

Energy Division in π^- - Nucleus Collisions at 1.5 GeV*

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The angular distributions and secondary spectra have been determined for protons and charged pions emergent from collisions of 1.5-GeV negative pions with emulsion nuclei. Expressed in terms of numbers of particles per collision, these results are compared to the predictions of the Monte Carlo intranuclear cascade, averaged over the light- and heavy-target nuclei. The charged-pion spectrum is found to be in large disagreement with that calculated. With included observations on neutral-pion emission, the fraction of primary energy carried away by pions is $(16 \pm 4)\%$, where the calculation predicts 48%. Possible reasons for this discrepancy are discussed, and a long-standing question of apparent energy imbalance in these collisions is revived.

INTRODUCTION

Collisions of energetic particles with complex nuclei are of interest because of the increasing success of Monte Carlo intranuclear cascade models,¹⁻³ and also because of the possibility of studying processes unattainable in a hydrogen target. In foregoing work⁴ we have examined the inelastic nuclear collisions of 3.5-GeV negative pions in emulsions, and the neutral-pion emission from those collisions. That study was made in an attempt to resolve an old question regarding apparent energy imbalance when fast pions interact with target nuclei. A satisfactory result was not obtained at 3.5 GeV, primarily because of uncertainties in identifying the lightly-ionizing products. The inquiry has therefore been pursued to a lower-beam energy where the fast outgoing particles are identifiable with greater assurance.

1-2-GeV π^- on nuclei. Among the papers which report observations of nuclear interactions of 1-2-GeV negative pions in emulsions, those of Crew and Hill⁵ and of Ronne and Danielsson⁶ are concerned primarily with collisions in complex nuclei and compare experimental results with predictions of an intranuclear cascade calculation. We note that these authors reach divergent conclusions, but that both groups find a charged-pion component at forward angles which is significantly smaller than expected from the cascade model. A preliminary report of this work has confirmed this effect.⁷ From this it is seen that in repeating the experiment, particular attention must be given to systematic errors in collecting the data and identifying the cascade products.

In the above-mentioned experiments^{5, 6} information regarding the charged products of the collisions was obtained through observations of all the lightly-ionized tracks emergent from fewer than 500 beam stars. In this paper we describe obser-

vations on the lightly-ionized ($v \gtrsim 0.39c$) tracks arising from 2607 beam stars. From this sample, those 704 tracks which showed a visible path length exceeding 3.0 mm in the immediate plate containing the parent star were subjected to further measurement. Particle identification was thus confined to tracks of dip angles $< |11^\circ|$. Then, following Abrahamson *et al.*,⁸ a geometric correction was applied to the numbers of identified particle tracks in each interval of emission angle to take account of those particles at large dip angles. The corrected observations lead to angular distributions and spectra for protons and charged pions. These results are compared to the corresponding distributions predicted by the histories of 8000 incident negative pions followed through intranuclear cascade calculations⁹ in which account has been taken of the creation and decay of the $(\frac{3}{2}, \frac{3}{2})$ isobar.

Uncharged products. For a balance of energy, it is necessary to consider not only the visible outgoing tracks, but also the emission of neutrons, uncharged strange particles, and neutral pions. Above the range of evaporation energies, neutron emission is apparently similar to proton emission with intensity 80% greater.^{3, 10} Strange-particle production is of sufficiently low cross section that its exclusion represents a small source of error in the experiment. This error can be minimized by removing from the data that $\approx 1\%$ of the collisions which show charged strange-particle emission. Neutral-pion emission has been studied by three methods:

(i) The "background" pairs and associated electromagnetic cascades found throughout the emulsion stack.^{11, 12} These observations give photon spectra and angular distributions which are sensitive to the methods employed in collecting the data, and unavoidably include a component which entered the stack with the beam.

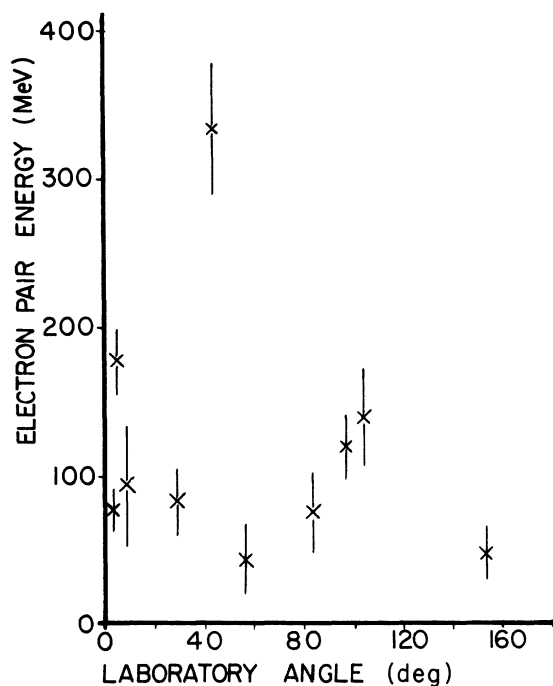


FIG. 1. The energies of 10 electron pairs, which convert between 10 and 100 μm from a beam collision, show angle of dip $< 13^\circ$, and exhibit an alignment with the collision consistent with a photon origin at a point closer than 1 μm to the collision. This alignment criterion corresponds to a time resolution $\lesssim 10^{-14}$ sec.

(ii) Those electron pairs materialized close to, and showing a precise alignment with, a nuclear collision of a beam particle. An averaged conversion length of 4.6 cm in emulsion causes these data to be greatly restricted in number.⁴ Despite poor statistical significance, this method provides the surest measure of π^0 emission, since short-lived objects with competing photon decays are of low occurrence among the products.

(iii) The "direct" electron pairs arising from the alternate π^0 decay mode and materialized at the point of π^0 decay. With no visible displacement of the pair origin from the beam star, and with a wide range of opening angle, these pairs have been employed for a determination of the π^0 lifetime.¹³ It is necessary to distinguish the "direct" pairs from other pairs of lightly-ionized tracks which are evidently not due to electrons. With this precaution, a pattern of π^0 production generally similar to that derived from the related pairs of (ii) is found, but differing widely from results found through method (i). The high spatial resolution of emulsion, in conjunction with the short π^0 lifetime, provides 10^{-14} sec resolution in the quantitative determination of π^0 emission from these collisions.

Emulsion nuclei, light and heavy. In previous

emulsion experiments, attempts have been made to distinguish those collisions which occur in Ag or Br nuclear targets by requiring a number of evaporation prongs exceeding seven. However, in attempting to exclude those collisions that take place in C, N, and O nuclear targets, a significant fraction of the Ag and Br collisions—those of low residual nuclear excitation—are also removed from the data. Since the low-excitation Ag, Br stars might be expected to show a larger probability of energetic pion emission in the forward direction, this removal could lead to systematic error. In the present work, therefore, observations of inelastic nuclear collisions have been accumulated without regard to star size. For comparison with experiment, the calculated angular distributions and spectra are accordingly formed by combining two components, appropriately weighted.¹⁴ The heavy (Ag, Br) component derived from 73% of the histories represents the products of the intranuclear cascade in ^{100}Ru , while the light (C, N, O) component, based on 27% of the histories, represents the products from ^{16}O . The results of the Monte Carlo intranuclear cascade calculations in ^{16}O are as valid⁹ as those derived from ^{100}Ru .

EXPERIMENTAL

Emulsion

The experiment has been carried out with Ilford K5 emulsions in which the structure of the developed grains can provide an improved consistency in ionization measurements. This consistency has been checked—

- over the range of development gradient with emulsion depth, and
- between different plates of the stack, using the tracks from energetic electron pairs as a useful indicator for the relativistic plateau of ionization. Likewise,
- the level of minimum to plateau¹⁵ has been evaluated under the conditions of our photographic processing.

The average grain density of beam tracks is taken as the reference level of ionization.

Following these checks an ionization calibration has been built up for singly-charged-particle velocities in the range of $0.39-0.82c$. This characteristic is based on grain counts over long tracks of arrested pions. The emulsions, 600 μm thick, were exposed as glass mounted plates, since measurements were to be made only on tracks inclined at small dip angles. The atmosphere of the laboratory has been maintained at relative humidity of 55%.

Beam

The stack was exposed at the first focus of the Chamberlain beam at the Lawrence Radiation Laboratory Bevatron.¹⁶ The beam was directed parallel to the emulsion plane and the exposure timed to accumulate 2.4×10^5 beam particles/cm² on the upstream face of the stack. The angular divergence of the beam has been found to be $\pm 1.5^\circ$, increasing to $\pm 2.2^\circ$ along the beam direction in the stack. This rather large divergence is a slight disability in comparison with the satisfactorily low photon contamination obtainable near the first focus. The momentum spread and charged-particle contamination of the beam were $< 2\%$ and $\leq 5\%$, respectively.

The energy of the beam particles has been determined by two independent methods.

- (a) Multiple Coulomb scattering measurements on beam tracks with visible path exceeding 2.0 cm before collision. The beam divergence has not precluded some differential beam-track scattering determinations.
- (b) Dynamical analysis of those evident (π^- , p) collisions which satisfy the coplanarity condition for elastic beam-particle collisions with free hydrogen targets. These measurements concur in giving the kinetic energy of the beam particles 1.50 ± 0.05 GeV.

Collection of Data

Charged Products

Nuclear collisions of beam particles are found both by following beam tracks and by systematic search through a certain volume of emulsion. A close scrutiny has been carried out through 0.50 cm³ of these emulsions, using high-resolution objectives at magnification 1000 \times and covering only that emulsion depth between $30 \mu\text{m}$ below the surface and $20 \mu\text{m}$ above the glass. A search with such magnification allows detection of stars with low nuclear excitation and elucidation of chance juxtaposition of thin tracks. Among the 2607 collisions produced by beam particles which were recorded in this search, a fraction $(13 \pm 2)\%$ with ≤ 2 evaporation tracks were found, in satisfactory agreement with the results of beam following. It is expected from the work of Walker and Crussard¹⁷ that another 13% of the collisions in complex nuclei will show no visible nuclear evaporation.

The outgoing tracks from each collision were examined closely, and those with ionization < 4.6 times that of the beam tracks were recorded. Of these, the 704 that showed a path length exceeding 3.0 mm in the same plate were subjected to multi-

ple Coulomb scattering measurement, yielding 416 protons of kinetic energies > 70 MeV and 281 charged pions with kinetic energies > 12 MeV. No proton with $p\beta > 1400$ MeV/c, or ionization corresponding to velocity $> 0.87c$, has been found. Seven tracks have been attributed to charged kaons and excluded from the data, together with the parent collisions.

Neutral Pions

A volume of radius $200 \mu\text{m}$ centered on each of 26 000 beam stars has been searched for electron pairs, again using magnification 1000 \times . Among a large number of electron pairs which are evidently (a) primary to a cascade and (b) with origin significantly displaced ($> 10 \mu\text{m}$) from the nuclear collision, 39 effectively "point" to the star. The pair bisector thus satisfies a criterion of three-dimensional alignment to the star consistent with the π^0 lifetime, and with negligible probability of chance occurrence for background pairs.

The search efficiency for these related pairs is highest for close pairs at small dip angles. Search efficiency tests with different observers have shown no failure to refind any related pair with displacement between 10 and $100 \mu\text{m}$ and dip angle $< |13^\circ|$. 10 such pairs fall within these limits. Multiple Coulomb scattering measurements have been made on the tracks of eight of these and for two the photon energy has been estimated by the less reliable opening angle. The results shown in Fig. 1 suggest a photon emission of rather small energy despite the generally enhanced conversion probability toward larger photon energies. This conclusion has been checked by tracing backward 0.2 mm from the origins of another 300 electromagnetic cascades of the background, with no additions to the data.

To estimate the π^0 emission intensity, it is necessary to determine the geometric loss factor for the dip angle interval, $< |13^\circ|$, and to assume that the decay photons are distributed in angle like the charged pions. On this basis, the 10 related pairs correspond to a geometrically corrected total of 34 pairs converted between 10 and $100 \mu\text{m}$ from a star. For an averaged conversion length (30–400 MeV) of 4.6 cm, the probability of conversion in the $90\text{-}\mu\text{m}$ path interval is 2.0×10^{-3} /photon. If an average n neutral pion is emitted from each of 26 000 stars, we find $n = 0.33 \pm 0.10\pi^0$ per star.

Cross Sections

An average cross section for nuclear collision of 1.50-GeV negative pions in complex emulsion nuclei is obtained by determining the mean free path of beam particles to cause a visible nuclear

evaporation. This quantity provides a basis for comparison with the calculated cross sections of secondary product spectra from the intranuclear cascade and excludes that 5% of all collisions in emulsions which occur on free-hydrogen nuclear targets.^{14, 17} The nonelastic³ cross section takes no account of diffraction scattering. The collision mean free path is measured by two methods, beam-track following and determination of the total beam-track length in a given searched volume. The respective results, 49 ± 3 and 51 ± 2 cm are consistent with foregoing work,⁵ and give an average non-elastic collision cross section of 430 ± 22 mb, corresponding to 740 ± 30 mb for the heavy emulsion nuclei and 200 ± 10 mb for the O, N, and C nuclei. Since the averaged calculated cross section is 640 ± 12 mb, there is a disagreement which is only partly explained by our failure to include edge collisions of no visible evaporation. In comparing theory and experiment, the consequences of this disagreement may be avoided by extracting the data in terms of the average number of particles per collision.

COMPARISON WITH CASCADE MODEL

Taking account of the geometric factors, 416 measured proton tracks are equivalent to 4030

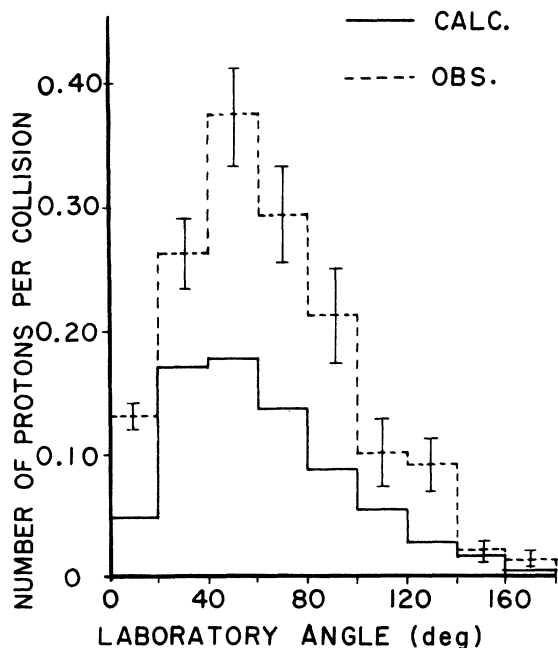


FIG. 2. The angular distribution for protons of kinetic energies exceeding 70 MeV compared with the corresponding prediction of the intranuclear cascade calculation. The larger experimental uncertainties at angles 60–120° reflect smaller numbers of observed tracks and larger geometric correction factors.

± 330 emergent protons with kinetic energy > 70 MeV, or 1.55 ± 0.13 cascade protons per star. The calculation yields 0.86 protons (> 66 MeV) per star, the averaged contribution from ^{100}Ru and ^{16}O . Likewise, the 281 measured pion tracks are equivalent to 2420 ± 170 emergent charged pions with kinetic energy > 12 MeV, or 0.93 ± 0.10 charged pions per star while the prediction of the calculation is 1.12 charged pions (> 0 MeV) per star.

In Figs. 2 and 3 the observed angular distributions of protons and charged pions are compared with those calculated from the cascade model. The proton histograms are superficially similar in form, but the disagreement in proton emission intensity becomes evident. The charged-pion histograms, although not widely divergent with respect to over-all intensity, suggest a missing experimental pion component for the forward direction.

The proton spectra for all angles, and angular intervals $0-30^\circ$, $30-90^\circ$, are compared in Fig. 4. It is of interest that the experimental proton enhancement is distributed widely in energy. For the charged pion spectra of Fig. 5, however, the disagreement between theory and experiment becomes large. The calculated histograms represent the summation of positive and negative pion

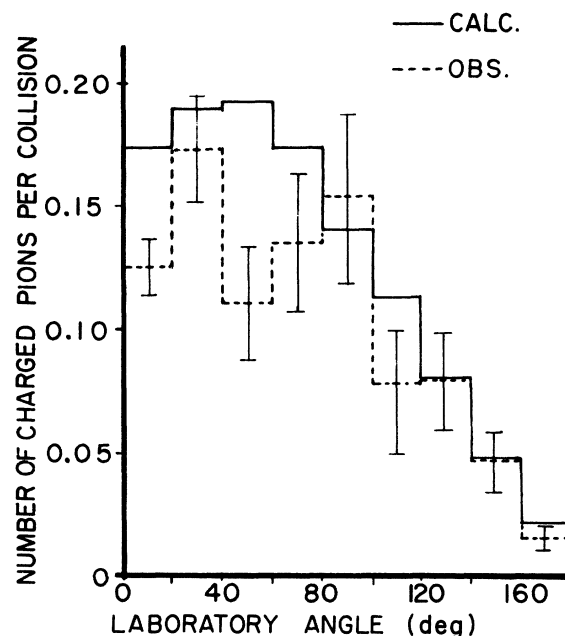


FIG. 3. The comparison of experiment and theory for the angular distribution of outgoing charged pions. The observed pions are of kinetic energies exceeding 12 MeV, while the calculated distribution derived by summation of the separate distributions for positive and negative pions, is for all pion energies. The 0–12-MeV contribution is rather small.

contributions from collisions in ^{100}Ru and ^{16}O , extracted for angular intervals $0-180^\circ$, $0-25^\circ$, and $25-75^\circ$. It is evident that a sustained pion intensity at energies >600 MeV predicted by the cascade calculation is not found in the experiment, and that this effect is attributable to the emission both at small and at large angles. The sparse electromagnetic observations of Fig. 1 suggest a neutral-pion emission of similar characteristics.

The charged-pion results have prompted an inquiry into possible sources of systematic error which might result in such spectral disagreement. A failure to include the majority of collisions of low nuclear excitation would be a conceivable cause. For this reason a sample of beam stars of small visible nuclear evaporation, comprising $\sim 12\%$ of the data has been separated from the main body. The pion spectrum exhibited by this sample is indeed more energetic than the residuum, by $\sim 10\%$ in the average but the effect is insufficient to account for the observed disagreement. This

conclusion still holds when observations on the tracks from a sample of collisions of no visible evaporation are considered.

ENERGY DIVISION

A numerical integration performed over the charged-pion spectrum gives a geometrically corrected total kinetic energy 455 ± 61 GeV carried away from 2600 beam collisions by charged pions. From the crude data of Fig. 1 a further 0.33 ± 0.10 neutral pions per star are added with an average π^0 kinetic energy of 100 ± 30 MeV for a total pion emission 1.26 ± 0.20 pions per star and total kinetic energy 206 ± 33 MeV/star carried away by pions. Taking account of the outgoing-pion mass excess 36 ± 28 MeV/star, the pion emission is found to account for 240 ± 60 MeV/star or $(16 \pm 4)\%$ of the primary kinetic energy.

The calculated pion emission ($1.57\pi^{+-0}$ from ^{100}Ru , $1.75\pi^{+-0}$ from ^{16}O) gives an average 1.62

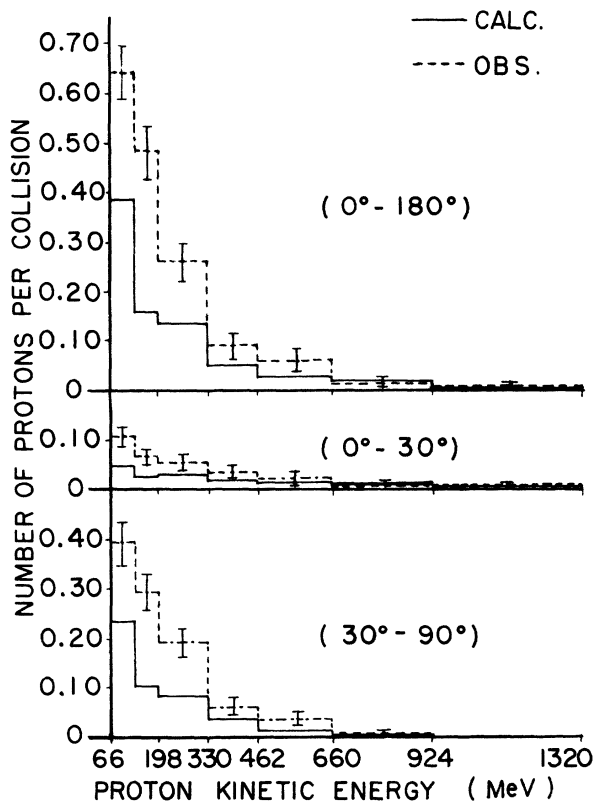


FIG. 4. The proton spectra compared for different ranges of laboratory emission angle. The $0-30^\circ$ experimental result is based on 168 tracks, the $30-90^\circ$ result on 180 tracks. The calculated spectra are derived by extracting the corresponding proton spectra from collisions in ^{100}Ru and ^{16}O , and combining these predictions with appropriate weighting.

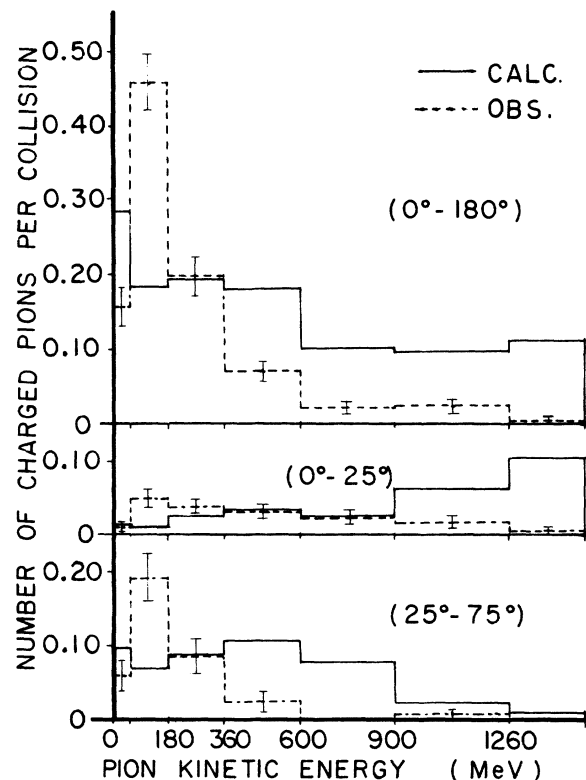


FIG. 5. The charged-pion spectra for different ranges of laboratory emission angle. The $0-25^\circ$ experimental result is based on 122 tracks, the $25-75^\circ$ result on 77 tracks. The calculated spectra are derived as for the protons, with summation of the separate positive- and negative-pion contributions. The evident disagreement between experiment and theory is the primary cause for the energy division discrepancy found in this work.

pions per collision with a total kinetic energy 641 MeV per star and a pion mass imbalance of 84 MeV per star. Thus the calculation predicts an over-all 725 MeV/collision in pion emission, or 48% of the primary energy with an uncertainty of ~2%.

A similar comparison yields an experimental proton emission 283 ± 33 MeV per collision and a calculated result (kinetic energy > 66 MeV) of 151 MeV per collision, with the difference being almost entirely attributable to the greater emission intensity found in the experiment. The disparity—a factor of about 2—in the kinetic energy carried away by cascade protons is less subject to experimental uncertainty than the pion disparity and leads to conclusions regarding possible nuclear processes which have not been considered in the cascade calculation.

POSSIBLE ENERGY IMBALANCE

The average total energy carried away from the collisions includes the pion and proton components above, a cascade neutron component which is dependent on an assumption of fast neutron/proton ratio, and an excitation energy of the residual nucleus which takes account of both nuclear binding and evaporation. Comparison of this outgoing energy with the incident kinetic energy should provide information on a possible energy imbalance.

On the basis of the intranuclear cascade calculation the neutrons of kinetic energy > 66 MeV are assumed to be distributed similarly in energy to the protons and the fast neutron/proton ratio is taken to be 1.8. For the purpose of this evaluation, therefore, the fast neutrons are considered to carry away an average 510 ± 50 MeV from the collisions. The average nuclear excitation energy U is evaluated from the number N of heavily-ionized tracks displayed by the disintegrating nuclei using the empirical relation,¹⁸ $U = 42(N+1)$ MeV. Among the tracks of more massive fragments “heavily ionized” here includes protons with energies up to 70 MeV. With observed $N = 6.1 \pm 0.2$ the average excitation energy is 300 ± 10 MeV.

Summation of the four outgoing energy terms thus yields 1330 ± 150 MeV. It is of interest to consider the effect on this figure of different assumed values for the fast neutron/proton ratio, 1.8 ± 0.3 . The sum of the outgoing energies can vary between 1250 and 1420 MeV. Considering the uncertainties of this estimate the question of a possible energy imbalance is inconclusive.

DISCUSSION

The considerable divergences between experiment and theory described here can be discussed

under four broad headings as follows:

Experimental uncertainties.

- (a) The beam pion energy may be much lower than determined.
- (b) A large fraction of the lightly-ionized tracks from stars might have escaped detection.
- (c) The close-converted electron pair component may have been underdetermined by a large factor.
- (d) Large systematic errors in the measurements on lightly-ionized tracks might have led to gross underevaluation of the particle energies.
- (e) In the collection of beam-pion nuclear collisions, some stars with energetic secondary particles may have selectively escaped inclusion.

These effects have all been discussed in the course of the experiment, together with the appropriate observational checks. It is not considered likely that these uncertainties could be the cause of the energy division discrepancy.

Isobar absorption processes.

- (f) Secondary interactions of the $(\frac{3}{2}, \frac{3}{2})$ isobar N^* ,

$$N^* + N \rightarrow 2N$$

would have the effect of both reducing the pion emission and enhancing the intensity of nucleon emission. Although such a mechanism not included in the cascade calculation might partially explain the observed proton enhancement, it could hardly occur on such scale as to double the fast proton intensity. Furthermore, the effect on pion emission should be negligible in the upper half of the spectrum.

(g) Fast cascade neutron emission should be partly dependent on the level of isobar absorption. The cascade calculation predicts that for kinetic energies > 66 MeV, the neutron spectrum is similar to the proton spectrum, with neutron/proton ratio 1.8. If isobar absorption results in strongly preferred neutron emission then the fast neutron/proton ratio might go above 2.0 when isobar absorption exceeds 50%. This effect would still be inadequate to explain the divergence between theory and experiment.

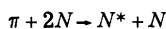
Alternative energy transfer process.

- (h) The theory invokes the inelastic pion-nucleon interaction,

$$\pi + N \rightarrow N^* + \pi$$

as a major process in the development of the intranuclear cascade. If cascade nucleons are carrying away nearly twice as much energy as expected from the theory, it is necessary to find an alternative process by which this may be achieved. The simultaneous interaction of the

fast pion with two target nucleons,



is suggested as a means by which this could occur, and incidentally, result in diminished pion emission. For such an alternative process to be an explanation of the results, the cross section must be supposed as large as the pion-nucleon cross section.

Unrecognized meson emission.

(i) If the experimental results on charged pions are accepted, the over-all pion discrepancy might be explained by invoking a neutral pion emission much enhanced in both energy and intensity. It is noted that foregoing measurements of the electromagnetic emission with resolution $\approx 10^{-11}$ sec, have indeed led to the conclusion that neutral pions carry away a large fraction of the primary energy.¹² If further observations on π^0 emission show a continuing disagreement between low- and high-resolution measurements of electron pairs, the origin of the photons may be more complicated than presently considered.

CONCLUSIONS

The results of a laborious experiment have been compared with the predictions of exhaustive calculations. Pion emission, expected to account for nearly half the primary energy, is found to be rather feeble while the proton emission is stronger than expected. If the comparisons had been made on an absolute cross-section basis, the protons would have been in better agreement but the pion disagreement would be worse. It is not considered likely that refinements in the intranuclear cascade model—higher isobars, secondary interactions of isobars—will lead to an appreciably closer agreement between theory and experiment.

Apparently some interaction mechanism for more effective transfer of primary pion energy to the target nucleons is required and a cross section much larger than presently considered would have to be invoked. There is insufficient information regarding the cascade neutrons in the experiment and for this reason the question of whether any of the incident pion energy is unaccounted for remains unanswered. A much larger experimental effort will be required to determine this possible imbalance more accurately.

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¹⁶We are grateful to the operating crew of the Lawrence Radiation Laboratory Bevatron, and to T. Ypsilantis and W. W. Chupp for their great assistance in monitoring the beam.

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