_1 and j_2 represent the total angular momentum of x in the two bound states. Letting $l_1 \to 0$ reproduces the familiar (d,p) selection rules. There is no parity selection rule limiting the possible l values if l_1 and l_2 are nonzero. Neglecting recoil changes this.

Without approximation we may rewrite Eq. (2) as:

$$T_{\mathrm{DWBA}}^{lsj} \propto \int d\vec{r}_a \int d\vec{r}_b\, \chi_a^{(+)}(\vec{r}_a) G^{lsj}(\vec{r}_a, \vec{r}_b)\chi_b^{(-)}(\vec{r}_b)\,, \tag{4}$$

where now the G^{lsj} contains the nuclear-structure information, and l, s, j, are defined above. The angular parts of Eq. (4) are all proportional to spherical harmonics[11]:

$$\chi_a^{(+)}(\hat{r}_a) \propto Y_{L_a}^{m_a}(\hat{r}_a)\,,$$
$$\chi_b^{(-)}(\hat{r}_b) \propto Y_{L_b}^{m_a}(\hat{r}_b)\,,$$

and

$$G^{lsj}(\hat{r}_a, \hat{r}_b) \propto Y_l^m[f(\hat{r}_a, \hat{r}_b)],$$

where spin-orbit coupling is ignored. Now if $\hat{r}_b$ is set equal to $\hat{r}_a$, the integration over $\hat{r}_a$ and $\hat{r}_b$ is simple and gives a Clebsch-Gordan coefficient

$$\langle L_b l 00 |\, L_a 0\rangle;$$

that is to say $L_a + L_b + l = \text{even}$. Since the parity change in a reaction is given[12] by $(-1)^{L_a+L_b}$ this means that

$$(-1)^l = \Delta\pi\,,$$

which is the "normal" parity.[11] For example, the reaction $^{12}\mathrm{C}(^{14}\mathrm{N}, ^{13}\mathrm{C})^{13}\mathrm{N}$ has

$$\vec{j} = 0 - \tfrac{1}{2} = \tfrac{1}{2}\,, \qquad (p_{1/2} \text{ state in } ^{13}\mathrm{N}),$$
$$\vec{s} = \vec{1} - \tfrac{1}{2} = \tfrac{1}{2}\,, \qquad (p_{1/2} \text{ state in } ^{14}\mathrm{N}),$$
$$\vec{l} = \vec{1} - \vec{1} = 0, 1, 2\,,$$

but

$$\vec{j}=\vec{l}+\vec{s};$$

therefore

$$l=0,1.$$

The normal parity is given by:

$$(-1)^l=(+)(+)(-)(-)=(+).$$

So, in nonrecoil $l=0$ only is allowed, but with recoil $l=0$ and $l=1$ are allowed.

We see that the normal parity requirement is due to the symmetry in the nonrecoil (or zero-range) DWBA amplitude which results from an effective restriction on the position of the particle x. The semiclassical theory of Brink[13] makes a similar assumption and arrives at the same result for the possible l values.

5. EXACT RECOIL DWBA AND THE PROGRAM LOLA

Austern *et al.*[9] have developed a DWBA theory which exactly includes recoil. This formalism is used in the program LOLA[14] and is partially reproduced here. The differential cross section for the stripping reaction defined above is:

$$\frac{d\sigma}{d\Omega}=\frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{2J_B+1}{2J_A+1}\sum_{lsj}\frac{|A_{lsj}|^2}{2s_a+1}\sum_m|\beta_m^l(\theta)|^2, \tag{5}$$

where μ_a is the reduced mass of the pair a,A, etc. The β_m^l are defined by

$$\beta_m^l(\theta)=4\pi/k_ak_b\sum_{L_aL_b}i^{L_a+L_b-l}(2L_b+1)^{1/2}\langle L_blm-m\,|\,L_a0\rangle[(L_b-m)!/(L_b+m)!]^{1/2}P_{L_b}^m(\theta)I_{L_aL_b}^l, \tag{6}$$

where the radial integral $I_{L_aL_b}^l$ is

$$I_{L_aL_b}^l=\int_0^\infty dr_a\,r_a\int_0^\infty dr_b\,r_b\,\chi_b^{L_b}(r_b)F_{L_aL_b}^l(r_ar_b)\chi_a^{L_a}(r_a). \tag{7}$$

The two-dimensional form factor $F_{L_aL_b}^l$ is:

$$\begin{aligned}F_{L_aL_b}^l(r_ar_b)=&\tfrac{1}{2}\sum_{\lambda_1\lambda_2K}(v_1r_a)^{l_1-\lambda_1}(t_1r_b)^{\lambda_1}(v_2r_a)^{\lambda_2}(t_2r_b)^{l_2-\lambda_2}\\&\times g_K(r_ar_b)\sum_{\Lambda_a\Lambda_b}(-1)^{L_b-l_2+\lambda_2-\lambda_1}(2l_1+1)(2l_2+1)(2K+1)\\&\times[(2\Lambda_a+1)(2\Lambda_b+1)]^{1/2}\langle\Lambda_aK00\,|\,L_a0\rangle\langle\Lambda_bK00\,|\,L_b0\rangle\\&\times\langle l_1-\lambda_1,\lambda_200\,|\,\Lambda_a0\rangle\langle l_2-\lambda_2,\lambda_100\,|\,\Lambda_b0\rangle W(L_aL_b\Lambda_a\Lambda_b;lK)\\&\times\binom{2l_1}{2\lambda_1}^{1/2}\binom{2l_2}{2\lambda_2}^{1/2}\begin{Bmatrix}l & l_1 & l_2\\ \Lambda_a & l_1-\lambda_1 & \lambda_2\\ \Lambda_b & \lambda_1 & l_2-\lambda_2\end{Bmatrix},\end{aligned} \tag{8}$$

where the "kernels" $g_K(r_ar_b)$ are defined by

$$g_K(r_ar_b)=\int_{-1}^{+1}d\mu\,\psi_B(r_1)V_i(r_i)\psi_a(r_2)P_K(\mu) \tag{9}$$

and $V_i=V_a$ in the post approximation and $\mu\equiv\hat{r}_a\cdot\hat{r}_b$. The natural coordinates of ψ_B and ψ_a are expanded:

$$r_1=\alpha[r_a^2+\gamma^2r_b^2-2\gamma r_ar_b\mu]^{1/2},$$

$$r_2=\alpha[\delta^2r_a^2+r_b^2-2\delta r_ar_b\mu]^{1/2},$$

where

$$\alpha=aB/x(A+a),\quad \gamma=b/a,\quad \delta=A/B.$$

Combinations of these form v_i, t_i:

$$v_1=\alpha,\quad v_2=-\alpha\delta,\quad t_1=-\gamma\alpha,\quad t_2=\alpha.$$

The values of Λ_i, λ_i, and K are limited by the following triads which must obey triangular inequalities: $(L_a\Lambda_aK)$, $(L_b\Lambda_bK)$, $(\Lambda_al_1-\lambda_1,\lambda_2)$, $(\Lambda_b,l_2-\lambda_2,\lambda_1)$, (l,L_aL_b), (ll_1l_2), and $(l\Lambda_a\Lambda_b)$, of which the first four sums must be even.

The program LOLA first computes the bound-state wave functions $\psi_i(r_i)$ which are interpolated and integrated [Eq. (9)] by the Gauss-Legendre method. These "kernels" contain the radial-nuclear-structure information and are stored on permanent files since they are independent of the incident energy and optical-model parameters used. Their calculation takes about half of the total computation time for an $l=0$ transfer, less for higher l transfers.

The angular momentum tensors of Eq. (8) are computed and directly integrated with the dis-

torted waves using Simpson's rule in Eq. (7). The distorted waves and bound states are calculated with the subroutines used in DWUCK.[15]

The A_{lsj} are given by[10]

$$A_{lsj} = i^{l}[(2l+1)(2s_a+1)]^{1/2}\mathcal{S}_1^{1/2}\mathcal{S}_2^{1/2}W(l_1 j_1 l_2 j_2; s_x l), \tag{10}$$

with the $\mathcal{S}$'s representing spectroscopic factors. The final cross section for a stripping reaction is:

$$\frac{d\sigma}{d\Omega} = \frac{2J_B+1}{2J_A+1}\sum_{lsj}(2l+1)W^2(l_1 j_1 l_2 j_2; s_x l)\mathcal{S}_1\mathcal{S}_2\sigma_{\mathrm{LOLA}}(\theta). \tag{11}$$

For a pickup reaction $B(b,a)A$, the factor $(2J_B+1)/(2J_A+1)$ is replaced by $(2s_a+1)/(2s_b+1)$, the other factors being included in the program. Note that different values of l, s, and j are added *incoherently*.[9]

The program uses about 300 000 double-precision words on the Saclay IBM 360/91 and less than 4 min to compute any of the cases in this article ($2l$ transfers with 55 partial waves) to 5% precision.

6. CALCULATIONS

A. Post-Prior Representations

Buttle and Goldfarb have shown that if recoil is neglected the post and prior representations [see Eq. (9)] do not give the same results, differing in some cases by factors of several hundreds.[4] By changing the lengths of $\vec{r}_a$ and $\vec{r}_b$ in Eq. (1), but not modifying the $\hat{r}_a = \hat{r}_b$ condition, they were able

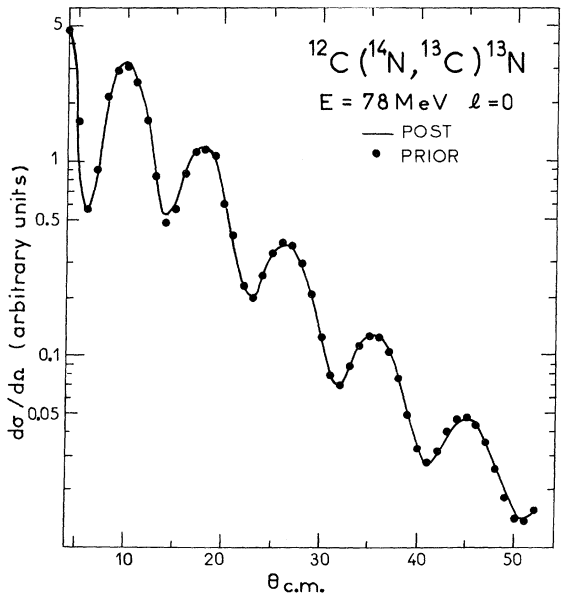

FIG. 2. Predictions for the $^{12}C(^{14}N,^{13}C)^{13}N$ reaction using the post and prior representations.

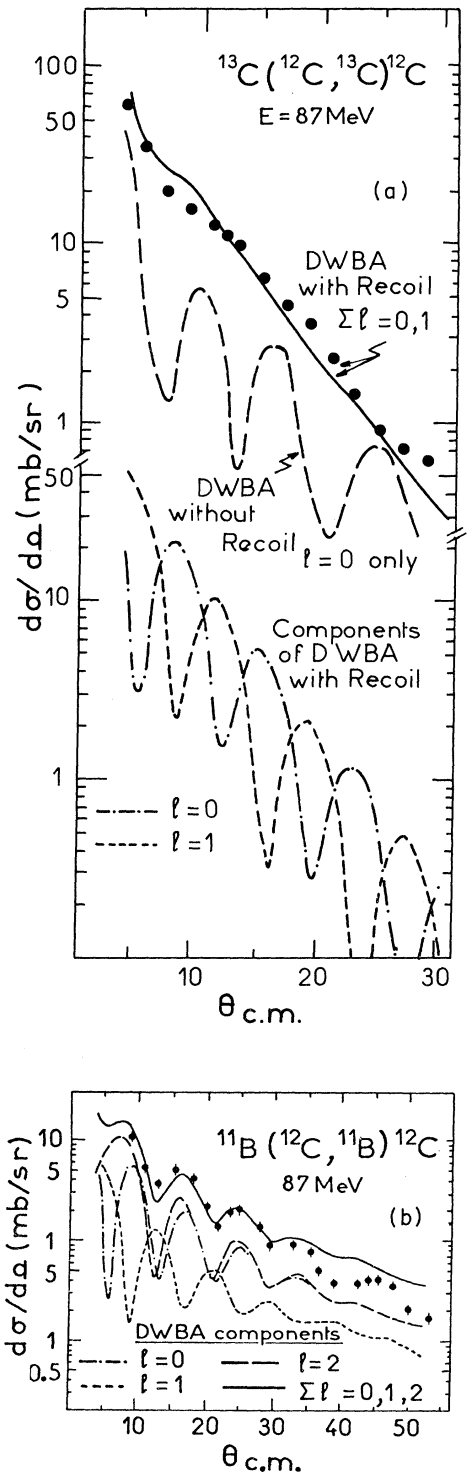

FIG. 3. Comparison of experiment and theory for the reactions $^{13}C(^{12}C,^{13}C)^{12}C$ and $^{11}B(^{12}C,^{11}B)^{12}C$ which correspond to backward-angle elastic scattering. The elastic scattering predicted by the optical model in this region is completely negligible compared with the experimental data.

TABLE I. Optical potentials.

Reaction	Energy (MeV)	V_0	W_0	r_0	r_I	a_0	a_I	Ref.
$^{12}C+^{13}C$	87	50	25	0.95	1.18	0.70	0.53	20
$^{12}C+^{11}B$	87	100	26	1.02	1.10	0.70	0.44	20
$^{14}N+^{12}C$	78	100	38.5	0.92	1.29	0.77	0.26	17
$^{14}N+^{11}B$	77	50	25.	0.86	1.30	1.11	0.50	20
		100	10.85	0.90	1.39	0.85	0.49	20
$^{14}N+^{11}B$	41	50	25.	1.01	1.32	0.95	0.60	20
		100	8.30	0.80	1.46	0.985	0.62	20
$^{14}N+^{11}B$	113	50	25.	1.04	1.40	0.74	0.45	20
		100	9.64	1.02	1.41	0.78	0.47	20
$^{16}O+^{11}B$	60	100	30	1.19	1.26	0.48	0.26	19
$^{16}O+^{42}Ca, ^{26}Mg$	42	100	40	1.22	1.22	0.49	0.60	16

to produce a satisfactory agreement between the two representations. As shown in Fig. 2 for the $^{12}C(^{14}N, ^{13}C)^{13}N$ reaction, if recoil is included exactly there is an almost perfect overlap between the two theoretical predictions. It should be noted that the Coulomb interaction terms included by Buttle and Goldfarb[14] are not included in the calculations presented here. However, for reactions above the Coulomb barrier the extra terms are probably not important.

B. $^{12}C+^{13}C$ and $^{11}B+^{12}C$

Low-energy heavy-ion reactions ($\eta \gtrsim 10$) produce angular distributions with no structure[16] but, at higher energies ($\eta \gtrsim 3$), the angular distributions oscillate strongly in some cases but not in others.[17-20] Semiclassical descriptions[21] and nonrecoil DWBA theories[17,20] predict, however, strongly oscillating angular distributions in all cases. Good examples are the backward angles of the $^{12}C+^{13}C$ and $^{11}B+^{12}C$ elastic scattering, which correspond to one-nucleon transfer. The data,[20] measured at 87 MeV, are shown in Fig. 3. The $^{13}C(^{12}C, ^{13}C)^{12}C$ reaction shows *no* structure whereas the $^{11}B(^{12}C, ^{11}B)^{12}C$ reaction shows clear diffraction structures. As noted by the authors, "the large difference between the two cases is very unexpected and should provide a severe test for any reaction theory for one-nucleon-transfer reactions at high energies."

In the reaction $^{13}C(^{12}C, ^{13}C)^{12}C$

$$j=\tfrac{1}{2}, \quad s=\tfrac{1}{2}$$

$$\vec{l}=\vec{I}-\vec{I}=0, 1, 2,$$

but

$$\vec{j}=\vec{l}+\vec{s};$$

therefore

$$\vec{l}=0, 1.$$

In the nonrecoil DWBA theory $(-1)^l=(+)$, therefore only $l=0$ is allowed. The nonrecoil $l=0$ prediction of Ref. 20 is in Fig. 3(a). In exact recoil DWBA, the $l=1$ is also allowed giving a component of the same magnitude as, but oscillating beautifully out of phase with, the $l=0$ component. The incoherent sum produced the result shown in Fig. 3(a). For the reaction $^{11}B(^{12}C, ^{11}B)^{12}C$

$$j=\tfrac{3}{2}, \quad s=\tfrac{3}{2}, \quad l=0, 1, 2.$$

Again in nonrecoil theory, $l=1$ is not allowed. As shown in Fig. 3(b), the $l=1$ component oscillates out of phase with the $l=0$ (and $l=2$), and the sum reproduces the observed oscillations. It should

TABLE II. Spectroscopic factors.

Reaction	$\mathcal{S}_1\mathcal{S}_2$	Yields	Cohen-Kurath (Ref. 22)
(a) $^{13}C(^{12}C, ^{13}C)^{12}C$	0.43	$\mathcal{S}(^{13}C=^{12}C+p_{1/2})=0.66$	0.61
(b) $^{11}B(^{12}C, ^{11}B)^{12}C$	6.7	$\mathcal{S}(^{12}C=^{11}B+p_{3/2})=2.6$	2.85
(c) $^{12}C(^{14}N, ^{13}C)^{13}N$	0.53	Using(a) we get $\mathcal{S}(^{14}N=^{13}C+p_{1/2})=0.80$	0.69
(d) $^{11}B(^{14}N, ^{13}C)^{12}C$	2.3	Using(b) we get $\mathcal{S}(^{14}N=^{13}C+p_{1/2})=0.88$	0.69
(e) $^{11}B(^{14}N, ^{15}O)^{10}Be$	0.48	...	0.46
(f) $^{11}B(^{16}O, ^{15}N)^{12}C^*$ (2^+)	3.5	Using $\mathcal{S}(^{16}O=^{15}N+p)=2$ $\mathcal{S}(^{12}C^*(2^+)=^{11}B+1p_{1/2})=1.7$	1.1

also be noted that the $l=0$ and $l=2$ have *equal* strength, this *despite* the factor $(2l+1) \times W^2(l_1 j_1 l_2 j_2; s_x l)$ in Eq. (11) which is $\frac{1}{12}$ for $l=0$; $\frac{5}{12}$ for $l=2$.

In semiclassical theories and nonrecoil DWBA calculations, the highest l transfer is always the most important, in complete disagreement with the exact recoil DWBA. As will be shown later, however, the rule is more nearly valid at lower energies.

In the above calculations (and all those to follow), the nucleon is bound to the core in a Woods-Saxon potential with $r_0=1.25$, $a_0=0.65$ in both bound states. For each reaction, the same optical-potential parameters were used in the incoming and outgoing channels. The values were those which were found in the experimental study to give a good fit to the elastic scattering in the incident channel and are listed in Table I.

For these elastic transfer reactions, the product $S_1 S_2$ is just S^2, making the reactions good spectroscopic tools. The spectroscopic factors found are listed in Table II and are in excellent agreement with the theoretical values of Cohen and Kurath.[22] It should be noted that analyses of light-ion reactions between ^{12}C and ^{11}B or ^{11}C have some difficulties fitting the experimental data.[23]

C. ^{12}C(^{14}N,^{13}C)^{13}N

A preliminary account of the effects of recoil on single-nucleon-transfer heavy-ion reactions used this reaction as an example.[24] Similar to the ^{13}C(^{12}C, ^{13}C)^{12}C reaction, the experimental data show no sign of structure whereas nonrecoil DWBA calculations oscillate rapidly. Again we have

$$j=\tfrac{1}{2}, \quad s=\tfrac{1}{2}, \quad l=0,1,$$

but only $l=0$ is allowed in nonrecoil theories. As

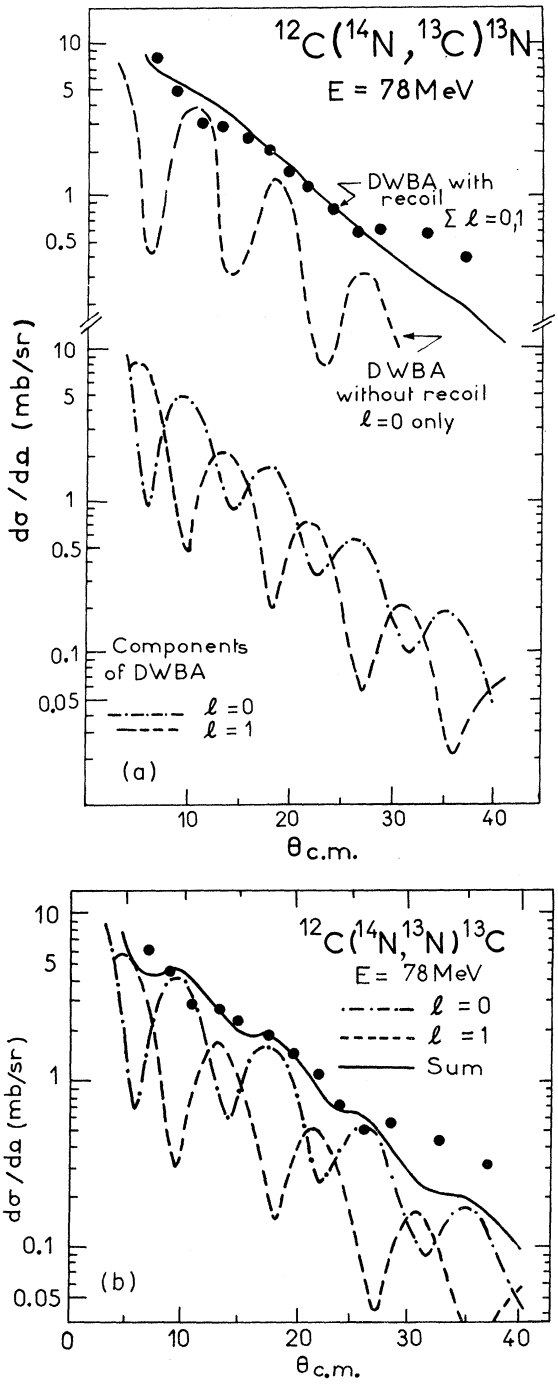

FIG. 4. Comparison of experiment and theory for the reaction $^{12}C+^{14}N \rightarrow {}^{13}C+{}^{13}N$.

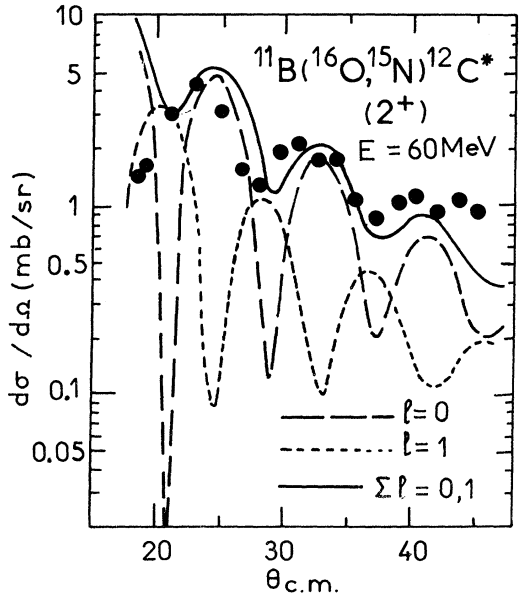

FIG. 5. Comparison of theory and experiment for the ^{11}B(^{16}O, ^{15}N)^{12}C reactions.

shown in Fig. 4(a), the recoil l-transfer component is absolutely necessary to obtain agreement with the experimental data. The reaction ^{12}C-(^{14}N, ^{13}N)^{13}C [Fig. 4(b)] occurs at complementary angles ($\pi - \theta$) but does not interfere significantly with the proton-transfer reaction. Both reactions also have $s = \frac{3}{2}$ components from the transfer of a $p_{3/2}$ nucleon from ^{14}N, but when multiplied by the very small Cohen-Kurath spectroscopic factors, this component is found to be completely negligible.

The spectroscopic factors found in both reactions were the same and are listed in Table II. Using the S (^{13}C = ^{12}C + n) found in Sec. 6 B, we arrive at a spectroscopic factor for ^{14}N in good agreement with Cohen and Kurath.

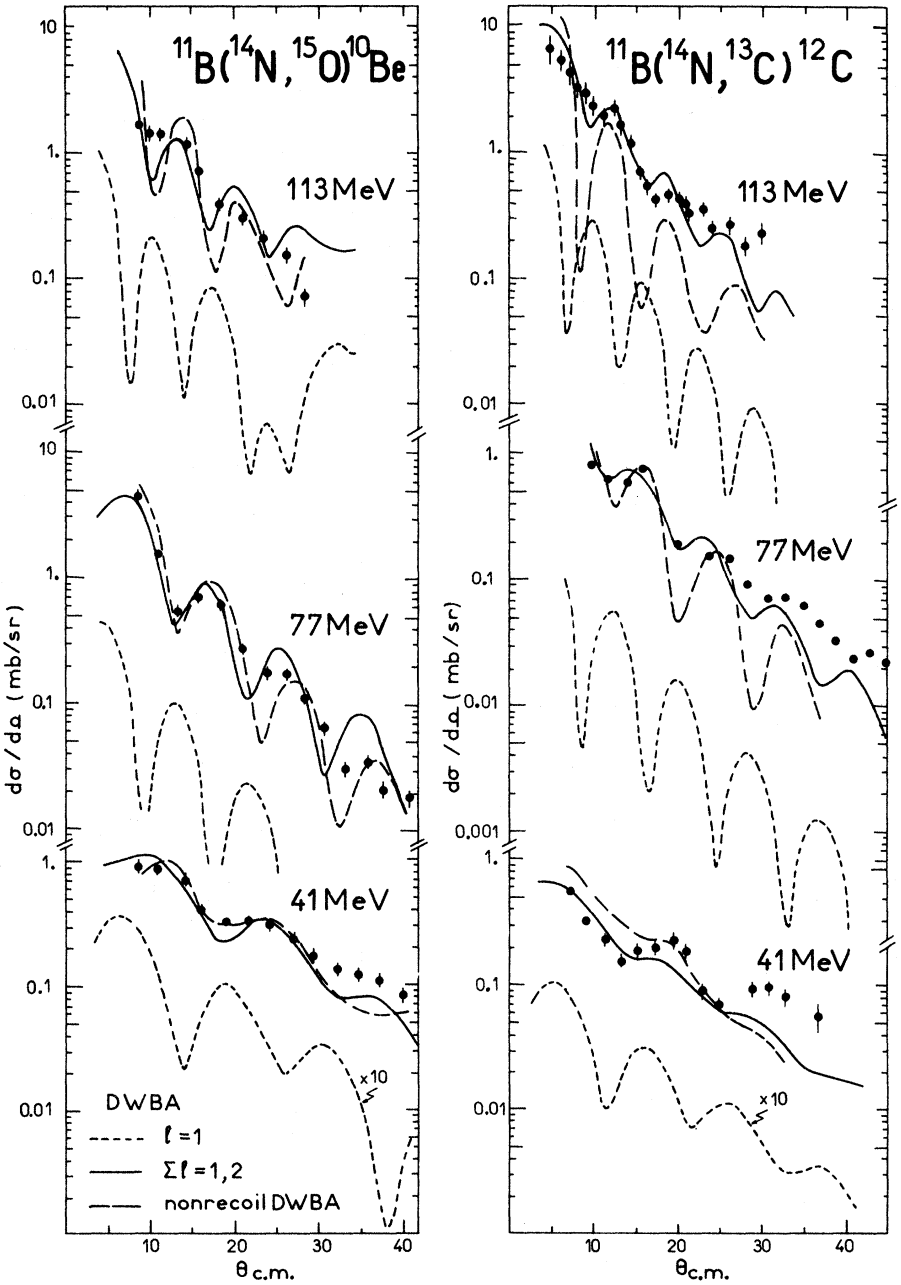

FIG. 6. Experimental data and theoretical predictions for the ^{11}B(^{14}N, ^{15}O)^{10}Be and ^{11}B(^{14}N, ^{13}C)^{12}C reactions where the "normal" l transfers dominate.

D. $^{11}B(^{14}N,^{13}C)^{12}C$ and $^{11}B(^{14}N,^{15}O)^{10}Be$

For both of these reactions the selection rules are

$$j=\tfrac{3}{2},\quad s=\tfrac{1}{2},\quad l=1,2,$$

and the factor $(2l+1)W^2(l_1 j_1 l_2 j_2; s_x l)$ is: $\frac{1}{12}$ for $l=1$; $\frac{5}{12}$ for $l=2$.

This result favors the "normal" $l=2$ component particularly at lower energies. The experimental data[20] were taken at 41, 77, and 113 MeV and allow us to see the effects of recoil on "normal" parity transfers as a function of energy, as shown in Fig. 6. In Ref. 20 an attempt was made to approximately include recoil with the result that the minima of the nonrecoil DWBA partially filled in, particularly at higher energies. The exact recoil DWBA has considerably reduced minima although the prediction still oscillates more than the experimental data at the higher energies. Interestingly enough the (^{14}N, ^{15}O) seems to be less affected by recoil than the (^{14}N, ^{13}C) reaction, although the two exact recoil DWBA curves are similar, the two nonrecoil predictions are rather different. Unfortunately, the 113-MeV predictions are rather sensitive to the optical parameters chosen. The (^{14}N, ^{13}C) reaction produced the best fits with the $V_0=50$ MeV sets, while the (^{14}N, ^{15}O) reaction preferred the $V_0=100$ MeV sets. The spectroscopic factors found at 41 and 77 MeV were identical and are listed in Table II, but the values found at 113 MeV were as much as 2 times larger depending on which optical set was used.

It is also interesting to note that the $l=1$ component is completely negligible at 41 MeV but would be quite important at 113 MeV were it not for the $(2l+1)W^2$ factor.

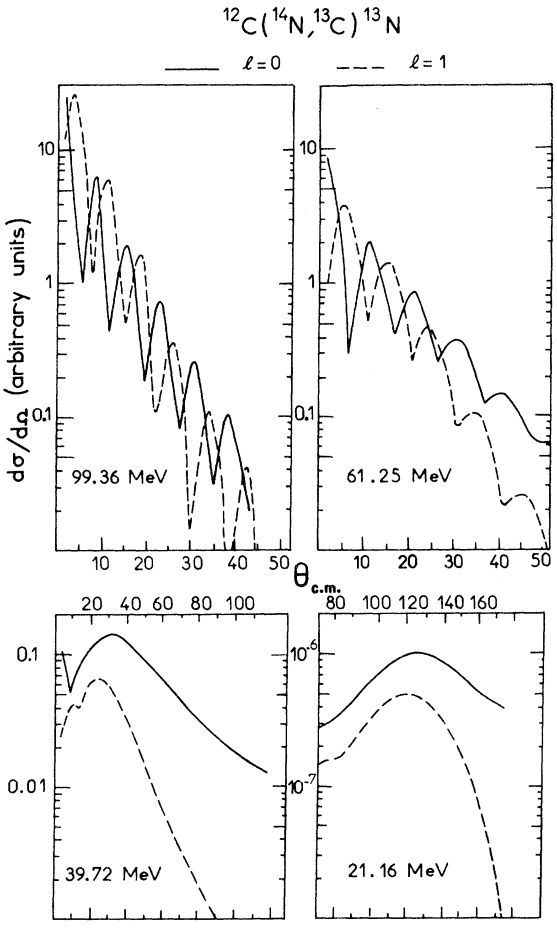

FIG. 7. Predicted cross sections for the $l=0$, 1 components of the $^{12}C(^{14}N,^{13}C)^{13}N$ reaction as a function of incident energy using the same 78-MeV optical-model potentials.

E. $^{11}B(^{16}O,^{15}N)^{12}C^*$ (2^+)

The Heidelberg group measured one-proton-transfer heavy-ion reactions induced by 60-MeV ^{16}O ions on ^{11}B targets. These data were analyzed with a slightly modified Buttle-Goldfarb nonrecoil approximation.[19] Using the optical potentials determined from the incident-channel elastic scattering they were able to obtain a reasonable fit to the $^{11}B(^{16}O, ^{15}N)^{12}C$ ground-state reaction. In this case

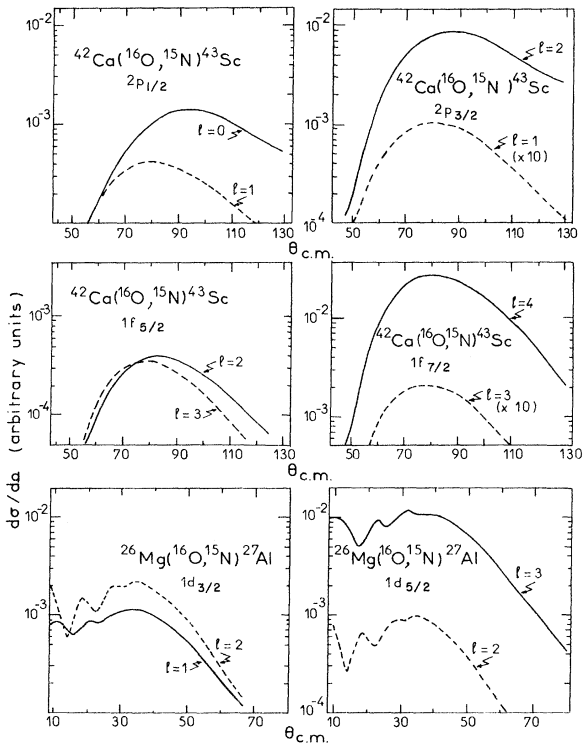

FIG. 8. Theoretical predictions for (^{16}O, ^{15}N) at $E_{^{16}O}$ =42 MeV for heavier targets and larger l transfers.

the selection rules are:

$$j=\tfrac{3}{2}, \quad s=\tfrac{1}{2}, \quad l=1,2,$$

with the $l=2$ being the "normal" parity component. At these energies we would expect an $l=2$ "normal" component to be much stronger than an $l=1$ "nonnormal" component as was seen in Sec. 5 D.

For the $^{11}B(^{16}O, ^{15}N)^{12}C^*(2^+)$ reaction, however, they found that angular distributions calculated with parameters which fitted the elastic scattering showed too pronounced structure as compared with the experimental data. In order to get better agreement, the optical parameters were arbitrarily changed in that study. In this case the selection rules are:

$$j=\tfrac{1}{2}, \quad s=\tfrac{1}{2}, \quad l=0,1,$$

with the $l=0$ as the "normal" component. The exact recoil DWBA prediction using the unmodified optical parameters is shown in Fig. 5 and clearly shows why the experimental data do not have deep minima. A reanalysis of the data from that experiment is in progress.

F. Extrapolations

Figure 7 shows the effect of changing the energy on the reaction $^{12}C(^{14}N, ^{13}C)^{13}N$ of Sec. 6 C using the same optical parameters as were used at 78 MeV. As the incident energy decreases, the nonnormal components become less important in agreement with previous estimates.[25]

In order to study the effect of increasing the target mass, calculations were performed for the hypothetical reactions $^{42}Ca(^{16}O, ^{15}N)^{43}Sc$ ($2p_{3/2}$, $2p_{1/2}$, $1f_{7/2}$, $1f_{5/2}$ final states) at 42 MeV and $^{26}Mg(^{16}O, ^{15}N)^{27}Al$ ($1d_{5/2}$ and $1d_{3/2}$ final states) at 42.3 MeV. The results are shown in Fig. 8.

We may summarize things as follows: Consider a heavy-ion reaction which transfers a $1p_{1/2}$ nucleon from the projectile to a $j_{\gtrless} \equiv l_2 \pm \frac{1}{2}$ orbit around the other core (vice versa for a pickup reaction) with an l-transfer value of $l^{\pm}$ corresponding to the upper or lower possible value.

$j_>$ States

Recoil will affect the shapes of the l^+ (normal) components probably by reducing the diffraction structures (if they exist), but the l^- (nonnormal) components will be small.

$j_<$ States

Recoil will affect the shapes of the l^- (normal) components and will introduce l^+ (nonnormal) components which should never be ignored and often will seriously affect the shapes and spectroscopic factors of the DWBA predictions.

It is expected that these results will be more pronounced as the target mass is increased, as the l_2 value is raised, and as the incident energy goes up.

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