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Fission in Stellar Plasma*

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The fission of heavy nuclides is examined in a dense stellar plasma. The correction in the Coulomb energy due to the interaction between the ion and the induced-electron charge density affects the process of fission and reduces the fission barrier depending upon the physical situation. Shell- and surface-symmetry effects are included in the calculation of the fission barrier as a function of the fissility parameter χ and surface energy of a spherical nuclear drop E_s^0. The fissionability of some of the heavy elements that are likely precursors of superheavy elements in an r-process chain may adversely affect the formation of superheavy elements in stellar situations.

I. INTRODUCTION

At high temperatures and densities in highly evolved stars such as supernova, the matter is fully ionized. Burbidge *et al.*[1] hypothesized that spontaneous fission of ^{254}Cf was responsible for the energy in the exponential part of the light curve of supernova type I. This suggestion led to an understanding of r-process nucleosynthesis of transuranic elements in stellar situations.[2] It was later suggested[3] that the proper conditions for the r process were nearly met in massive objects associated with quasistellar radio sources. Objects with masses in the range 10^4–$10^6 M_\odot$ are found to provide the necessary environment for r-process nucleosynthesis.[4] More recently there was a suggestion that both supernova type I and type II conditions were necessary for r-process nucleosynthesis of superheavy elements.[5]

The heavy ionized nuclei in stellar interiors draw around them the free electrons in the assembly. As a result the electron charge density affects the Q value of reactions[6] and modifies the Coulomb energy of the nucleus. So far there seems to be no attempt to study the effect of the induced charge on the Coulomb energy of the fissioning nuclei for high densities and temperatures. It is the purpose of this paper to calculate the Coulomb corrections at the high temperature and density conditions generally available in massive stars and supernova. We use the method[6] of a modified cluster expansion, where the effective electron-ion Hamiltonian is constructed on the basis of the electrons forming a linear dielectric medium with the positive ions forming a uniform neutralizing background.

The fissility parameter is defined as the ratio of the Coulomb energy to twice the surface energy of a spherical drop. Therefore, any correction in the Coulomb energy due to the physical environment will certainly modify the fissility parameter. As a consequence the rate of fission will be modified.

It was pointed out[2] that neutron-rich nuclei were more susceptible to fission due to the surface-energy term. The possible superheavy-element synthesis in r process is related to the strong influence of the shell effect as well. The shell effect arises due to bunching of single-particle levels in nuclei. We have included the surface-symmetry term as well as the shell-correction effects in our calculation.

The superheavy elements are supposed to be formed in the r process[5,7] at various astrophysical situations. Some of the heavy elements in the transuranic region may be important seed nuclei for the continuation of the r-process chain to the superheavy region. The fissionability of these nuclei may be important in fully understanding the r-process synthesis of superheavy elements in stellar situations.

In Sec. II the method for calculating the Coulomb correction is given. The method for calculating the fission barrier is given in Sec. III. Section IV summarizes and concludes the results of our study.

II. METHOD OF CALCULATION

The interaction energy of an ion with the induced-electron charge density, to a sufficient approximation, is given by[6]

$$\delta M c^2 = -\tfrac{1}{2} q_\alpha{}^2 k_e, \tag{1}$$

where $q_\alpha = Ze$, the charge of the αth ion and

$$k_e{}^2 = 4\pi e^2 \int \frac{d^3p}{(2\pi\hbar)^3} \frac{1}{E}\left(1+\frac{E^2}{p^2c^2}\right)[f_-(E)+f_+(E)], \tag{2}$$

with $E=(c^2p^2+m^2c^4)^{1/2}$, $f_\mp(E)=[\exp\beta(E\mp\mu)+1]^{-1}$, and $\beta = 1/kT$ (k is the Boltzmann constant and T the temperature).

In what follows we have taken $E=cp$, i.e., neglected the electron rest energy.

A. High-Temperature Condition

To a sufficient approximation, it may be assumed that equal numbers of electrons and positrons are created[8] in equilibrium with radiation at a sufficiently high temperature. Then the electron and positron chemical potentials are zero. Integration of Eq. (2) then gives the value of k_e. Numerically the interaction energy is

$$\delta Mc^2 = -4.5\times10^{-4}(kT)Z^2 \text{ MeV}. \tag{3}$$

B. High-Density Condition

As the density increases, the energy of the electron far exceeds its rest energy and we have to consider relativistic effects. We consider here a completely degenerate relativistic electron gas. The energy of the electron in that limit is $E=cp$. We assume here that no positron is present in the assembly and so the f_+ term is dropped from Eq. (2). Then,

$$k_e{}^2 = \frac{4e^2}{\pi\hbar^3 c}\int \frac{E\,dE}{e^{\beta(E-\mu)}+1}. \tag{4}$$

The limiting energy, i.e., the chemical potential of the relativistically degenerate electron gas, is

$$\mu = (3\pi^2)^{1/3}\hbar c n_e{}^{1/3}, \tag{5}$$

where n_e is the number density of the electrons for which the spin multiplicity factor $g=2$. Substituting $\beta E = x$ and $\beta\mu = y$ in Eq. (4) one gets,

$$k_e{}^2 = \frac{4e^2(kT)^2}{\pi\hbar^3c^3}\int_0^\infty \frac{x\,dx}{e^{x-y}+1}. \tag{6}$$

Integrating Eq. (6) (neglecting $\pi^2/6$ compared to $y^2/2$ for $y \gg 1$) gives

$$k_e{}^2 \approx \frac{4e^2(kT)^2}{\pi\hbar^3c^3}\frac{y^2}{2}. \tag{7}$$

The interaction energy is then

$$\delta Mc^2 \simeq -Z^2\frac{\alpha^{3/2}}{\sqrt{2\pi}}y(kT), \tag{8}$$

where α is the fine-structure constant. Numerically,

$$\delta Mc^2 = -1.565\times10^{-5}Z^2\rho^{1/3} \text{ MeV g}^{-1/3}\text{ cm}. \tag{9}$$

C. Astrophysical Situation

The r-process environment leading to the synthesis of heavy elements is available[4] for objects with masses of the order of $(10^{5\pm1})M_\odot$. Fowler and Hoyle[9] give an equation which reduces to

$$\rho = 2\times10^5(M_\odot/M)^{1/2}T_9{}^3 \text{g cm}^{-3} \tag{10}$$

for T_9 near about 2.5. T_9 is the temperature in °K divided by 10^9. The densities are calculated for $M=10^4 M_\odot$, $10^2 M_\odot$, and $1.5 M_\odot$ from Eq. (10).

Ohnishi[5] has recently observed that both type I and type II supernovas are the likely sites for the r-process synthesis. In that case a lower mass ~$1.5 M_\odot$ has also to be taken into account. The density calculated from Eq. (10) for this mass is found to be 2.55×10^6 g cm^{-3} consistent with the density conditions assumed for explosive oxygen burning in a supernova model.[10] It is also believed that the surface of a neutron star may be the site for heavy-element synthesis by r processes. In that case, a still higher density may be available for the r process.

III. SURFACE-SYMMETRY ENERGY AND SHELL EFFECT

In the calculation of the fissility parameter χ, we need the surface energy E_s^0, which includes an asymmetry term to account for the difference between the neutron and proton numbers. The fissility parameter, following Bolsterli *et al.*[11] is

$$\chi = \frac{Z^2/A}{50.88[1-1.7826(I/A)^2]}, \tag{11}$$

where $I=N-Z$ is the neutron excess. Fission-stability estimates now include this symmetry effect which makes the heavy nuclei with high neutron excess more susceptible to fission with the increase of the fissility parameter χ.

Fission half-lives of the even-even isotopes of the heaviest elements exhibit a sharp maximum at $N=152$ and decreases rapidly at neutron numbers not only above 152 but also below it. The result due to Nurmia *et al.*[12] serves to illustrate the dramatic effect of the 152-neutron subshell in the fission half-lives of the isotopes of the heaviest elements. Beside such subshell effects, there are major shell effects between regions of various shell closures. Thus the shell-correction term has to be included in the fission-barrier calculation.

The experimental fission barrier can be obtained from the liquid-drop barrier after correcting for shell and pairing effects.[13] Taking the approximation that the shell correction at the saddle point is negligible and pairing corrections are the same at the ground state and the saddle point, the experimental fission barrier can be written as

$$E_b^{\text{exp}} - E_b^{\text{liq. drop}} = \Delta E_s, \tag{12}$$

where ΔE_s stands for the shell correction. Using Myers and Swiatecki's data[14] we plot ΔE_s versus the fissility parameter χ. This is approximated by a linear function using the method of least squares. The coefficient of Cohen and Swiatecki's expression[15] for the liquid-drop barrier is changed to account for the change of χ and E_s^0 values. We thus approximate the fission barriers by

$$E_b \approx 0.93(1-\chi)^3 E_s^0 + 36.87\chi - 29.80 \text{ MeV}, \quad (13)$$

where E_s^0, the surface energy of a spherical drop is given by[11]

$$E_s^0 = 17.9439[1 - 1.7826(I/A)^2]A^{2/3} \text{ MeV}. \quad (14)$$

When the fissioning nuclei are considered in various astrophysical situations, the effects of temperature and density have to be taken into account. The Coulomb force within the nucleus is repulsive and the attractive interaction in (3) and (9) facilitates the process of fission. The fissility parameter is then modified to

$$\chi_m = \chi - \frac{\delta M c^2}{2E_s^0}, \quad (15)$$

where χ is the usual fissility parameter defined in Eq. (11) and the second term on the right-hand side is the correction for the temperature and density effects. This modified fissility parameter is then used in Eq. (13) to calculate the fission barriers for a number of nuclei from U to Fm for various astrophysical situations.

TABLE I. Fission barrier.

Z	A	E_b (exp) (MeV)	E_b (calc) [a] (MeV)
92	236	5.8 [b]	5.59
	238	5.8 [c]	5.70
	240	···	5.80
94	240	4.7 [b]	4.81
	242	4.90 [d]	4.90
	244	4.8 [d]	4.99
	246	···	5.07
96	244	4.4 [d]	4.24
	246	4.5 [d]	4.31
	248	4.4 [d]	4.37
98	248	4.0 [d]	3.86
	250	4.1 [d]	3.90
	252	3.8 [d]	3.94
100	250	···	3.63
	252	···	3.65
	254	3.5 [d]	3.67

[a] Calculated from Eq. (13) of the text.

[b] I. Halpern, Ann. Rev. Nucl. Sci. 9, 245 (1959).

[c] E. K. Hyde, *The Nuclear Properties of Heavy Elements* (Prentice Hall, Englewood Cliffs, N. J., 1964), Part III, p. 23.

[d] Barriers obtained from the empirical relation due to Myers and Swiatecki (Ref. 14).

IV. RESULTS AND DISCUSSIONS

We have calculated the fission-barrier energies from Eq. (13) and compared with the values tabulated by Myers and Swiatecki.[14] It is found that the equation gives values which compare well with the experimental values. Therefore, it is quite natural to suggest that an approximate shell-correction term has been included in the calculation of the fission-barrier energies for the mass region considered here. These values are shown in Table I for comparison.

We now take this as the standard equation for the calculation of fission barriers. We consider first a high-temperature condition $kT = 1$ MeV and then obtain various densities consistent with the mass range from $1.5M_\odot$ to $10^4 M_\odot$ for $T_9 = 2.5$. Neutrons are produced at about $5\times10^9\,°\text{K}$ in supernova conditions. Neutron addition to seed nuclei, however, takes place during the expansion as the temperature falls to about $10^9\,°\text{K}$.[2] We assume here a temperature $T_9 = 2.5$ for r-process buildup of heavy fissioning nuclei in stellar situations.

We calculate the modified fissility parameter

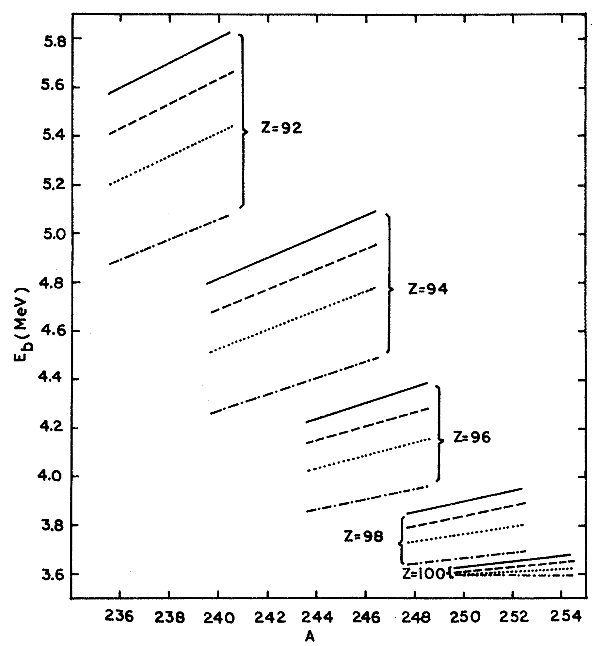

FIG. 1. Fission barrier (in MeV) as a function of the mass number of various elements. The solid line is drawn with barriers calculated from Eq. (13). The dashed line, dotted line, and the dot-dashed line give the barrier energies for $kT = 1$ MeV, and $\rho = 3.12\times10^5$ g cm^{-3}, 2.55×10^6 g cm^{-3} at $T_9 = 2.5$, respectively. The barrier energies calculated for $\rho = 3.12\times10^4$ g cm^{-3} for a $10^4 M_\odot$ object give similar results as shown by the dashed line in the figure.

from Eq. (15). The fission-barrier energies for a number of even-even nuclei are calculated for these modified fissility-parameter values. The resulting fission barriers are then plotted against mass number in Fig. 1. The temperature-density effect is much more drastic for light transuranic elements. The results obtained for $kT = 1$ MeV nearly equal those for the densities in $10^4 M_{\odot}$ objects and hence the latter case is not plotted in the figure. Moreover, the change in the barrier energy for $kT = 1$ MeV lies within the possible scatter of the available data. The effect of high density alone seems to be important for these shifts.

There are several uncertainties in present calculations of shell corrections to fission barriers; e.g., those resulting from the zero-point energy problem alone are of the order of 1 MeV.[16] Thus the temperature/density corrections calculated for ordinary r-process situations appear to be relatively unimportant. However, effects considered here might become very critical for certain high-density models.

Many of the light transuranic elements may undergo spontaneous fission because of the barrier reduction resulting from density/temperature effects, hence r-process synthesis to the heavier region may not be feasible in certain cases. Brueckner *et al.*[16] pointed out that Myers and Swiatecki's conventional mass law might not be suitable for extrapolation to the region of unknown nuclei. In that case, the calculation of barrier energies from Eq. (13) for the heavier mass region will be of doubtful validity. Viola[17] estimated that the rapid process cannot produce elements heavier than $A \sim 275$ as a result of the fast spontaneous-fission rates of elements with $N \geq 185$. Thus perhaps different mechanisms[16,18] for the production of superheavy elements in stellar situations may be necessary. Perhaps, the mechanism of heavy-ion reaction on transuranic elements during the collapse of supermassive objects may be important as a source of superheavy elements.[19]

At the temperature and density considered here, electron-capture fission may be excluded. Estimates of Nakazawa *et al.*[20] show that electron capture begins at much higher density conditions for a star of $10 M_{\odot}$.

Some nuclei in stellar situations may be so highly excited that the shell effects are reduced.[21] While discussing the synthesis of superheavy elements in stellar conditions, this effect may be quite important.

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