Channel Spin Components of p-Wave Neutron Widths in Niobium*

R. E. Chrien, M. R. Bhat, and G. W. Cole
Brookhaven National Laboratory, Upton, New York 11973
(Received 27 February 1973)

Measurements of the angular distribution of γ rays following p-wave capture in niobium have been made for resonances at 35.8, 42.2, and 94.3 eV. The differential cross section is the sum of the incoherent contributions from each of the possible channel spin states describing capture in those resonances. From the angular distributions, the ratio of neutron widths corresponding to the two channel spin states can be determined. The results are consistent with what is expected from assuming that each channel spin-dependent width independently follows a Porter-Thomas distribution.

[NUCLEAR REACTIONS ^{93}Nb$(n,\gamma_i)^{94}$Nb, E_n = 30–100 eV; measured $\sigma_{n\gamma}(E,\theta)$, Ge(Li); deduced channel spin components Γ_n.]

In the conventional description of nuclear reactions, it is customary to introduce the concept of channel spin S:

$$\vec{S} = \vec{s} + \vec{I},$$

where s is the spin of the incident particle, and I the target spin. Where the incident particle is a nucleon, $s = \frac{1}{2}$ and in general the reaction may proceed through either of the two channel spins, $I \pm \frac{1}{2}$, available. In the case of resonance reactions proceeding through a specific resonance $\vec{J} = \vec{I} + \vec{S}$, where both I and $l \neq 0$, both channel spins S may contribute to the same resonance of given J. To make these ideas specific, consider p-wave neutron capture on an $I = \frac{9}{2}$ target. Here $I = \frac{9}{2}$, $l = 1$, $s = \frac{1}{2}$, and $S = 4$ or 5. Resonances of total angular momentum $J = 3$, 4, 5, and 6 may be formed; for these resonances $S = 4$ for $J = 3$ and $S = 5$ for $J = 6$. For the resonances of $J = 4$ or 5, both S values 3 and 4 may be present.

In the case of an unpolarized beam on an unpolarized target, the two channel spins will contribute incoherently.[1] Thus the so-called "neutron width," Γ_n, measured in such an experiment is actually the sum of two independent widths, one for each channel-spin value. For ^{93}Nb resonances with $J = 4$ or 5 we can write

$$\Gamma_n(J = 4, 5) = \Gamma_n(S = 4) + \Gamma_n(S = 5).$$

Since in this case the "neutron widths," as they are conventionally called, actually are the sums of two widths, they exhibit unusual distribution properties. Specifically, if we assume independent channels, with equal mean strengths, these widths obey a χ^2 distribution with two degrees of freedom. This situation is in contrast to the one degree of freedom (or Porter-Thomas distribution) exhibited by neutron widths resulting from $l = 0$ capture, or $l = 1$ capture on spin-zero target nuclides.

There have been several early attempts to[2-4] measure the relative sizes of reaction widths corresponding to the different channel spins; these have always been carried out with charged-particle induced reactions. Up to now no such measurements have been carried out for neutron resonances. In the present experiment we have measured the angular distributions for primary γ rays following p-wave neutron capture in the three lowest energy resonances in niobium. Each channel spin is characterized by its own angular distribution. Where both spins participate, the observed angular variation is the sum of the two independently contributing components. From the experiment, the relative size of these components can be determined.

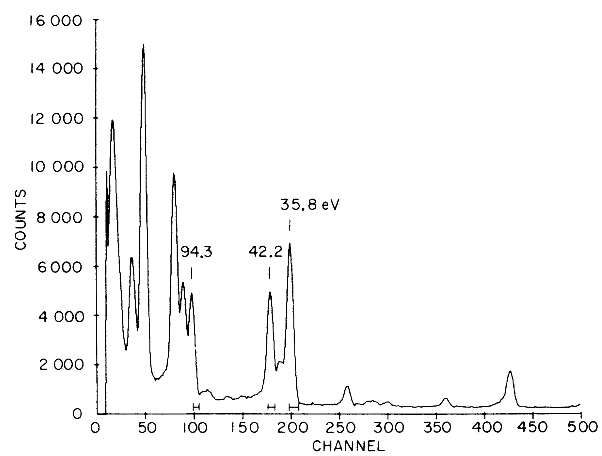

FIG. 1. The time-of-flight spectrum for niobium with a flight path of 22 m and a chopper speed of 15 000 rpm. The three labeled resonances were used in the present experiment. The smaller peaks at low energies represent impurities in the sample.

These measurements were made at the fast-chopper facility of the High Flux Beam Reactor at Brookhaven National Laboratory. The sample of monoisotopic niobium weighed 798 g. The measurements were made with a flight path of 22 m and a chopper speed of 15 000 rpm. The γ rays were detected with two Ge(Li) detectors of volume 12 and 35 cm^3 positioned, respectively, at 90 and 135° with respect to the direction of the incident neutron beam.

The time-of-flight spectrum obtained is shown in Fig. 1. In it are shown the 35.8-, 42.2-, and 94.3-eV p-wave resonances in Nb. The horizontal lines under the peaks of the resonances indicate the time-of-flight scan limits used to obtain the corresponding pulse-height γ spectra. γ-ray spectra due to these resonances are shown in Fig. 2. The two intense peaks in the 35.8-eV spectrum at 7229.3 and 6917.3 keV correspond to the 6^+ ground state and the 5^+ excited state at 312.0 keV in ^{94}Nb. In this experiment we could use only the most intense γ-ray peaks for the angular distribution measurements. These are given in Table I along with the initial- and final-state spins and the resonance spins. This information is taken from an earlier paper[5] which gives more extensive data regarding the spin assignment of the Nb resonances, the corresponding primary capture spectra, and the spins of the states in ^{94}Nb populated by these transitions. The intensities of the γ rays measured at 90 and 135° using two detectors were suitably normalized using the γ rays due to capture in the s-wave resonances at 105.8, 119.2, and 193.8 eV, as their angular distributions are isotropic.

TABLE I. The energies and final-state spins and parities for the transitions studied in this experiment. The resonance energy, in eV, in which the transition appears with useful intensity is given in parentheses.

E_γ (keV)	E_x (keV)	J^π
7229 (35.8)	0	6^+
7188 (94.3)	41	3^+
7171 (35.8)	58	4^+
6917 (35.8)	312	5^+
6272 (35.8, 42.2)	958	5^+
6259 (42.2, 94.3)	971	4^+
5948 (42.2)	1281	4^+
5907 (94.3)	1322	4^+

The intense γ rays used for the angular distribution studies are known from our previous experiment[5] to be electric dipole. The angular correlation can be calculated using a well-known expression of the following form[6]:

$$W(\theta)=\sum\left[(-1)^{a-c}/16\pi^2\hat{a}^2\right]\bar{Z}(l_1 b l_1' b'; ak)\times\bar{Z}_1(L_2 b L_2' b'; ck)\langle b\,\|\,l_1\,|\,a\rangle\langle b'\,|\,l_1'\,\|\,a\rangle^*\times\langle c\,|\,L_2\,\|\,b\rangle\langle c\,|\,L_2'\,\|\,b'\rangle^* P_k(\cos\theta),$$

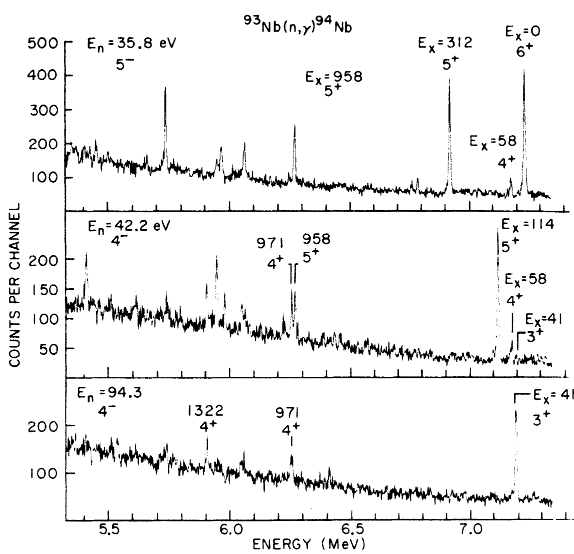

FIG. 2. Portions of the γ-ray spectra for the 35.8-, 42.2-, and 94.3-eV resonances, as observed at $\theta=90°$ to the neutron beam. The labeled peaks were used in the angular-distribution analysis.

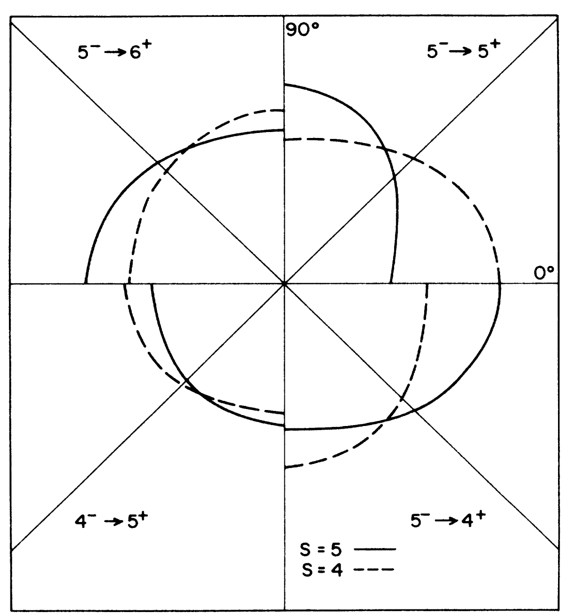

FIG. 3. Angular-correlation plots for some of the transitions considered. The distributions are fore-and-aft and left-to-right symmetric, hence only one quadrant is shown for each.

where l_1, l_1' are the incoming particle orbital angular momenta; $L_2 L_2'$ are the outgoing photon multipolarities; a the channel spin, $(\hat{a}^2 = 2a+1)$; b, b' the resonance spin; c the final-state angular momentum; $P_k(\cos\theta)$ the Legendre polynomial of degree k describing the angular distribution of photons emitted at an angle θ with respect to the incident beam. This expression can be used for interfering states b, b'; however, in the present experiment we have well-separated resonances with $l_1 = l_1'$, $L_1' = L_1$, and $b' = b$.

The angular distribution resulting from substitution in the above expression, for a single spin state, is of the form

$$W(\theta) = a + b \sin^2\theta .$$

According to the incoherence property obeyed by the various channel-spin components, the angular correlation for a mixed resonance will be just the sum of the individual channel-spin correlations:

$$W(\theta) = W(S=4) + W(S=5) .$$

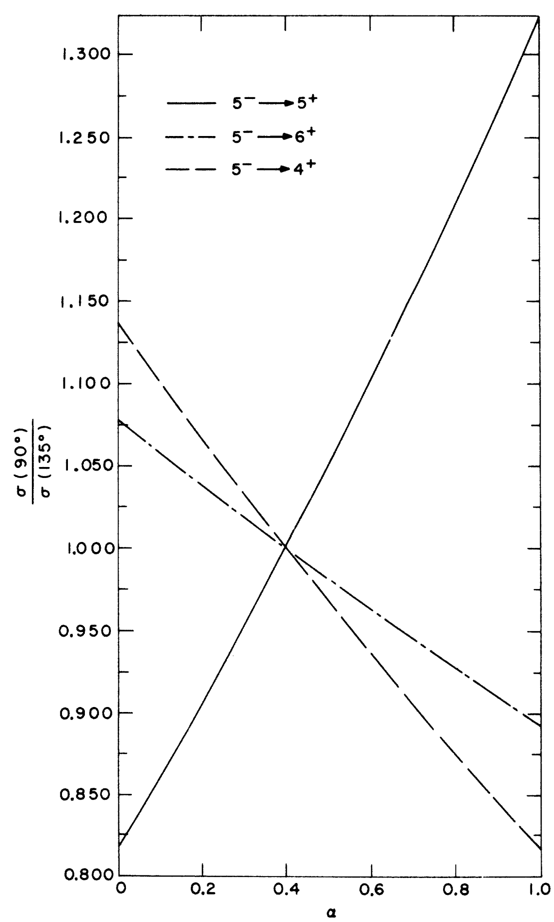

FIG. 4. The "theoretical" dependence of $\sigma(90°)/\sigma(135°)$ on α for the transitions of the 35.8-eV 5^- resonance.

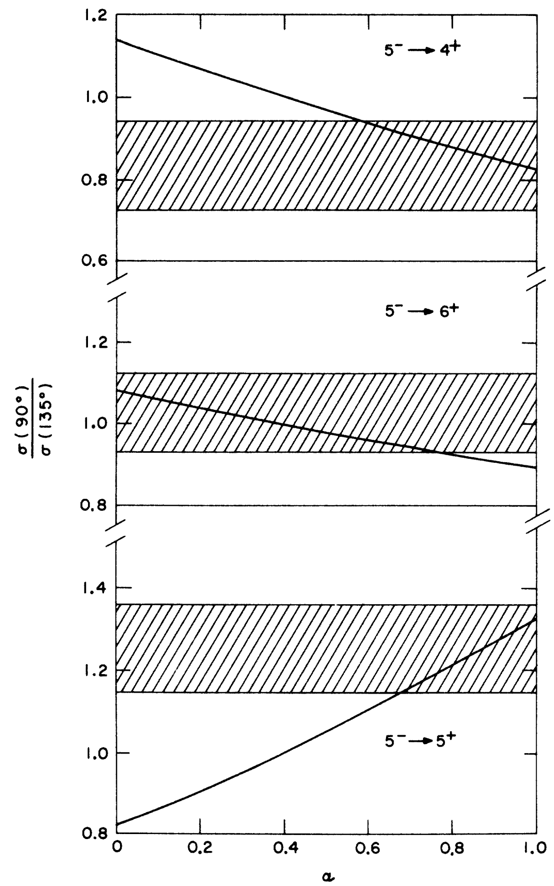

FIG. 5. The measured cross-section ratios for the 7171-, 7229-, and 6917-keV transitions in the 35.8-eV resonance, reading from top to bottom. The error bands shown are 2 standard deviations wide.

Some of the angular correlations are shown in Fig. 3 for the two channel-spin values for various combinations of resonance and final-state spin and parity. It is sufficient to measure the γ-ray emission at two angles, 90 and 135° to the beam. Since only relative angular correlations are measured in the experiment, we compare the ratio

TABLE II. A summary of results in this experiment. The values of $\alpha = \Gamma_n(5)/[\Gamma_n(4) + \Gamma_n(5)]$ shown are weighted averages, for each resonance, for the γ rays of Table I.

E_n (eV)	J^π	α
35.8	5^-	0.70 ± 0.08
42.2	4^-	0.27 ± 0.17
94.3	4^{-} [a]	0.84 ± 0.13

[a] The 4^- assignment for this resonance was described as "tentative" in Ref. 5. The results of the present experiment support the 4^- assignment.

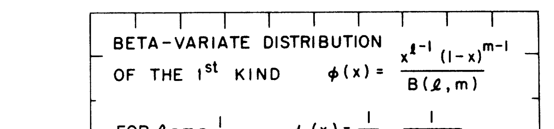

FIG. 6. The theoretical distribution function for $\alpha = \Gamma_n(5)/[\Gamma_n(4) + \Gamma_n(5)]$ assuming normally distributed width amplitudes.

$\sigma_{n\gamma}(90°)$ to $\sigma_{n\gamma}(135°)$ as a function of the parameter α where

$$\alpha = \Gamma_n(5)/[\Gamma_n(5) + \Gamma_n(4)].$$

The parameter α thus measures the relative proportion of the higher channel spin 5 in the resonance mixture. Figure 4 shows this ratio as a function of α for dipole emission from a 5^- resonance.

The results of the measurement on the 5^- resonance at 35.8 eV resonance for three separate γ rays leading to 4^+, 5^+, and 6^+ states are shown in Fig. 5. The shaded bands represent the measured α values within 1-standard-deviation error limits (assuming statistical independence). The resulting values of α were combined to produce a single value of α for the resonance. Similar results were obtained for the 42.2- and 94.3-eV resonances and are shown in Table II. The values of α given there were derived from four primary transitions in the case of 35.8-eV resonance and from three transitions in the case of the 42.2- and 94.3-eV resonances.

What values of α are reasonable to expect from this experiment? If the interaction is strongly S-dependent, we would expect values of α close to 0 or 1. Evidence for a spin-spin term in the optical-model potential has been cited by Firk and Melkonian.[7] There is little expectation, however, for a large S dependence, since the observed values of the s-wave neutron strength function show little, if any, spin dependence. If the usual assumption of the Gaussian distribution of width amplitudes is made, we can predict the distribution law for the ratios α. It is easy to show that if $\Gamma_{n\lambda}(I+\frac{1}{2})$ and $\Gamma_{n\lambda}(I-\frac{1}{2})$ are drawn from χ^2 distributions with $2l$ and $2m$ degrees of freedom, respectively, then

$$\Phi(\alpha) = \frac{\alpha^{l-1}(1-\alpha)^{m-1}}{B(l,m)},$$

where B is the beta function.[8] Thus for Porter-Thomas distributions for $\Gamma_{n\lambda}(I+\frac{1}{2})$ and $\Gamma_{n\lambda}(I-\frac{1}{2})$, and the assumption of equal mean widths (no channel-spin dependence for the interaction),

$$\Phi(\alpha) = \pi^{-1}(\alpha)^{-1/2}(1-\alpha)^{-1/2}.$$

The distribution function $\Phi(\alpha)$, shown in Fig. 6, has a mean of 0.5 and a variance of $\frac{1}{8}$. In the present experiment limitations on available flux and resolution have limited us to only three resonances and their corresponding values of α. Considering the small sample size, the results appear to be consistent with the proposed distribution for $\Phi(\alpha)$. In view of the form for $\Phi(\alpha)$, it appears that a very large quantity of resonance data would be required to uncover a significant departure of $\bar{\alpha}$ from the expected value of 0.5. Such a collection of data is well beyond the possibilities available with the present state of neutron-measurement technology.

---

*Work supported by the U. S. Atomic Energy Commission.

[1]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), p. 422 *et seq.*.

[2]S. Devons and G. R. Lindsey, Proc. Phys. Soc. Lond. **A63**, 1202 (1950).

[3]D. A. Liberman and R. F. Christy, Phys. Rev. **92**, 1085(A) (1953).

[4]D. A. Liberman, Ph.D. thesis, California Institute of Technology, 1955 (unpublished).

[5]R. E. Chrien, K. Rimawi, and J. B. Garg, Phys. Rev. C **3**, 2054 (1971).

[6]A. J. Ferguson, *Angular Correlation Methods in Gamma-ray Spectroscopy* (North-Holland, Amsterdam 1965).

[7]F. W. K. Firk and E. Melkonian, *Experimental Neutron Resonance Spectroscopy*, edited by J. A. Harvey (Academic, New York, 1970). .

[8]C. E. Weatherburn, *A First Course in Mathematical Statistics* (Cambridge, U. P., 1949), p. 177.