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Proton-Proton Final-State Interaction in the ^{2}H(p, n)2p Reaction*

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Neutron spectra from the ^{2}H(p, n)2p reaction have been measured at nine proton energies between 16 and 26 MeV. The high-energy portions of the neutron spectra taken at a laboratory angle of 3.5° have been analyzed with impulse approximation calculations of the proton-proton final-state interaction to extract the proton-proton scattering length. The average value of the scattering length determined from these measurements is within approximately 1 fm of the −7.8-fm result of pp scattering but is not independent of the details of the comparison of experimental and calculated spectra. Experimental requirements for a determination of the neutron-neutron scattering length from the ^{2}H(n, p)2n reaction are discussed.

I. INTRODUCTION

Reactions leading to three particles in the final state have been extensively studied to obtain information about two-body forces not readily observable in two-particle scattering experiments. A variety of reactions producing two neutrons has been examined under kinematic conditions such that the neutrons have low relative energy. The goal of such studies has been to obtain the parameters of the neutron-neutron interaction from analysis of this final-state interaction. Most analyses to extract the nn parameters, principally the scattering length, make simplifying approximations, *e.g.*, use of the impulse approximation, representing the nucleon-nucleon interaction by S-waves only, ignoring other possible final-state interactions, among others. These approximations must be justified by equivalent analyses of charge conjugate reactions to obtain the well established proton-proton or neutron-proton parameters in order to establish the validity of the nn results. This comparison technique has been reviewed recently by several authors.[1-3]

The simplest reactions which allow parallel treatment of nn and pp final-state interactions are the nucleon-induced deuteron breakup reations, ^{2}H(n,p)2n and ^{2}H(p,n)2p. Enhancement of the yield of high-energy neutrons from the ^{2}H(p,n)2p reaction resulting from the final-state interaction of the two protons has been observed in measurements at this laboratory[4] for proton bombarding energies between 6 and 14 MeV, at 20 MeV by Slobodrian, Conzett, and Resmini,[5] at 30 and 49.5 MeV by Clough *et al.*,[6] and at 45.5 MeV by Margaziotis, Wright, and van Oers.[7] For laboratory angles less than 5°, the shape of the high-energy portion of the neutron spectra reported in Refs. 4,6, and 7 is well represented by impulse approximation calculations using the correct pp scattering length and effective range. However, no attempt was made to optimize the fits to the data to extract values for the pp scattering length. As this procedure has been used in several attempts[8-13] to obtain the nn scattering length from forward-angle ^{2}H(n,p)2n spectra for neutron bombarding energies between 8 and 24 MeV, effort to improve the available ^{2}H(p,n)2p data in this energy range and to apply an equivalent treatment is worthwhile. The Livermore cyclograaff neutron time-of-flight facility makes possible measurements with improved neutron energy resolution over most of the range of previous measurements.

II. EXPERIMENTAL PROCEDURE

The Livermore cyclograaff time-of-flight facility utilizes the post-acceleration sweeper and multiple detector array[14] formerly used with the 90-in. variable energy cyclotron. A 15-MeV ^{1}H$^-$ beam is extracted from the 80-cm fixed energy cyclotron and post-acceleration swept to reduce the burst rate from 25 to 5 MHz. This beam is injected into the EN tandem and accelerated to the desired energy. An average target current of 1 μA was obtained with a 2.5-ns burst width.

Deuterium gas at a pressure of 1.5 atm was contained in a double foil gas target 2.5 cm long with 10-mg/cm^2 Ta windows. Neutrons were detected with a 5-cm-diam × 5-cm-long NE 213 scintillator 10.7 m from the target at an angle of 3.5° to the proton beam. The scintillator was shielded by 1 m of concrete and 4 m of earth. A 2-m-long water collimator limited the area visible by the scintillator to the target region so that neutrons from reactions on slits, collimators, and the beam dump were not detected. Beam-associated backgrounds were determined by placing Ta foils identical to those on the gas target in the proton beam at each energy.

Neutron time-of-flight spectra were obtained with conventional fast-timing methods. Pulse-shape discrimination was used to separate proton and electron recoils in the scintillator. A γ-ray time spectrum was accumulated simultaneously with the neutron spectrum to monitor the time width of the beam pulses by observing the time width of the prompt γ burst from the target. Energy resolution of the cyclograaff proton beam is 60 keV or less when the accelerators are operated for time-of-flight measurements. Proton energy loss and straggling in the target gas and entrance foil contributed less than 50-keV spread to the neutron energy. The largest contributions to the neutron energy resolution were the proton burst duration and the neutron transit time across the scintillator. The resolution of the time-of-flight spectrometer was determined at each energy by measuring the width of the ground-state neutron group from the $^{11}B(p,n)^{11}C$ reaction using a thin target. As the Q value for this reaction is −2.7 MeV, the ground-state neutron goup occurs within 100 keV of the peak of the $^2H(p,n)2p$ neutron spectrum at all bombarding energies. The measured neutron energy resolution deteriorated from 0.5 MeV for 16-MeV proton energy to 1.1 MeV for 26-MeV proton energy.

Neutron spectra were taken with the bias on recoil pulse height set sufficiently high that only scintillations from proton and electron recoils would be accepted for analysis. The detector efficiency as a function of neutron energy could then be calculated directly from the neutron-proton differential cross sections and the measured pulse-height response of the NE 213 scintillator.[15] The biases were set at equivalent proton recoil energies of 3.5 MeV for proton bombarding en-

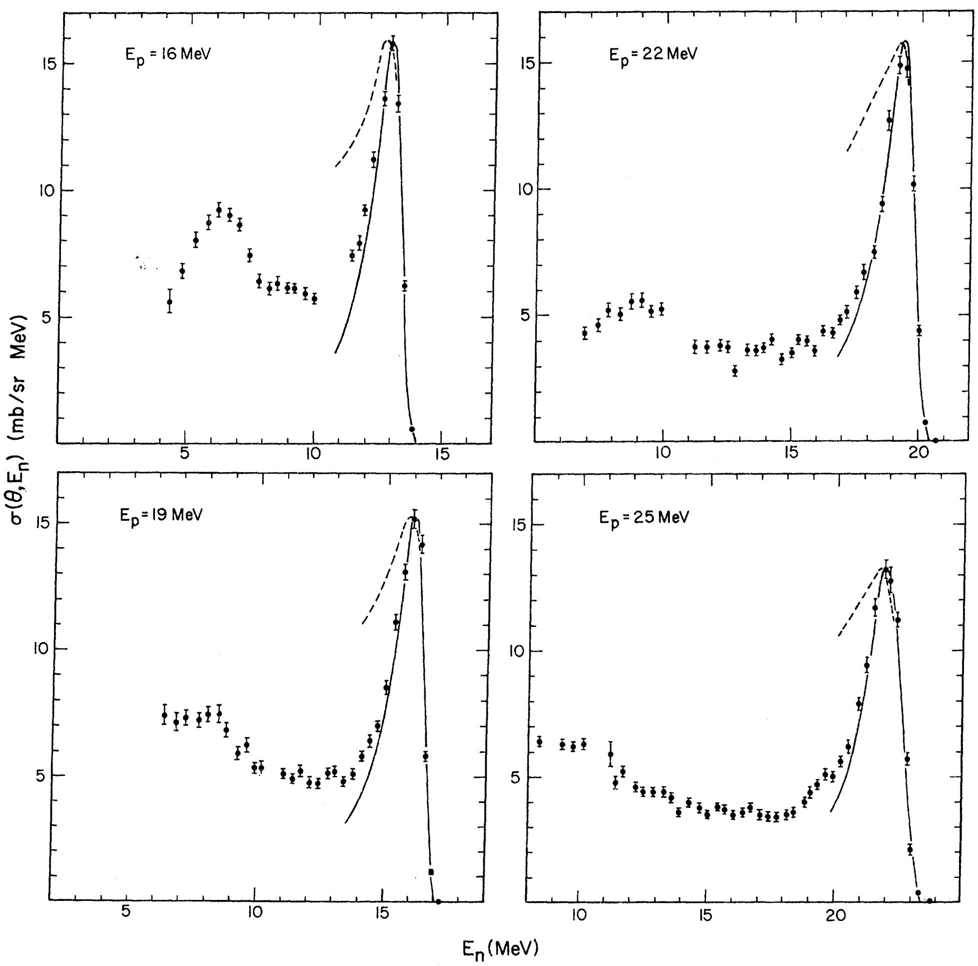

FIG. 1. Neutron energy spectra from $^2H(p,n)2p$ reaction for several proton bombarding energies measured at 3.5° in the laboratory system. The data have been averaged in 300-keV bins to reduce the number of points. Results of impulse approximation and Watson-Migdal calculations of the pp final-state interaction peak are shown as solid and dashed lines, respectively. The scattering length and effective range used in the calculations are those determined from pp scattering. Gaps in the spectra result from omission of channels containing γ rays.

ergies of 16 and 17 MeV and at 5.5 MeV for bombarding energies of 18 MeV or greater. The absolute normalization is known with an uncertainty of approximately 7%.

III. MEASUREMENTS AND CALCULATIONS

A. Neutron Spectra

Neutron energy spectra from the $^2H(p,n)2p$ reaction taken at a laboratory angle of 3.5° are shown for several proton energies in Fig. 1. The enhancement of the high-energy neutron yield resulting from the *pp* final-state interaction is evident. This neutron yield enhancement is separated by 7 to 12 MeV from the enhancement at lower neutron energies resulting from the *np* final-state interaction. Adequate separation is needed if the assumption that the shape of the high-energy end of the neutron spectrum is dominated by a single final-state interaction is valid. Also shown in Fig. 1 are spectra calculated with the Watson-Migdal final-state-interaction formalism[16,17] and the impulse approximation. The spectra for both formalisms were calculated with the formulas given by Phillips[18] for values of the $^1S_0$ *pp* parameters[19] $a_{pp} = -7.82$ fm, $r_{pp} = 2.79$ fm and normal-

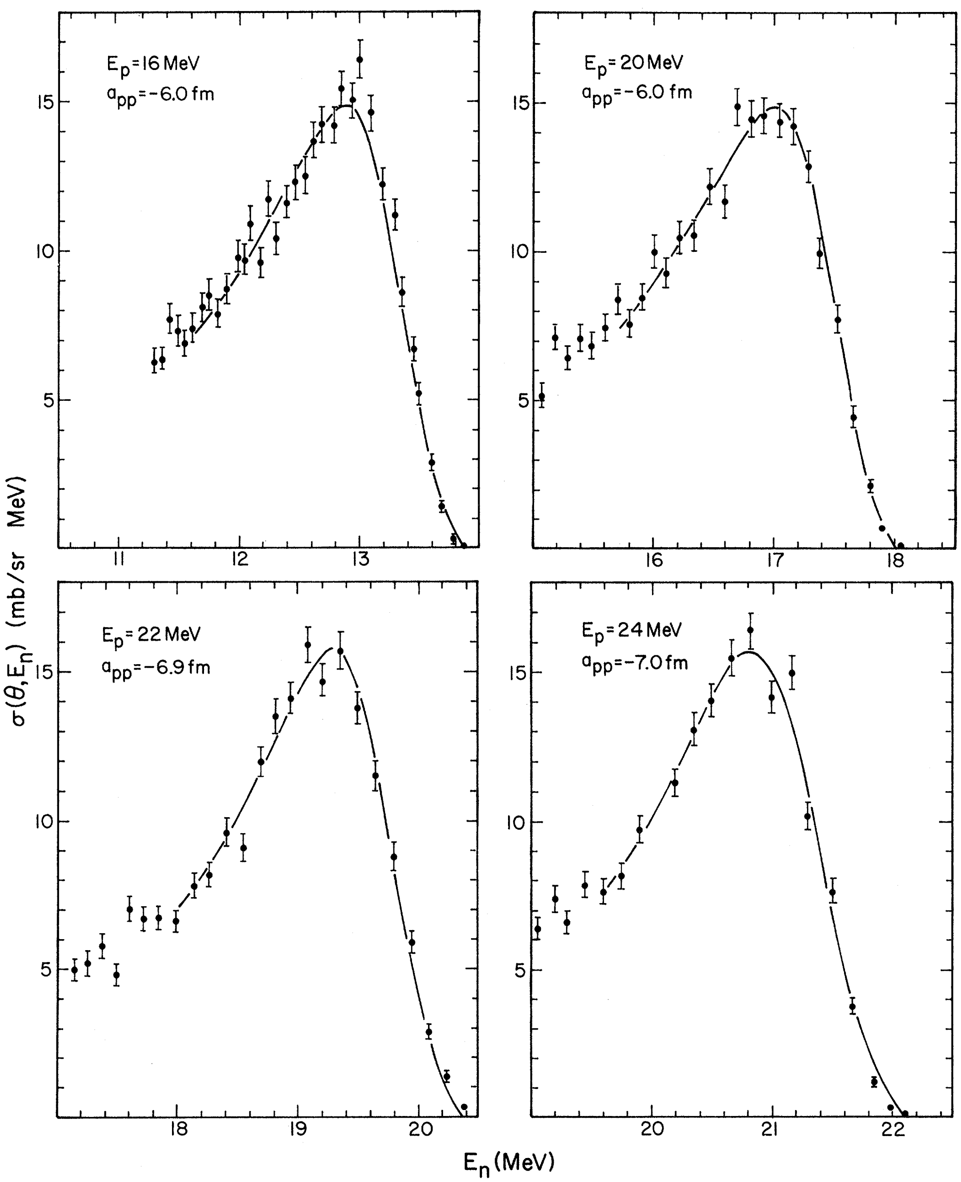

FIG. 2. Fits to the highest 2 MeV of the neutron spectra obtained by optimizing the fit of the impulse approximation calculation of the final-state-interaction peak.

ized to the peak height of the data. Breakup of deuterons by nucleons is more appropriately described by the impulse approximation than the Watson-Migdal formalism because the extended structure of the deuteron results in breakup proceeding through long range processes. While the impulse approximation calculations reproduce the shape of the upper end of the data quite adequately, no agreement between data and Watson-Migdal calculations is possible for realistic values of a_{pp}. The inadequacy of the simple Watson-Migdal approach for deuteron breakup has been examined in some detail by Aitchinson.[20]

B. Spectrum Fitting

The data obtained at 3.5° lab were fitted with impulse approximation calculations to obtain values of the pp scattering length. Changes in the pp effective range of the order of 10% resulted in negligible changes in the shape of the calculated spectra. For all calculations, the effective range was fixed at 2.79 fm while the scattering length was varied in 0.5-fm steps from −5.5 to −9.5 fm to determine a value which best fitted the data. The internal energy of the two-proton system is related to the neutron energy E in the center-of-mass system by

$$E_{pp} = \tfrac{3}{2}(E_{\max} - E)_{\text{c.m.}} ,$$

where $E_{\max}$ is the maximum possible neutron energy. After folding the measured neutron energy resolution into the calculated spectra, small shifts in energy were made to align the rise from threshold of the experimental and calculated spectra. These energy adjustments were usually less than the separation of neighboring data points corresponding to adjacent time channels. This separation is the inherent uncertainty of the time-of-flight energy determination.

TABLE I. Scattering length values from area normalization and peak normalization.

E_p (MeV)	Area normalization a_{pp} (fm)	Δa_{pp} (fm)	$\chi^2_{\min}$	N	Peak normalization a_{pp} (fm)
16	−6.0	±0.2	41	25	−7.3
17	−6.5	±0.2	40	22	−7.0
18	−6.2	±0.2	25	21	−7.2
19	−6.8	±0.2	28	18	−8.1
20	−6.0	±0.3	18	15	−6.4
22	−6.9	±0.3	23	13	−7.5
24	−7.0	±0.4	16	10	−7.3
25	−6.5	±0.4	19	11	−7.6
26	−7.5	±0.5	9	11	−8.2
Average	−6.6	±0.5			−7.4 ± 0.7

The spectra calculated with the impulse approximation formalism were normalized to the integral of the experimental data over the neutron energy interval corresponding to $0 \leq E_{pp} \leq 2$ MeV. This energy range includes the upper 2.2–2.5 MeV of the laboratory neutron spectra when resolution effects are included. The value of the scattering length giving the best fit to the data at each proton bombarding energy was determined by minimizing χ^2. Experimental data and normalized spectra for several scattering length values are shown for four proton bombarding energies in Fig. 2.

C. Scattering Length Results

In Table I, the value of the scattering length corresponding to the best fit obtained by normalizing to the integrated cross section at each bombarding energy is given. The minimum value of χ^2 obtained at each energy, the number of degrees of freedom, and the statistical uncertainty in the scattering length corresponding to increasing χ^2 by unity are also given. At several energies, the minimum value of χ^2 is much larger than the number of degrees of freedom (taken to be the number of data points less three). Averaging pairs of data points and repeating the χ^2 calculations produced significantly better agreement between the mini-

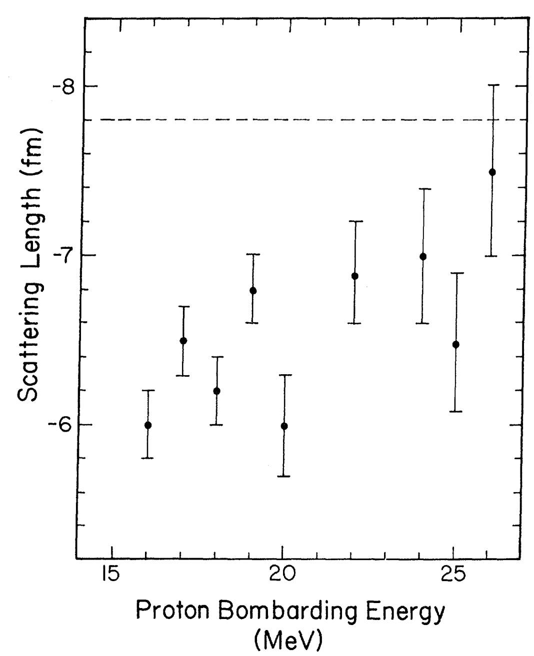

FIG. 3. Values of the pp scattering length obtained by optimizing the fit of the impulse approximation spectra to the final-state-interaction peak observed in the 3.5° neutron spectrum at each proton bombarding energy. The dashed line is the result obtained from analysis of pp scattering.

mum value of χ^2 and the suitably reduced number of degrees of freedom, indicating that some channel-sharing problems were present in the analog-to-digital converter used in data taking. However, the best fit value obtained for the scattering length was unaltered.

A mean value of -6.6 ± 0.5 fm is obtained when the scattering lengths determined separately are averaged together. The uncertainty quoted for the average is the standard deviation of individual values from the mean and is not consistent with the statistical uncertainties quoted in Table I. This discrepancy results from the energy dependence of the scattering lengths. A systematic trend toward more negative values with increasing proton bombarding energy is apparent when the results are plotted against proton bombarding energy as in Fig. 3. The contribution of processes other than the *pp* final-state interaction (fsi) is the most probable cause of this behavior. For a fixed-incident proton energy, the probability that rescattering and the *np* final-state interaction contribute to neutron production increases with decreasing energy of the observed neutron. In the present analysis, the comparison of measured and calculated spectra is performed for the same 2-MeV interval in E_{pp} at all incident proton energies. As the proton bombarding energy increases, the *pp* final-state interaction peak becomes better separated from the other allowed processes. The contribution of these processes to the cross section in a fixed interval near the peak should then decrease with increasing incident proton energy. These contributions would result in the extraction of a too-positive value of the scattering length, since for a simple model one would expect the cross section in the tail of the final-state-interaction peak to be increased more than the peak cross section itself. The trend in the deduced scattering length values is consistent with the above hypothesis.

An alternative normalization of the calculated spectra to the data which reduces the sensitivity of the fitting process to contamination of the tail of the fsi peak may be used. If the calculated spectra are adjusted to fit the experimental peak

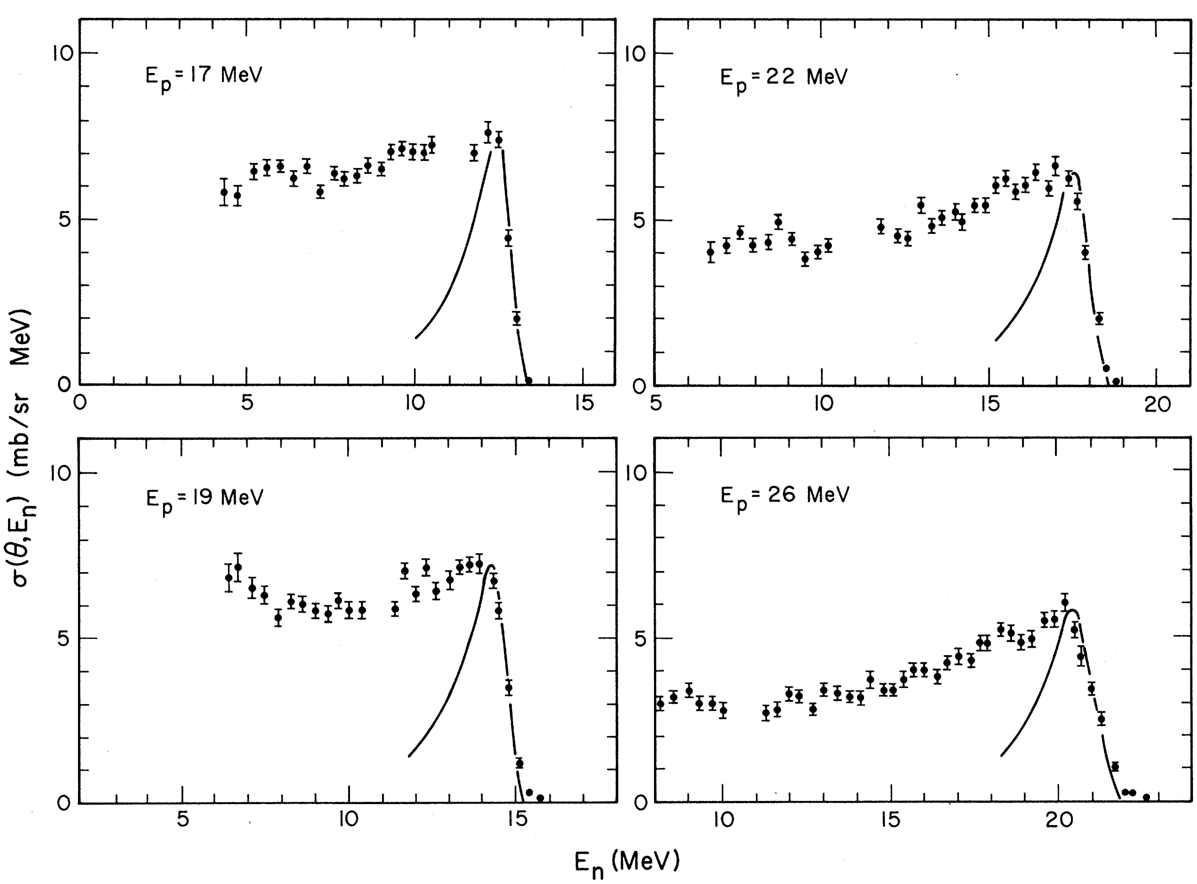

FIG. 4. Neutron spectra measured at 23.8° in the laboratory system. The solid line is the impulse approximation result calculated for the scattering length value which gave the best fit to the 3.5° spectrum at each bombarding energy.

height rather than the integrated cross section, data points near the peak are weighted more heavily than those at lower neutron energy. The best fit to the spectrum is determined by minimizing the mean square deviation of the calculated spectra with respect to the data over $0 \leqslant E_{pp} \leqslant 2$ MeV interval used previously. The scattering length values obtained with this procedure are listed in Table I. As expected, the scattering lengths determined are more negative. However, they still show the trend with energy exhibited by the results obtained from area normalization. The average value is -7.4 ± 0.7 fm. Although the fits produced with this normalization method are not statistically as good as those produced by the area method, they may be physically more reasonable. For all values of the scattering length, spectra normalized to the peak height fitted the high-energy side of the fsi peak equally well. This is to be expected since this portion of the spectrum represents only the experimental resolution. The best fit spectra produced by normalizing to the integrated cross section do not fit the rise of the fsi peak as well as those obtained with peak height normalization: The fit to the high-energy rise is poorer to compensate for a better fit at lower energy where contributions to the cross section from competing processes are present. The difference between the average values obtained using the two normalization methods probably represents a reasonable estimate of the theoretical uncertainty involved in the extraction of the scattering length from kinematically incomplete data.

IV. DISCUSSION

The description of proton-induced deuteron breakup in terms of the impulse approximation gives a reasonable fit to the high-energy portion of the neutron spectrum at 3.5° lab for proton energies between 16 and 26 MeV. Although the values of the pp scattering length extracted from the data are somewhat dependent upon the details of the comparison of calculated and experimental spectra, the average value of a_{pp} obtained is within approximately 1 fm of the free scattering result. The deviation of the deduced scattering length values from the free scattering result is qualitatively consistent with the presence of contributions from the np final-state interaction and rescattering.

The present treatment is inadequate for angles beyond the extreme forward ones. Neutron spectra measured at 24° lab are compared with impulse approximation calculations in Fig. 4. No agreement is possible for realistic values of the scattering length. Similar failure of the impulse approximation beyond the forward angles has been observed at higher bombarding energies.[6,7] The momentum transfer increases very rapidly with the neutron laboratory angle; the probability of rescattering effects increases as a consequence, reducing the dominance of the pp final-state interaction.

The limited success of the present extraction of the pp scattering length from ^{2}H$(p,n)2p$ spectra satisfies a necessary but not sufficient condition for the extraction of the nn scattering length from small angle ^{2}H$(n,p)2n$ spectra. Assuming that the impulse approximation treatment of the deuteron breakup process is valid, one may investigate the experimental requirements for a determination of a_{nn} with statistical accuracy of the order of ± 2 fm. The most probable value of a_{nn} is obtained by assuming charge symmetry of the nucleon-nucleon interaction and calculating a Coulomb corrected value of a_{pp}. The result of this calculation[19] is that a_{nn} should be -17 ± 1 fm. An alternate value may be obtained by assuming charge independence and requiring that a_{nn} be the same as the np singlet scattering length, -24 fm. The further assumption that the -17-fm result for the Coulomb corrected value of a_{pp} is in error must then be made. Analysis of the ^{2}H$(\pi^-, \gamma)2n$ reaction[21] and several recent kinematically complete neutron-induced deuteron breakup experi-

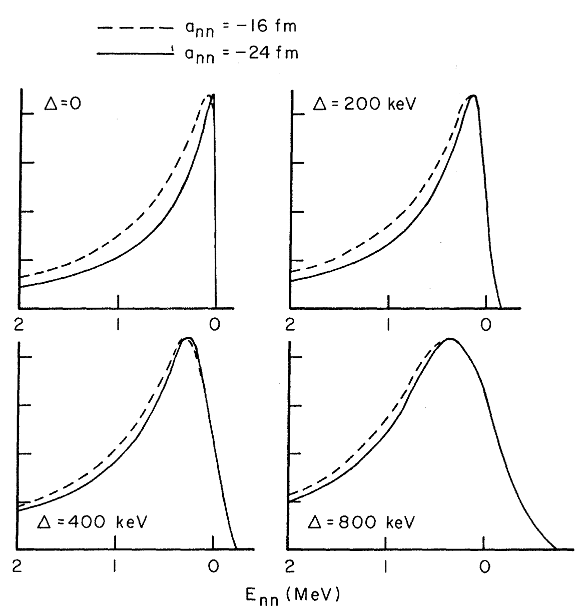

FIG. 5. Impulse approximation calculations for the nn final-state-interaction peak in the ^{2}H$(n,p)2n$ reaction. Spectra for two extreme values of the scattering length are compared when observed with different values of the experimental resolution.

ments[22-24] yield values of a_{nn} near −16 fm. However, attempts to fit ^{2}H(n,p)2n spectra in a manner similar to the present work have resulted in scattering lengths grouped near −16[12,13] and −22 fm.[8-11]

In Fig. 5 we show impulse approximation calculations of the high-energy portion of the ^{2}H(n,p)-2n proton spectrum. The spectra were calculated for the extreme scattering length values −16 and −24 fm. Instrumental resolution has been simulated by folding in a Gaussian function trucated at 10% of peak height. The spectra for the two values of a_{nn} have been adjusted to have the same peak height. The difference between the curves diminishes rapidly as the instrumental resolution broadens. A rough experimental figure of merit may be devised by determining the statistical accurancy required to differentiate between the curves to ±2 fm at E_{nn} = 1 MeV. For the ideal case in which no instrumental broadening occurs, the accuracy required is 10%. As the spectra are measured with poorer resolution, the difference decreases to approximately 5.6% for 200-keV resolution, 3.3% for 400-keV resolution, and 2.5% for 800-keV resolution. The instrumental resolution obtained in all measurements of ^{2}H(n,p)-2n spectra reported to date has been 500 keV to 1 MeV and the statistical uncertainties have been typically 5 to 10%. In view of the experimental requirements suggested above, the difference between the values near −16[12,13] and −22 fm[8-11] deduced from these measurements does not appear statistically significant.

Although quite difficult, measurement of the ^{2}H(n,p)2n spectra with proton energy resolution near 200 keV should be possible with a magnetic spectrograph. The results obtained here for the ^{2}H(p,n)2p reaction suggest that such a measurement performed at or above 30 MeV would yield a value for a_{nn} whose uncertainty would be more dominated by the uncertainty of the model calculation than by experimental considerations.

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