Experimental Evidence for Repulsive Cores in Heavy-Ion Reactions*

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The simultaneous study of the $^{12}C+^{12}C$, $^{12}C+^{16}O$, and $^{16}O+^{16}O$ total reaction cross sections at low energy strongly suggests that the optical potential of the heavy ions has a central soft repulsive core. Conventional Woods-Saxon potentials can fit the $^{12}C+^{12}C$ and $^{16}O+^{16}O$ reactions on the one hand or the $^{12}C+^{16}O$ reaction on the other but only the potentials with the repulsive core can fit all three reactions with the parameters varying smoothly from reaction to reaction. These reactions also determine accurately the surface thickness of the imaginary part of the potential; it varies from 0.35 fm for the $^{16}O+^{16}O$ reaction to 0.55 fm for the $^{12}C+^{12}C$ reaction. At very low energies most of the absorption takes place while the two nuclei are far from each other.

1. INTRODUCTION

The optical model is used here to gain a first understanding of the phenomena involved in low-energy heavy-ion reactions and to allow an extrapolation to the energies of interest in astrophysics.[1]

Twenty years ago the optical model was successfully applied to nucleon-nucleus reactions[2] even though why it worked was then not understood. The main properties are now understood but work is still going on to relate the nucleon-nucleus[3] optical model to the nucleon-nucleon interaction.

The last few years have seen a large increase of activity in experimental work on heavy-ion[4] reactions, not only for their own intrinsic interest and for their importance in astrophysics but also because a thorough understanding of heavy-ion reactions is required as a basis for exploiting multinucleon transfer reactions to obtain spectroscopic factors. To understand phenomenologically the wealth of experimental results now coming out it is still necessary to rely on optical models. Efforts are being made to justify the use of optical models and to calculate their properties for heavy ions.[5] They are still in a preliminary state and they still need to be guided by a phenomenological analysis of experimental results while, at the same time, giving some indications on the important characteristics to be included in the optical models.

We will analyze the total reaction cross section for the ^{12}C-^{12}C, ^{12}C-^{16}O, and ^{16}O-^{16}O systems. Low-energy measurements have recently been made for all these systems.[6-8] They were done partially to measure the cross sections at energies far below the Coulomb barrier, where reactions take place in stars. They cover the energy range from slightly above the Coulomb barrier to as low as current experimental techniques allow them to go. This analysis will cover the same energy range. These systems were chosen because they involve very similar particles; they are all spinless bosons, and they could be made of α-particles, that is, they have even and equal numbers of protons and neutrons. The parameters of the optical models that reproduce the data are then expected to vary smoothly from system to system since the systems are very similar and only the numbers of nucleons involved change from one to the other. The requirement that the reaction parameters vary smoothly will be seen to impose the existence of a soft repulsive core in the optical potential. For most of our analysis, we will use the ^{12}C-^{16}O system. We will also present new results for the ^{12}C-^{12}C and ^{16}O-^{16}O systems, but the ^{12}C-^{12}C system has been analyzed previously in detail[9] and it is not necessary to repeat the analysis. The comparison of the three systems will however be important.

We will see that low-energy measurements determine the interior properties of nuclei. The experimental cross section will be compared in Sec. 2 to nonresonant Woods-Saxon potentials:

$$V = V_{\mathrm{WS}} + V_{\mathrm{Coul}} = \frac{-(V_0 + iW_0)}{1 + e^{(r-r_0)/a}} + V_{\mathrm{Coul}}\,, \tag{1}$$

where V_0 and W_0 are the real and imaginary parts of the potential, R_0 is its radius, and a its surface thickness. W_0 will be chosen large enough to eliminate resonances. As discussed below, the nonresonant potentials are those for which the absorption (W_0) is sufficiently large to damp out any trace of the single-particle resonances. With such nonresonant potentials it will then be found impossible to reproduce the total reaction cross section. Giant resonances appear in the data for all three of the ^{12}C-^{12}C, ^{12}C-^{16}O, and ^{16}O-^{16}O systems. The resonances are similar in the three cases. This makes difficult a simultaneous analysis of all three reactions in terms of giant resonances in Woods-

Saxon potentials: whereas in the ^{12}C-^{12}C and ^{16}O-^{16}O cases the particles are identical bosons, they are different in the ^{12}C-^{16}O case. For identical bosons only even partial waves come in, while for different bosons all partial waves come in. Because all the even partial waves of Woods-Saxon wells tend to resonate together while the odd partial waves resonate together in between the resonances of the even waves, the resonance behavior of the ^{12}C-^{16}O system is then very different from the resonance behavior of the other systems. It will be necessary to add a soft repulsive core to the Woods-Saxon potential to reproduce the measured cross sections. The behavior of waves in repulsive-core potentials will be analyzed in Sec. 3, and in Sec. 4 the analysis of the data in terms of optical potentials will be presented.

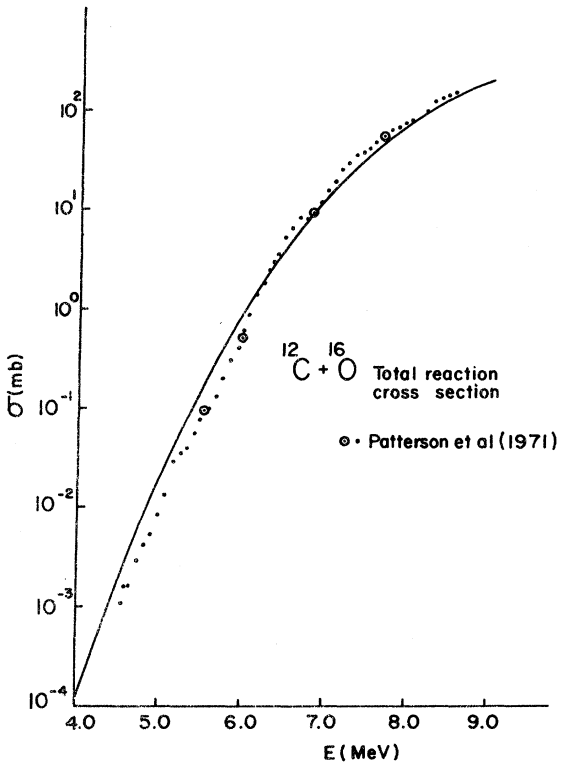

FIG. 1. Experimental and nonresonant values for the $^{12}C+^{16}O$ total reaction cross sections as a function of energy. The points refer to values obtained by detecting γ rays. They were normalized by the circled points where particles were detected, as described by Patterson *et al.* (1971). The full line is obtained from a nonresonant Woods-Saxon potential (see Table I and the text). On all other figures the experimental results and the theoretical cross sections are divided by these nonresonant cross sections to eliminate the order-of-magnitude variations due to the Coulomb barrier and so emphasize the nuclear structure.

2. EXPERIMENTAL RESULTS AND WOODS-SAXON POTENTIALS

The measured cross sections are first compared to theoretical cross sections obtained with Woods-Saxon potentials. The total experimental reaction cross sections are shown in Figs. 1 and 2. In Fig. 1 are shown the experimental cross sections for $^{12}C+^{16}O$ along with the reaction cross sections calculated with a nonresonant potential, V_{NR}. Figure 2(b) shows the experimental points divided by cross sections (σ_{NR}) calculated with V_{NR}. The resonant structure is then evident. Dividing by σ_{NR} takes out the orders of magnitude of variation due to the Coulomb barrier but cannot eliminate an additional one order of magnitude oscillation. Similar oscillations are even more evident in the ^{12}C-^{12}C data shown in Fig. 2(a) and somewhat less in the ^{16}O-^{16}O data of Fig. 2(c). All three reactions show energy variations which cannot be explained with V_{NR}. Resonances of some sort must be present in the nuclear potentials. These resonances have a width of 1.0 MeV or so and are separated by at least 4.0 MeV as can be seen from the ^{12}C-^{16}O and ^{12}C-^{12}C cases. We will now indicate how V_{NR} was chosen for each of the three reactions. It will then be possible to understand how the results are modified by the presence of resonances and later of repulsive cores.

The resonant potentials are of the Woods-Saxon type [Eq. (1)]. All our theoretical cross sections were calculated by solving numerically the radial part of Schrödinger's equation for a particle with reduced mass μ:

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi_l+\left(V(r)+\frac{\hbar^2 l(l+1)}{2\mu r^2}\right)\psi_l=E\psi_l\,, \qquad (2)$$

with the boundary condition $\psi_l=0$ at $r=0$, and by matching the calculated wave to incoming and outgoing Coulomb wave functions. The matching radius, a, was chosen so that the nuclear potential had no effect beyond it. The matching conditions are:

$$\begin{aligned} &I_l+\beta O_l=\gamma\psi_l\,,\\ &I_l'+\beta O_l'=\gamma\psi_l\,, \end{aligned} \qquad (3)$$

and they determine β and γ.

The absorption cross section was then calculated with

$$\sigma_c(\text{abs})=\frac{2\pi}{k^2}\sum_l(2l+1)T_l(c) \qquad (4)$$

and

$$\begin{aligned} T_l&=\frac{2|\gamma|^2M}{\hbar^2k}\int_0^a W(r)\psi_l\psi_l^*dr\\ &=\frac{0.048|\gamma|^2\mu}{k}\int_0^a W(r)\psi_l\psi_l^*dr\,, \end{aligned} \qquad (5)$$

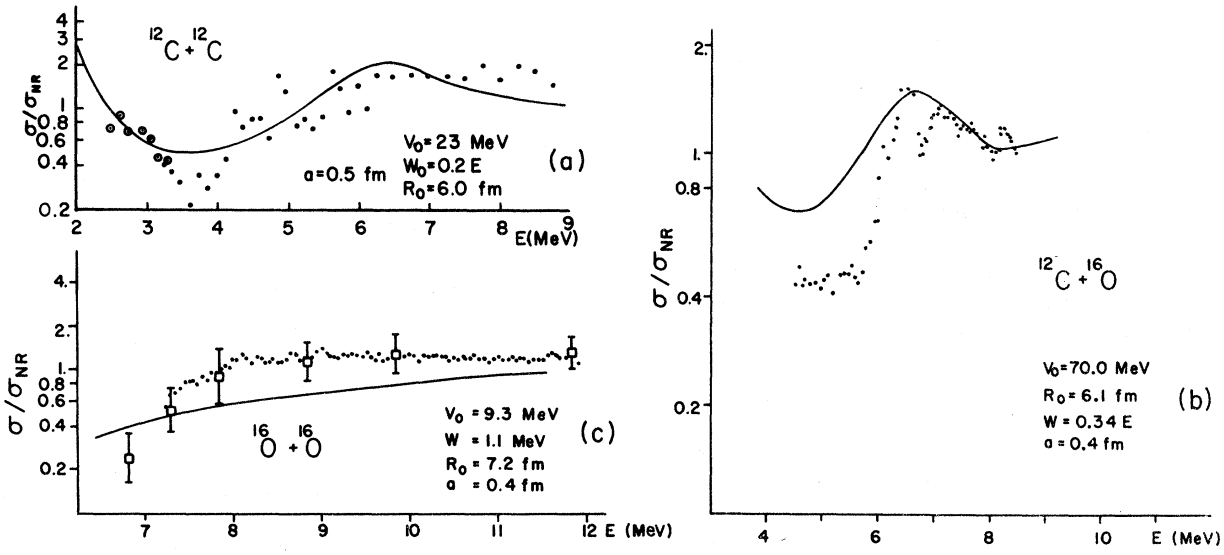

FIG. 2. Fits obtained with resonant Woods-Saxon potentials for the three cases of interest. It is possible to obtain reasonably good fits for all three systems with these potentials. However, they are not quite so good as those obtained with repulsive cores (see Fig. 6) and the required potentials show discontinuous variations from system to system. Note that Woods-Saxon potentials should not be used to try to reproduce intermediate structure with resonances every 300 keV in the $^{12}C+^{12}C$ system but only the gross structure or giant-resonance structure (see Ref. 9). On (a) the points are from Patterson *et al.* (Ref. 6), and the circled points from Mazarakis and Stephens (1970) and Stephens (1972). On (b) the points are from Patterson *et al.* (Ref. 7) and on (c) the experimental results are from Spinka and Winkler (Ref. 8). On (c) the points with error bars come from particle-detection measurements whereas the other points come from γ-ray-detection measurements.

which is obtained by using Green's theorem.[10]

T_l is called the transmission function and is equal to 1.0 if all incoming particles are absorbed, μ is the reduced mass, and k is the wave number. Obtaining T_l from Eq. (5) was found, at very low energies, to be easier numerically than the usual method of first obtaining the phase shift, and then T_l. We use c to specify the channel of interest.

We will now describe how the calculated absorption cross sections of Eq. (5) depend on the parameters of the optical potential of Eq. (1). So doing we will show that the V_{NR} we used (Table I) were really nonresonant and that the resonant behavior apparent in the experimental data is real and not introduced by our comparison potentials.

At energies below, or slightly above, the Coulomb barrier the Woods-Saxon potentials of Eq. (1) show the following behavior:

(1) a rapid decrease in absorption cross section as the energy is decreased. It is caused by the need to tunnel through the Coulomb barrier. For the reactions of interest it represents a seven order-of-magnitude variation in the cross section over the energy interval of interest to us.

(2) The spacing of giant resonances depends on the radius of the potential well and the reduced mass of the ions—for our systems, giant resonances are 5–10 MeV apart. As mentioned above, the resonances of even partial waves occur in groups, and the odd resonances also occur in groups but in between those of the even resonances.

(3) The size of W_0 determines whether or not the absorption cross section shows the effect of resonances. If $W \gtrsim 5$ MeV no resonance can be seen since the width of resonances is $2W_0$ and this is larger than the spacing of the resonances. When W_0 is large enough that no resonance can be seen, its exact value is unimportant so long as it is smaller than V_0. The exact value of V_0 does not matter either since nonresonant potentials are equivalent to a two-parameter family of equivalent square wells.[11] Changing V_0 can be mocked by changing R_0 and a appropriately. All the V_{NR} of Figs. 1 and 2 have W_0 large enough that no resonances appear. The exact value of V_0 is then unimportant. The most important parameter is R_0. It determines the energy dependence of the

TABLE I. Parameters of the nonresonant potentials.

Reaction	V_0 (MeV)	W_0 (MeV)	R_0 (fm)	a (fm)
$^{12}C+^{12}C$	50.0	10.0	5.77	0.4
$^{12}C+^{16}O$	50.0	10.0	6.1	0.40
$^{16}O+^{16}O$	50.0	10.0	6.65	0.40

cross section as well as its order of magnitude. The radius listed in Table I for V_{NR} was chosen so as to reproduce as well as possible the experimental behavior of the cross section. The surface thickness was then adjusted to improve the fit. It was, however, found that the surface thickness could not, for any of the reactions, eliminate the resonant structure shown in Fig. 2. Its effect, in first order, is to normalize the cross sections (just as the reflection factor of the equivalent square well[11] does). The values we obtained for the parameters of V_{NR} are similar to those determined in Reeves.[12]

On Fig. 2 are also shown cross sections obtained with Woods-Saxon potentials with resonant behavior ($W_0 \simeq 1$ MeV). The radius and surface thickness were adjusted to reproduce the average behavior while V_0 and W_0 were chosen to obtain resonances at the proper energy with the proper distance from each other and with the proper width. The experimental results of the ^{12}C-^{12}C system are well reproduced by the resonant Woods-Saxon well while those of ^{12}C-^{16}O and ^{16}O-^{16}O are not so well reproduced. Note also the large difference in the required depth of the real part of the potential: The giant resonances in ^{12}C-^{16}O are separated by the same energy as the giant resonances in ^{12}C-^{12}C, whereas in Woods-Saxon potentials of the same depth the resonances are twice as numerous in the ^{12}C-^{16}O case as in the $^{12}C+^{12}C$ case. In the ^{12}C-^{16}O case all partial waves come in, whereas only even partial waves come in the ^{12}C-^{12}C case since the two particles are then identical bosons. The odd partial waves usually resonate at energies in the middle between two even partial-wave resonances so reducing the distance between resonances by a factor of 2. This forces the difference in V_0. The potentials are not unique but the potential chosen for the ^{12}C-^{16}O case is the smallest to fit the data, and for ^{12}C-^{12}C it is not possible to increase V_0 appreciably without introducing a new phenomenon (Ref. 9) which we will describe below: absorption under the barrier (see also Ref. 11). So whereas the systems studied here are very similar, their absorption cross section cannot be reproduced by a continuous set of Woods-Saxon potentials. No Woods-Saxon potential reproduces the ^{12}C-^{16}O data very well. For these reasons, we will consider adding a repulsive core to the Woods-Saxon potential and we shall show that the introduction of the repulsive core has a simple effect on the grouping of giant resonances which facilitates our fits and allows the parameters of the potentials to vary smoothly from reaction to reaction. Thus low-energy total reaction cross sections determine the *interior* of the potential.

3. WAVE ANALYSIS OF OPTICAL POTENTIALS WITH REPULSIVE CORES

Calculations have recently been made to obtain the optical potential for heavy ions from the nucleon-nucleon interaction.[5] Those calculations are still preliminary but they strongly suggest the existence of a soft repulsive core in the heavy-ion potential—such a repulsive core is already believed to exist in the α-α interaction.[13] It is related to the exclusion principle. If two heavy ions get too close together, the particle density becomes larger and forces the added nucleus to have high momentum. The required energy leads to a repulsive core.

It is also possible to obtain the real part of the optical potential for the ^{12}C-^{12}C system by supposing the ^{12}C nuclei to be made of α particles using electron scattering to give the distribution of nucleons in the ^{12}C nucleus. One then obtains from the α potential, the ^{12}C-^{12}C potential. Such calculations (Ref. 9) lead to a soft repulsive core for the ^{12}C-^{12}C system, and the real part of the potential which is obtained without any arbitrary parameters gives an excellent fit to the total ^{12}C-^{12}C reaction cross section. There are then theoretical arguments in favor of the presence of repulsive cores in the optical potentials of heavy ions. Here we study the effect of the repulsive core on the wave behavior of a Woods-Saxon potential; in particular we show how it changes the resonance behavior so that in its presence the $^{12}C+^{16}O$ system resonates just like the $^{12}C+^{12}C$ and $^{16}O+^{16}O$ systems at low energy. We will also describe the effect of the imaginary part of the potential and specially how it may affect the low-energy extrapolation through absorption under the barrier.

To study the effect of repulsive cores we will add a Gaussian positive part to the potentials described by Eq. (1):

$$V = V_{WS} + V_R + V_{Coul} \tag{6}$$

with

$$V_R = Ue^{-cr^2}, \tag{7}$$

where U and c are positive arbitrary constants. U determines the strength of the repulsive core and c its range. Typically c will be chosen so that the potential V is repulsive for

$$r \leq 0.7R_0, \tag{8}$$

where R_0 is the radius of the Woods-Saxon potential. The real part of the potential is shown in Fig. 3. The inner region is dominated by the repulsive core. The repulsive core is larger over nearly all the well than the angular momentum barrier for partial waves with $l \leq 3$. It becomes negligible within the radius of the Woods-Saxon

potential. The total potential is then attractive in a spherical shell around the repulsive inner sphere. The attractive part is relatively shallow. The attractive potential is much weaker than the repulsive one. The depth of the attractive potential is of the same order as the height of the Coulomb potential at the nuclear radius.

The energy of the first resonance in such a well depends mainly on the depth of the attractive potential and the spatial width of the attractive region. The potentials we use below to fit experimental results have their first resonance at $E \simeq 6.0$–8.0 MeV. The value of the transmission functions for different partial waves is shown on Fig. 4 as a function of the depth of the attractive potential. For illustration purposes, we chose the ^{12}C-^{16}O system and used the parameters that give us a best fit for this system, except that we allowed V_0 of Eqs. (1) and (6) to vary. If the attractive potential has a depth smaller than 10.2 MeV, a $E = 6.25$ MeV particle is below all resonances.

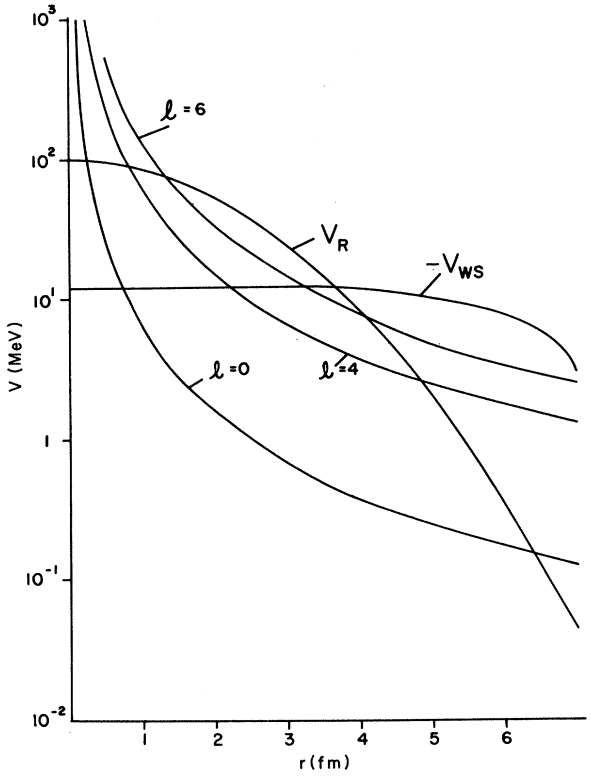

FIG. 3. Radius dependence of the different components of the real potential. The repulsive part of the potential is much larger than the angular momentum barrier for partial waves with $l < 4$. Resonances for all those partial waves then occur at essentially the same energy. Below the Coulomb barrier only the $l < 4$ partial waves are important. Even and odd resonances then occur at the same energy below the Coulomb barrier.

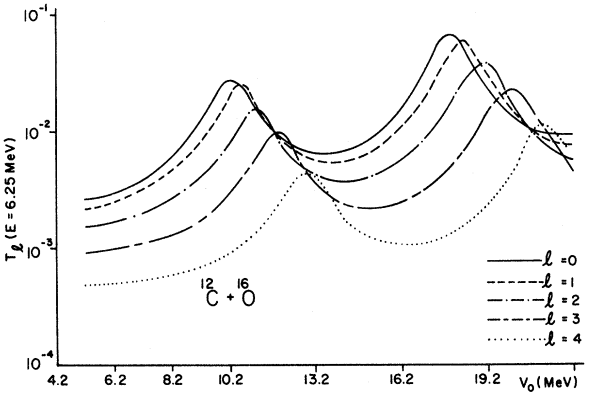

FIG. 4. Transmission functions for $^{12}C + ^{16}O$, at $E = 6.25$ MeV for a number of partial waves, as a function of the depth of the attractive part of the potential. In all cases the repulsive central part had $U = 100$ MeV and $c = 0.156$ fm^{-2} [Eq. (7)]. In repulsive potentials, even and odd partial waves resonate at nearly the same energy. Only as $l \geq 4$ does the angular momentum barrier start to change the resonance energy. Below the Coulomb barrier, only partial waves with $l < 4$ matter, however.

The different partial waves then appear in a non-resonant fashion. The ones with the larger angular momentum show the smaller transmission functions since they have to cross a higher barrier to get to the absorptive part of the potential. The $l = 0$ partial wave is of course the first to become resonant. But the p wave is resonant at very nearly the same energy and so is the d wave. This is very different from the behavior of Woods-Saxon, square or harmonic wells when the $l = 0, 2, 4, \ldots$ partial waves resonate at the same energy but the $l = 1, 3, \ldots$ resonate midway between the $l = 0$ resonances. This property will be very important for our discussion in Sec. 4. The "s" and "p"

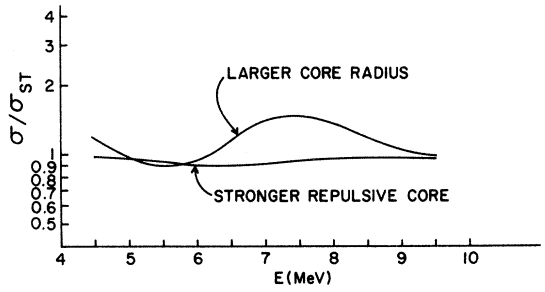

FIG. 5. Effect on the cross section of varying the parameters of the repulsive core. The σ_{ST} cross sections were obtained with the "standard" potential for $^{12}C + ^{16}O$ whose parameters were varied. Increasing the height of the repulsive core from 100 to 200 MeV affects the cross section by less than 10%, but changing the core radius from 3.7 to 4.7 fm causes variations by a factor of 1.5. By radius of the core we mean the radius at which $V_R = -V_{WS}$ (see Fig. 3).

waves resonate at the same energy since, for $l=1$, the angular momentum barrier is insignificant compared to the repulsive core (Fig. 3). It is of course larger than the repulsive core for $r \ll R_0$, but most of the region where, in a Woods-Saxon potential, the behavior of the partial wave is dominated by the angular momentum barrier is here dominated by the repulsive core which is the same for the $l=0$ and $l=1$ waves. As l increases, the relative importance of the angular momentum increases and the partial waves become resonant (at $E=6.25$ MeV) for deeper potentials. The height and radius of the repulsive core also influence the wave behavior. On Fig. 5 the ^{12}C-^{16}O system is used to compare wells with a resonance at $E=6$ MeV but with repulsive cores of different height and radius. The height has little effect. The repulsive potential with

$$U=100.0 \text{ MeV}$$

is already strong enough that increasing it does not alter its effect so long as the radius where

$$|V_R| = |V_{WS}|$$

is kept the same. When this radius is changed the resonance behavior changes. As it is increased the resonances of the different partial waves come in closer together. The $l=1$ resonance is at more nearly the same energy as the $l=0$ resonance. The repulsive core is larger than the angular momentum barrier over a larger fraction of the radius of the well and so the angular momentum barrier is less effective. The resonant properties of soft-repulsive-core wells are then modified according to the radius of the core. More and more partial waves resonate together as the radius of the core is increased. The properties of most attractive-only wells, that even partial waves resonate at the same energy and odd partial waves resonate at the same energy but different from that of the even waves, disappear with the presence of the repulsive core.

How will the imaginary part of the potential change the properties described above for the real part? The main change will be the introduction of absorption under the barrier. It is present in Woods-Saxon imaginary potentials but not in Gaussian ones. The experimental results described in Sec. 4 suggest that it is important at least for the $^{12}C+^{12}C$ system. To reduce, in our analysis, the number of arbitrary parameters to a minimum, we have always kept the radius and surface thickness of the imaginary part of the potential the same as those of the real part [Eq. (1)]. We have not added a central core to the imaginary part of the potential. It would be unphysical to add one since a large positive imaginary potential would imply the *creation* of a large number of particles in the center of the potential. One would wonder whether the imaginary part should have the Woods-Saxon shape close to $r=0$, but we will show that it does not matter. The only important part of the imaginary potential is where the real potential is attractive *and* under the Coulomb barrier.

Using Eq. (5) to determine the transmission function, it is possible to localize where the wave absorption takes place in a complex potential. The largest contribution to the cross section will come from the radius where $[W(r)\psi_l\psi_l^*]$ is largest. In Woods-Saxon potentials with repulsive cores, the function $[W(r)\psi_l\psi_l^*]$ may have maxima at two radii: close to the surface and far from the surface, below the Coulomb barrier. Within the repulsive core $|\psi_l|^2$ is extremely small, so unless W were unphysically large, the product $W|\psi_l|^2$ will be very small and the value of W does not matter there; that region contributes little to the integral in Eq. (5). In the attractive region of the potential the wave function $|\psi_l|^2$ reaches a plateau or has a local maximum; W is close to its maximum and so the product $W|\psi_l|^2$ may have a local maximum there. Outside the surface ($r \gtrsim R_0+2a$), $|\psi_l|^2$ will increase rapidly but W will decrease rapidly. From the WKB approximation,

$$|\psi_l|^2 \propto e^{2Kr}, \tag{9}$$

where

$$K \equiv [(2m/\hbar^2)(B-E)]^{1/2}.$$

However, for $r \gg R_0$, the absorbing potential varies as

$$W \propto e^{-r/a}. \tag{10}$$

At R_0+4a, the Coulomb barrier is highest, and we have for systems like $^{12}C+^{12}C$:

$$2K \gtrsim a^{-1}, \tag{11}$$

when the particles are 5.0 MeV or more below the Coulomb barrier. Then a second maximum appears in $(W|\psi_l|^2)$ since B and so K diminish as the radius increases. It corresponds to absorption of the wave taking place under the Coulomb barrier. This phenomenon is most prominent in Woods-Saxon potentials and has not been observed before. Compared to square-well behavior (Refs. 9 and 11) it leads to a slower decrease of the cross section as the energy is lowered. What the shape of the imaginary potential far in the tail should be is, in our opinion, unknown. It could in principle be determined theoretically by a detailed model of heavy-ion reactions. Such a model does not exist for the time being. The experimental results presented in the next section suggest a Woods-Saxon

shape for $r - R_0 \gg a$. The part of the imaginary potential that is determined by lowest energy measurements is not the part close to the center of the well, but is the part beyond the attractive potential. At those very low energies the cross section is then independent of the interior of the nucleus.

4. DISCUSSION

The low-energy total reaction cross sections for the $^{12}C + ^{12}C$, $^{12}C + ^{16}O$, and $^{16}O + ^{16}O$ systems show a similar resonant behavior whereas the resonant properties of Woods-Saxon potentials for the $^{12}C + ^{16}O$ system have been seen in Sec. 2 to be different from the resonant properties of the other two systems; only in the $^{12}C + ^{16}O$ system are there odd partial waves. In Sec. 3 we have found that adding a soft repulsive core would make the resonant behavior of Woods-Saxon potentials similar for all three systems by making the $l = 1$ and $l = 3$ partial waves resonate at essentially the same energy as the $l = 0$ and $l = 2$ waves. On Fig. 6 are found the fits that are obtained with soft-repulsive-core potentials [Eq. (6)]. On Table II are shown the parameters of the potentials. These parameters were not obtained by a least-squares fit. Such a fit is not too meaningful since, at least in the ^{12}C-^{12}C system, intermediate structure is present (Refs. 9 and 14) and the optical-potential fits should not reproduce those resonances.[15] Instead, the physical properties of Woods-Saxon and repulsive-core potentials described in Secs. 2 and 3 were used.

The radius of the Woods-Saxon potential was mainly used to increase or lower the value of the cross section. The real part of the potential was used to determine the position of the resonance. The height of the repulsive core was found in Sec. 3 to have little effect and was not varied. The size of the repulsive core was varied by varying c, so as to spread or condense the resonances (Fig. 5). The imaginary part of the Woods-Saxon potential was varied to determine the width of the giant resonance, and finally the surface thickness a to reproduce the low-energy behavior of the cross section.

The calculated cross sections of Fig. 6 are "best fits." The radius R_0 could be varied by about 0.2 fm either way and all other parameters varied at the same time to give reasonably good

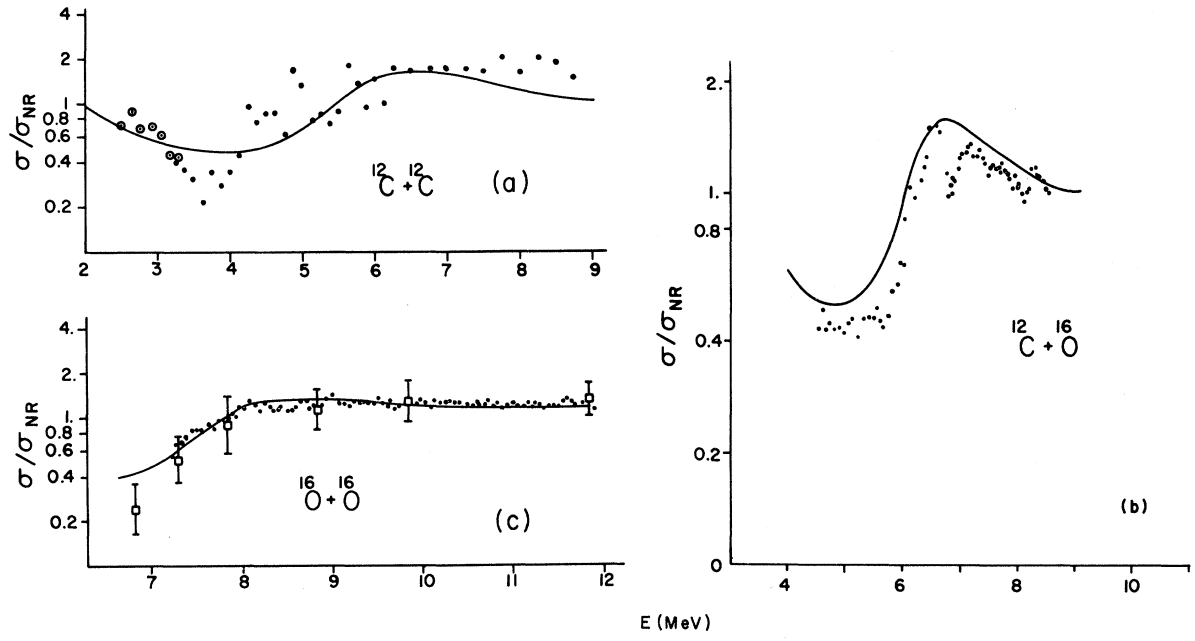

FIG. 6. Comparison of experimental cross sections to those obtained with repulsive-core potentials. The parameters of the potentials are listed in Table II. The three reactions are well reproduced with repulsive-core potentials. The parameters vary regularly from one reaction to the next. The $^{12}C + ^{12}C$ system shows low energy increase in the experimental cross section compared to the nonresonant cross sections. $^{12}C + ^{16}O$ shows a plateau whereas $^{16}O + ^{16}O$ shows a continuous decrease. This behavior can be explained by absorption under the Coulomb barrier in the $^{12}C + ^{12}C$ and $^{12}C + ^{16}O$ cases. The $^{16}O + ^{16}O$ system shows no indication of it. It is very sensitive to the shape of the imaginary part of the potential far from the nucleus. Continuing the cross-section measurements at even lower energies and studying other heavy-ion systems should yield important information on the shape of the imaginary parts of the potential far from the nucleus.

results. Also it would be possible to increase V_0 so that at $E=6.0$ MeV in $^{12}C+^{16}O$ we have the second $l=0$ resonance instead of the first. Good fits could still be obtained then. There is a family of values of potentials that reproduce the data, but the important result is that they all have repulsive cores and they all have absorption under the barrier.

The need for the repulsive cores has already been explained. Even with repulsive-core potentials it is not possible to reproduce the low-energy behavior of the cross section without absorption under the barrier. On Fig. 7 are shown the results obtained for $^{12}C+^{16}O$ when the second $l=0$ resonance is at 6.0 MeV and when no absorption under the barrier is allowed. The first $l=0$ resonance is at 2.0 MeV and it could be hoped to cause the low-energy behavior of the cross section. But it does not. Its effect starts being felt at too low an energy. The same is true for $^{12}C+^{12}C$. The only way to get the low-energy behavior is through absorption under the barrier and so the surface thickness of the Woods-Saxon potential must be chosen large enough. When a is large enough the cross section does not decrease so rapidly with energy, since the effect of the Coulomb barrier is partially compensated by shifting outwards the maximum of the integrand in the right-hand side of Eq. (5). For both $^{12}C+^{12}C$ and $^{12}C+^{16}O$ the surface thickness is determined by the existing data since compared to nonresonant data the cross section has started to increase at low energy. For $^{16}O+^{16}O$ the surface thickness obtained is rather an upper limit since the effects of absorption under the barrier are not evident in the data, even at the lowest energies. Lower energy measurements may settle this point. The values obtained for a range from 0.55 fm for $^{12}C+^{12}C$ to 0.35 for $^{16}O+^{16}O$. The change is continuous going from one reaction to another and physically can be understood by the ^{16}O magic nucleus being more tightly bound than the ^{12}C nucleus. When a nucleus is more tightly bound the tail of nuclear matter should not extend so far, and neither should the tail of its optical potential. Low-energy heavy-ion reactions give information on the interior of the potential appropriate to the system and on the tail of the imaginary potential at $r \gtrsim R_0+4a$. For instance, in the $^{12}C+^{12}C$ case the total reaction cross section between 3.5 and 8 MeV requires a resonance and implies that the nucleus is transparent so that the repulsive core may be seen. Below 3.5 MeV, however, the wave is mainly absorbed far from the surface and measurements permit a determination of the tail of the imaginary potential.

TABLE II. Parameters of the repulsive-core potentials.

Reaction	V_0 (MeV)	W_0 (MeV)	R_0 (fm)	a (fm)	U (MeV)	c (fm^{-2})
$^{12}C+^{12}C$	13.0	$0.22E$	6.2	0.55	100.0	0.100
$^{12}C+^{16}O$	10.2	$0.14E$	6.55	0.5	100.0	0.156
$^{16}O+^{16}O$	8.2	$0.123E$	7.5	0.35	100.0	0.190

All parameters vary in a continuous way, that is, the values for $^{12}C+^{16}O$ are in between those for $^{12}C+^{12}C$ and $^{16}O+^{16}O$. If the radii of the Woods-Saxon potentials are fitted with the relation

$$R_0 = r_0(A_1^{1/3} + A_2^{1/3}), \tag{12}$$

where r_0 is a constant and A_1 and A_2 are the atomic mass numbers of the nuclei involved, one obtains

$$r_0 = 1.42 \pm 0.07 \text{ fm}. \tag{13}$$

This is large compared to

$$r_0 = 1.09 \text{ fm}, \tag{14}$$

obtained from electron scattering on nuclei, and implies that most of the attraction occurs before there is an appreciable overlap of the two nuclei. This is easily understood in the picture presented

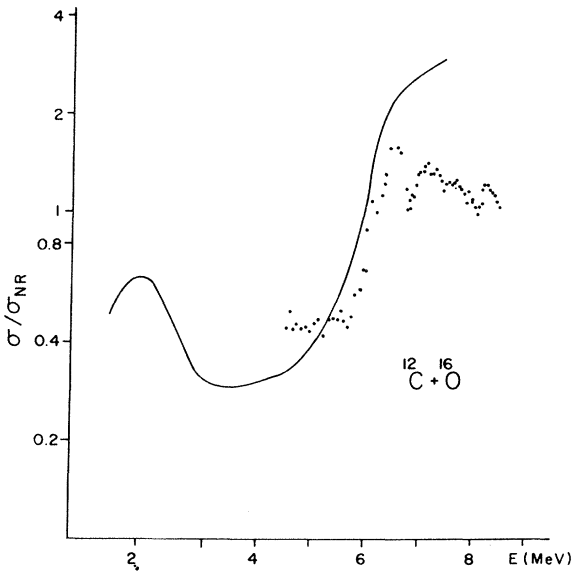

FIG. 7. Cross sections obtained with a repulsive-core potential when the second $l=0$ resonance is at $E=6$ MeV. The potential is given in Table II with V_0 changed from 10.2 to 17.2 MeV. The rise at $E=2$ MeV is due to the resonance there since absorption under the barrier was eliminated by taking $W=0$ for $R>8$ fm. The fit is poor. When a repulsive core is present, a good fit requires the first resonance to be at $E=6$ MeV and absorption under the barrier to explain the low-energy behavior.

here, since when the two nuclei overlap there is a repulsive core due to the exclusion principle. The nuclei attract each other, only through their surfaces. This was confirmed for the $^{12}C + ^{12}C$ system by calculations of the optical potential using the α-particle model (Ref. 9).

Three reactions have been studied here. What new information becomes available through the measurement of other systems? Systems like $^{12}C + ^{20}Ne$ or $^{16}O + ^{20}Ne$, which are expected also to be explained through an α-particle model, should behave similarly to the three systems studied here. They should have repulsive cores and their attractive parts should be shallow. They should show giant-resonance behavior. The radius of the attractive part should be given by Eq. (13). It is a little difficult to predict at exactly what energy the resonance should occur. The reaction $^{12}C + ^{20}Ne$ should show absorption under the barrier very prominently.

Systems like $^{13}C + ^{12}C$ might behave differently. One of the nuclei has an additional neutron. It is not as tightly bound as the other nucleons. By adding an attractive part of some 50.0 MeV to the potential it might, in the naive picture, lower the repulsive core by some 50.0 MeV. Our results were shown not to be sensitive on the height of the repulsive core. The ^{13}C-^{12}C reaction may then tell us whether the repulsive core in the ^{12}C-^{12}C reaction was larger or smaller than 50.0 MeV. If the repulsive core disappeared in the $^{13}C + ^{12}C$ case, the imaginary part of the potential would be relatively large, since two overlapping nuclei cannot have a long life. For neutrons and a nucleus, $W_0 \simeq 5$. MeV. ^{13}C overlapping with ^{12}C should last at most half as long, so $W_0 \gtrsim 10$ MeV. If the potential is attractive at $r = 0$, and $W_0 \simeq 10$ MeV there, then no resonance should be seen in the $^{13}C + ^{12}C$ total reaction cross section. It would be interesting to check this point experimentally.

Systems like $^{14}N + ^{14}N$ lie in between the two cases discussed. The nuclei are rather tightly bound but they cannot be represented by α particles. There should be qualitative differences with the systems discussed above. Resonance structure should be diminished but probably not wiped out.

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