PHYSICAL REVIEW C VOLUME 8, NUMBER 2 AUGUST 1973

Three-Nucleon Experiments and ^2S Defects*

D. P. Saylor and F. N. Rad

Cyclotron Institute, Texas A&M University, College Station, Texas 77843

(Received 16 April 1973)

Supported by the results of recent theoretical calculations of nucleon-deuteron scattering, it is suggested that most parts of the calculated three-nucleon scattering and breakup amplitudes are model-independent at low energies and, furthermore, that the ^2S-state amplitudes are the important exception to this rule. On the basis of the assumption that it is only differences between theoretical and physical ^2S amplitudes which will remain substantial as calculations improve, a new approach to the experimental three-nucleon problem is proposed. According to this approach, experiments would be chosen on the basis of the sensitivity to ^2S amplitudes and would be analyzed by using theoretical predictions for the amplitudes and empirically adjusted ^2S defects. Experiments expected to be sensitive to the ^2S amplitudes are discussed including angular distributions, spin-correlation measurement, and experiments of the triple-scattering type. Formulas for these parameters are given in terms of amplitudes having the simple structure predicted by the Amado model and it is shown how the ^2S defects will modify such predictions.

I. INTRODUCTION

The immediate goal of most three-body work has been the achievement of an understanding of how a three-nucleon system works qualitatively in terms of nucleons interacting through two-body forces. The long-range goal of three-nucleon experiments is to learn things about the interaction of nucleons which cannot be learned from the study of systems which consist of only two nucleons. We know that no interaction based solely on phenomenological two-nucleon forces should accurately describe the interaction of three nucleons when they all come close together simultaneously because phenomenological interactions are not complete descriptions of the two-nucleon force and also because explicitly three-body forces are present. Any prediction based on our knowledge of two-body forces will not fit accurate three-body experiments because of what can be called a "short-range defect." It is reasonable to assume that at low energies the observable "short-range defect" is important for only the lowest angular momentum state and is simply a "^2S defect." Calculations of p-d elastic scattering phase shifts have been made in the region below 24 MeV[1] using different models which fit the same low-energy parameters and they give very different predictions for the ^2S state, but differences between the ^4S phase shifts predicted by different models are so slight as to make no significant difference in comparisons with experiment. Presumably this is a consequence of the fact that when the identical particles have parallel spins the exclusion principle prevents all three particles from coming close together. Additional orbital angular momentum might be expected to have a similar effect.

We will assume that in a "low-energy domain" the short-range defect is negligible for all states except ^2S states. Our guess is that this "low-energy domain" will extend in energy to about 50 MeV.

II. STRUCTURE OF M MATRICES AT LOW ENERGIES

In order to extract information about the short-range defect from three-body experimental data, it is necessary to accurately include in the calculation what is known from two-nucleon scattering with adequate accuracy. A procedure which does give a good qualitative picture of p-d elastic scattering and breakup in the low-energy domain is one in which any interaction except that in relative S states of the nucleons is ignored. If this interaction is rank-one separable one has the Amado model,[2] the only model which has been used to date in breakup calculations. One promising procedure for getting sufficiently accurate two-body input into the three-body problem is to take a simplified force, solve the equations of motion exactly with this force, and then put in the more complicated features of the two-nucleon interaction by means of perturbation theory.[3]

Applied to the calculation of the two-nucleon M matrix,[4] where the perturbation scheme is unnecessary, one will get

$$M = M_0 + M_s , \tag{1}$$

where

$$M_0 = A + B\vec{\sigma}_1 \cdot \vec{\sigma}_2 \tag{2}$$

and the small correction due to the perturbation has the more complicated form

$$M_s = \alpha + \beta_p \sigma_{1p}\sigma_{2p} + \beta_k \sigma_{1k}\sigma_{2k} + \beta_n \sigma_{1n}\sigma_{2n} + \gamma(\sigma_{1n} + \sigma_{2n}) . \tag{3}$$

The zero-order M matrix M_0 takes on the simple form it does because with just S-wave interactions there is no mechanism by which angular momentum can be exchanged from spin to orbital motion. As a consequence of this, scattering is independent of the orientation of the total spin angular momentum in space and neither reorientation nor polarization of this total spin angular momentum results from a scattering. The preceding description of how the zero-order M matrix is simplified for the simplified Hamiltonian applies equally well to the case of p-d scattering and breakup. For elastic scattering we write the p-d M matrix as

$$M = M_0 + M_D + M_S , \tag{4}$$

where[5]

$$\begin{aligned} M_0 &= A + B\vec{\sigma}\cdot\vec{S} , \\ M_D &= K(\tfrac{1}{3} - \tfrac{2}{3}\vec{\sigma}\cdot\vec{S}) , \\ M_S &= a + b\sigma_n S_n + c\sigma_n + dS_n + e\sigma_p S_p + f\sigma_k S_k \\ &\quad + gS_{pp} + h\sigma_n S_{pp} + iS_{kk} + j\sigma_n S_{kk} \\ &\quad + k\sigma_p S_{np} + 1\sigma_k S_{kn} . \end{aligned} \tag{5}$$

The p-d M matrix is expressed in terms of three components: (1) the M matrix derived from the simplified zero-order Hamiltonian M_0; (2) the contribution of the doublet defect M_D which is still of the zero-order form; and (3) the small but complicated corrections which arise from including the complicated features of the two-nucleon interaction M_S.

This division of the M matrix suggests a corresponding grouping of possible experiments. There is a first group of experiments which is primarily useful for determining the amplitudes of the zero-order form and there is a second group which is sensitive to the additional structure in M_S. Experiments of the first group should be of particular value in determining the ^2S-defect parameter K which is a complex number and is independent of the scattering angle in the center of mass (though it is a function of energy). The amplitudes A and B are functions of angle because a number of orbital angular momentum states will contribute to scattering for the three-body system.

Experiments of the second group are very important. They provide a check on the methods used to compute the two-body input and they may show some three-body effects which are more subtle than the ^2S defect. The point of this paper being the ^2S defect, experiments which bear most directly on it are of primary interest to us. The first group of experiments is defined by the requirement that the predicted value of the measured quantity does not take on a trivial value (zero for example) if the M matrix has the zero-order form. The simplest experiments to perform are unpolarized differential cross-section measurements and they belong to the first group. At the next level of experimental difficulty are experiments in which one of the ingoing particles is polarized and an asymmetry is measured, or where the polarization of outgoing particles is measured with ingoing particles unpolarized. These experiments belong to the second group. This is so because all such asymmetries and polarizations vanish if the M matrix has the zero-order form M_0. At the third level of experimental difficulty are experiments which involve the spins of two particles, that is polarized-beam-plus-polarized-target experiments,[6] spin-correlation experiments,[4,5] and triple-scattering experiments.[4,5] If the axis along which the spin component is determined is the same for the two particles, the experiment lies in the first group; if on the other hand the axes are at 90° from one another, it is in the second group because the zero-order M matrix produces no spin rotations.

III. ELASTIC SCATTERING EXPERIMENTS

In writing expressions for experimental parameters we will consider the defect to be included in the zero-order amplitude; that is $A + \frac{1}{3}K \to A$, etc. Expressed in terms of the M-matrix amplitudes, six interesting experiments from the first group for elastic scattering are

$$\frac{d\sigma}{d\Omega} = |A|^2 + 2|B|^2 + \tfrac{4}{3}\mathrm{Re}B^*(b + e + f) + 2\,\mathrm{Re}A^*a , \tag{6a}$$

$$\frac{d\sigma}{d\Omega}D_n = |A|^2 + \tfrac{2}{3}|B|^2 + \tfrac{4}{3}\mathrm{Re}B^*b + 2\,\mathrm{Re}A^*a , \tag{6b}$$

$$\frac{d\sigma}{d\Omega}D_p = |A|^2 + \tfrac{2}{3}|B|^2 + \tfrac{4}{3}\mathrm{Re}B^*e + 2\,\mathrm{Re}A^*a , \tag{6c}$$

$$\frac{d\sigma}{d\Omega}D_k = |A|^2 + \tfrac{2}{3}|B|^2 + \tfrac{4}{3}\mathrm{Re}B^*f + 2\,\mathrm{Re}A^*a , \tag{6d}$$

$$\frac{d\sigma}{d\Omega}C_{nn} = \frac{d\sigma}{d\Omega}A_{yy} \tag{6e}$$

$$= \tfrac{2}{3}|B|^2 + \tfrac{4}{3}\,\mathrm{Re}A^*B + \tfrac{4}{3}\mathrm{Re}B^*(e + f + a) + \tfrac{4}{3}\mathrm{Re}A^*b . \tag{6f}$$

D_n is just the usual triple-scattering parameter D; D_p and D_k are similar measurements where the axis along which polarizations are defined are the usual $\vec{p}$ and $\vec{k}$ directions.[5] From the point of view of the dominant M_0 amplitude, the three D measurements are equivalent when viewed as experiments in the first group. A measurement of the difference between D_p and D_k is an experiment from the second group. The expressions in Eqs. (6a)–(6f) neglect the products of small terms which would be in a complete expression for the experimental parameters. A measurement of $d\sigma/d\Omega$, a D-type measurement, and A_{yy}, a polarized-beam–polarized-target measurement, would determine the magnitudes of A and B and the phase between them. It would also determine K the ^2S-defect parameter. It is also possible that K can be determined from the angular dependence of experiments of a single type if there is sufficient variation of the amplitudes A and B as a function of angle.

IV. BREAKUP REACTIONS

The M matrix which describes breakup is a 6×8 matrix. It can also be divided into three parts as in Eq. (4). The zero-order M matrix depends on three independent amplitudes,[7] one amplitude q for the quartet state, an amplitude d_1 for transitions to a doublet state in which the two like particles are in a triplet state, and a second doublet amplitude d_2 for transitions to a doublet state in which the two like particles are in a singlet state. The zero-order M matrix can be written with the spin-quantization axis chosen in an arbitrary direction; M_0 has the same form independent of this choice of quantization axis.

$$
M_0 = \begin{array}{ccc|cccccc}
 & & m & 1 & 1 & 0 & 0 & -1 & -1 \\
 & & \mu & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
\mu_1 & \mu_2 & \mu_3 & & & & & & \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & Q & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & G & J & 0 & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & H & K & 0 & 0 & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & I & L & 0 & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & L & I & 0 \\
-\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & K & H & 0 \\
-\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & J & G & 0 \\
-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & Q
\end{array} , \qquad (7a)
$$

where m is the deuteron magnetic quantum number, and μ is the incident-nucleon quantum number. μ_1 and μ_2 are the magnetic quantum numbers for the two identical nucleons, and μ_3 is the magnetic quantum number for the unlike nucleon in the final state.

The elements of the M matrix can be expressed in terms of the three independent amplitudes as follows:

$$
\begin{aligned}
Q &= q\,,\\
G &= \tfrac{1}{3}(q-2d_1)\,,\\
H &= \tfrac{1}{3}(q+d_1)+\sqrt{\tfrac{1}{3}}\;d_2\,,\\
I &= \tfrac{1}{3}(q+d_1)-\sqrt{\tfrac{1}{3}}\;d_2\,,\\
J &= \sqrt{\tfrac{2}{9}}\;(q+d_1)\,,\\
K &= (\sqrt{\tfrac{2}{9}}\,q-\sqrt{\tfrac{1}{18}}\,d_1)-\sqrt{\tfrac{1}{6}}\;d_2\,,\\
L &= (\sqrt{\tfrac{2}{9}}\,q-\sqrt{\tfrac{1}{18}}\,d_1)+\sqrt{\tfrac{1}{6}}\,d_2\,.
\end{aligned} \qquad (7b)
$$

The ^2S defects K_1 and K_2 must be added to the two doublet amplitudes d_1 and d_2 to account for the variation of M from its zero-order form. K_1 and K_2 are functions of the center-of-mass energy partition between the three particles, but are not functions of the additional three Euler angle variables necessary to completely define the directions of the outgoing particles. This opens up the possibility of determining the two defect functions K_1 and K_2 from a single type of experiment where the same partition of energy is observed for different directions of the outgoing particles. To M_0 a small correction (M_s) should also be added to permit transitions between states which are not permitted by M_0. The form of M_s will depend on the direction which is chosen as the quantization axis. Since the breakup case has not been the subject of a general analysis such as that made by Seyler[5] for the case of elastic scattering, we cannot write the general form of M_s in terms of the minimum number of independent amplitudes at this time.

It is expected that the d_2 amplitude will be large at low energies and that K_2 will be the large doublet defect. However, the two ^2S states are coupled and a significant K_1 may be required, particularly for energy partitions in which the neutron-proton relative energy is low.

Experiments belonging to the first group can be identified using the same rule as for elastic scattering. There are six experiments which are practical experimentally. They are

$$
\frac{d\sigma}{d\Omega_1 d\Omega_2 dE} = (\tfrac{2}{3}|q|^2+\tfrac{1}{3}|d_1|^2+\tfrac{1}{3}|d_2|^2)\times \text{phase-space factor}\,, \qquad (8a)
$$

$$
D(\vec{\mathrm{p}},\vec{\mathrm{n}}) = \frac{\frac{10}{9}|q|^2+\frac{1}{9}|d_1|^2-\frac{1}{3}|d_2|^2-\frac{16}{9}\,\mathrm{Re}\,q^*d_1}{2|q|^2+|d_1|^2+|d_2|^2}\,, \qquad (8b)
$$

$$
D(\vec{\mathrm{p}},\vec{\mathrm{p}}) = \frac{\frac{10}{9}|q|^2-\frac{2}{9}|d_1|^2+\frac{8}{9}\,\mathrm{Re}\,q^*d_1+(\frac{64}{27})^{1/2}\,\mathrm{Re}\,q^*d_2+(\frac{4}{27})^{1/2}\,\mathrm{Re}\,d_1^*d_2}{2|q|^2+|d_1|^2+|d_2|^2}\,, \qquad (8c)
$$

$$D(\vec{d}, \vec{n}) = \frac{10|q|^2 - 2|d_1|^2 + 6|d_2|^2 + 8\,\mathrm{Re}q^* d_1}{12|q|^2 + 6|d_1|^2 + 6|d_2|^2}, \tag{8d}$$

$$D(\vec{d}, \vec{p}) = \frac{10|q|^2 + 4|d_1|^2 - 4\,\mathrm{Re}q^* d_1 - 4\sqrt{3}\,\mathrm{Re}q^* d_2 - 4\sqrt{3}\,\mathrm{Re}d_1^* d_2}{12|q|^2 + 6|d_1|^2 + 6|d_2|^2}, \tag{8e}$$

$$A_{yy} = \frac{|q|^2 - |d_1|^2 - |d_2|^2}{2|q|^2 + |d_1|^2 + |d_2|^2}. \tag{8f}$$

The $D(\vec{a}, \vec{b})$ parameters are triple-scattering-type parameters like the D measurements for elastic scattering. The incident particle is polarized and the specified outgoing particle is analyzed along a common Z axis. Each observable in Eqs. 8 represents a number of possible "equivalent" experiments in the sense that D_p, D_n, and D_k are equivalent when M_s is ignored. A_{yy} is again the polarized-beam–polarized-target experiment. Among the variety of possibilities the cases where this Z axis is chosen to be the normal to the plane defined by the incident momentum and the momentum of the particle which is spin analyzed has the experimental advantage of not requiring magnets to reorient the nuclear spins.

If the M matrix had the zero-order form, the D parameters would be simply the ratio of the polarization of the outgoing particles to that of the incident particles. When polarization and analyzing power effects are present in the breakup reactions this ratio would still be interesting. It would seem preferable however to define the procedure for measuring the spin-transfer parameters in such a way that those polarization and analyzing power effects cancel in a manner analogous to the two-body D measurement.

In addition to the kinematically complete form the six experiments have versions in which the momenta of two of the three outgoing particles are left undetermined. The theoretical expressions for these are easily obtained by integration. For example, the inclusive version of $D(\vec{p}, \vec{p})$, in which the momentum of only one of the outgoing particles is determined, we call $D'(\vec{p}, \vec{p})$:

$$D'(\vec{p}, \vec{p}) = \left(\frac{d\sigma}{d\Omega_1 dE_1}\right)^{-1} \int \left[D(\vec{p}, \vec{p}) \frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1}\right] d\Omega_2 .$$

The expression for $D'(\vec{p}, \vec{n})$ is similar except for an extra factor of $\frac{1}{2}$ required to take account of the identity of the two protons.

V. CONCLUSION

There are a number of quite practical experiments (including the measurement of differential cross sections) which one can expect to be sensitive to the ^2S defects. Based on the general success of the Amado model it would seem reasonable to use this model as a basis for estimating the sensitivity of various possible measurements to the ^2S defects. Such estimates would be valuable and guides in planning experiments.

Finally, the ^2S defects themselves depend on the model used in the process of extracting them, but we expect the theoretical ^2S amplitude plus the defect to prove to be an experimental ^2S amplitude which is not particularly model-dependent. These experimental amplitudes could be used for comparison with predictions of models other than the one used to extract the defects.

ACKNOWLEDGMENT

We are grateful to Dr. J. G. Rogers for reading the manuscript and for suggesting improvements.

*Work supported in part by the National Science Foundation.

[1]F. A. McDonald and J. Nuttall, Phys. Rev. C 6, 121 (1972); I. H. Sloan, Nucl. Phys. A168, 211 (1971).

[2]R. Aaron, R. D. Amado, and Y. Y Yam, Phys. Rev. 140, B1291 (1965).

[3]E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B2, 167 (1967).

[4]L. Wolfenstein, Ann. Rev. Nucl. Sci. 6, 253 (1969).

[5]R. G. Seyler, Nucl. Phys. A124, 253 (1969).

[6]*Proceedings of the Third International Symposium on Polarization Phenomena in Nuclear Reactions, Madison, 1970,* edited by H. H. Barschall and W. Haeberli (University of Wisconsin Press, Madison, 1971).

[7]R. T. Cahill and I. H. Sloan, Nucl. Phys. A165, 161 (1971).