Calculation of Isospin Centroids in the Calcium, Nickel, and Zirconium Regions*

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Centroid energies of the residual nuclei from transfer of a single nucleon to $2p_{3/2}$ orbit in calcium-region targets, $2p_{3/2}$ and $1f_{5/2}$ orbits in nickel targets, $1g_{9/2}$ and $2d_{5/2}$ orbits in zirconium targets, have been calculated using the simple relationships connecting energy centroids of a residual nucleus to the residual interaction parameters, $\overline{W}^{T=1}_{j_1-j_2}$ and $\overline{W}^{T=0}_{j_1-j_2}$ of the two-body force. The general equations displaying these relationships have been derived using multipole-sum-rule techniques in an earlier paper. A reasonably good agreement between the calculated and experimental centroid energies demonstrates the usefulness of these equations for predicting the energy centroids knowing the residual-effective two-body force, on one hand, and extracting the effective two-body force from the transfer-reaction data, on the other.

I. INTRODUCTION

Most of the basic sum-rules methods developed within the framework of the shell model and applied in the field of nuclear spectroscopy (both, for predicting the strengths and the nature of the residual interaction, using data from stripping and pickup reactions) can be catagorized under two headings: (i) the non-energy-weighted[1] sum rules and (ii) the energy-weighted sum rules. Theoretical developments and application of the later category to single-nucleon transfer reactions, until recently, were limited largely to the simple target states, having active nucleons occupying only a single[2] shell-model orbit outside the inert core.

Very recently[3] it has been possible theoretically to extend the energy-weighted sum rules to predict the centroid energies of states excited in the stripping and pickup reactions involving target states, which are complex in the sense that outside the inert core the active nucleons occupy any number of shell-model orbits, but simple enough to have a definite value of the isobaric spin (this would exclude excited configurations for the target states). They can be even-even, even-odd, or odd-even types but not the odd-odd type (for such a category we do not, yet, have the answers to the centroid energies).

In the transfer reactions, the stripping takes place to any one of the active shells, but the recipient orbit (shell) is restricted to have only neutron occupancy; however the pickup is of the most general type.

II. THEORETICAL OUTLINE

In the present paper we simply state these[3] sum-rule equations and present their applications to stripping reactions in the calcium, nickel, and zirconium regions.

If we are given a nucleus with a target state of the even-even or odd-even type (which apart from the inert core has nucleons occupying any number of shell-model orbits), and a nucleon is transferred to the ith shell (containing only neutrons) of this target state, then the isospin centroid energies of the residual nucleus can be expressed as known functions of: (i) the occupancy and the angular momentum of the individual shell-model orbits; (ii) the isobaric spin of the target state; and (iii) the parameters of the nucleon-nucleon force operative in the isospin-zero and isospin-one states. In terms of strength[2] G_{JT} and the spectroscopic factors $S_{JT}(J_0T_0 \rightarrow JT)$ connecting target state $|J_0T_0X_0\rangle$ to a typical final state $|JT\rangle$

$$E^{(+)}_T - E^{(+)}(\mathrm{riz}) = \frac{\sum_J G^{(+)}_{JT} E^{(+)}_{JT}}{\sum_J (G^{(+)}_{JT})}$$

$$= \frac{\sum_J (2J+1) S^{(+)}_{JT} E^{(+)}_{JT}}{\sum_J (2J+1) S^{(+)}_{JT}}$$

$$= \Big[\sum_j p^{(+)}_T(i-j) + (N_i - \delta_{ij}) q^{(+)}_T(i-j)\, \overline{W}^{T=1}_{(i-j)} + (N_i + \delta_{ij}) r^{(+)}_T(i-j)\, \overline{W}^{T=0}_{(i-j)}\Big] \Big/ \Big[\sum_J (2J+1) S^{(+)}_{JT}\Big], \qquad (1)$$

TABLE I. Two-body-force parameters $\overline{W}^{T=1}_{j_1-j_2}$ and $\overline{W}^{T=0}_{j_1-j_2}$ for $1f_{7/2}$-$2p_{3/2}$ interaction (in MeV).

$\overline{W}^{T=1}_{7/2-3/2}$	$\overline{W}^{T=0}_{7/2-3/2}$	$\overline{W}^{T=1}_{3/2-3/2}$	$\overline{W}^{T=0}_{3/2-3/2}$
0.16	−1.7	−0.30	−1.3

TABLE III. Two-body-force parameters for $2p_{3/2}$-$1f_{5/2}$ interaction (in MeV).

$\overline{W}^{T=1}_{3/2-3/2}$	$\overline{W}^{T=0}_{3/2-3/2}$	$\overline{W}^{T=1}_{3/2-5/2}$	$\overline{W}^{T=0}_{3/2-5/2}$	$\overline{W}^{T=1}_{5/2-5/2}$	$\overline{W}^{T=0}_{5/2-5/2}$
−0.30	−1.3	0.42	−1.8	−0.21	−1.0

where

$$p_T^{(+)}(i-j) = -\frac{1+\delta_{ij}}{2}E_t(i-j)\left(1+\frac{f(T)}{T_0}\right),$$

$$q_T^{(+)}(i-j) = \left(\frac{3}{4}n_j + \frac{f(T)T_{0j}}{2T_0}\right), \tag{2}$$

$$r_T^{(+)}(i-j) = \left(\frac{1}{4}n_j - \frac{f(T)}{2T_0}T_{0j}\right).$$

In Eqs. (2):

(i) $n_j = \langle n_j(\mathrm{op})\rangle_j$ is the number of nucleons in the jth active shell.

(ii) $E_t(i-j)$ is the two-body energy due to the interaction of the nucleons in the ith active shell (i.e. the shell to which the particle has been added) with those in the jth active shell of the target state (here j labels any one of the active shells, including the ith, in the target state).

(iii) T_0 is the isobaric spin of the target state and T refers to that of the centroid of states of the residual nucleus.

(iv) T_{0j} is the isospin contribution due to nucleons in the jth shell. We also define,

$$N_i = (2j_i+1)\,; \quad f(T) = T(T+1) - \tfrac{3}{4} - T_0(T_0+1)\,.$$

(v) $E(\mathrm{riz})$, the energy of the residual-interaction zero state, is defined as the energy of that state of the final nucleus which results when the interaction between the transferred nucleon and the nucleons in all the active shells is switched off. It includes, essentially, the energy of the target state (assumed to be known) plus the interaction energy of the incoming particle (hole) with respect to the inert core.

(vi) $\overline{W}^{T=1}_{(i-j)}$ and $\overline{W}^{T=0}_{(i-j)}$ are the two-body interaction energies between one nucleon in the ith and the other in the jth orbit (averaged over states of different angular momentum), when the residual two-body force is operative in the isospin-one and isospin-zero state, respectively. The superscript (+) reminds us that the quantities involved refer to the reactions involving the addition of a nucleon.

III. APPLICATIONS

We have applied Eqs. (1) and (2) to titanium, chromium, iron, nickel, and zirconium isotopes. The target states in all these cases have either one or at the most two active shells and the transfer reaction adds a proton either to one of the active shells or to the next higher empty shell. In all such cases we apply Eqs. (1) and (2) and thus predict isospin centroids of the spectra of the final nuclei. We illustrate the application, by cal-

TABLE II. Calculated and experimental energy centroids (with respect to ground state) of ^{47,49,51}V, ^{51,53,55}Mn and ^{55,57}Co states excited via transfer of a proton to $2p_{3/2}$ orbit.

Residual nucleus (Target + ith proton)	$\overline{E}_{T_<}$ Calc. (MeV)	Expt. (Refs. 4, 5) (MeV)
^{46}Ti + $2p_{3/2}(p)$	2.84	2.10
^{48}Ti + $2p_{3/2}(p)$	2.87	2.40
^{50}Ti + $2p_{3/2}(p)$	2.63	3.10
^{50}Cr + $2p_{3/2}(p)$	2.44	2.40
^{52}Cr + $2p_{3/2}(p)$	2.16	2.80
^{54}Cr + $2p_{3/2}(p)$	2.27	2.52
^{54}Fe + $2p_{3/2}(p)$	1.68	2.45
^{56}Fe + $2p_{3/2}(p)$	1.25	1.43

TABLE IV. Calculated and experimental energy centroids (with respect to ground state) of ^{59,61,63,65,67}Cu states excited via transfer of a proton to $2p_{3/2}$ and $1f_{5/2}$ orbits.

Residual nucleus (Target + ith proton)	$\overline{E}_{T_<}$ Calc. (MeV)	Expt. (Refs. 6, 7) (MeV)
^{58}Ni + $2p_{3/2}(p)$	0.28	0.32
^{60}Ni + $2p_{3/2}(p)$	0.40	0.51
^{62}Ni + $2p_{3/2}(p)$	0.34	0.50
^{64}Ni + $2p_{3/2}(p)$	0.28	0.62
^{66}Ni + $2p_{3/2}(p)$	−0.03	...
^{58}Ni + $1f_{5/2}(p)$	0.98	0.91
^{60}Ni + $1f_{5/2}(p)$	0.99	0.98
^{62}Ni + $1f_{5/2}(p)$	1.37	...
^{64}Ni + $1f_{5/2}(p)$	1.73	1.55
^{66}Ni + $1f_{5/2}(p)$	1.78	...

TABLE V. Two-body-force parameters for $1g_{9/2}$-$2d_{5/2}$ interaction (in MeV).

$\overline{W}^{T=1}_{9/2-9/2}$	$\overline{W}^{T=0}_{9/2-9/2}$	$\overline{W}^{T=1}_{9/2-5/2}$	$\overline{W}^{T=0}_{9/2-5/2}$	$\overline{W}^{T=1}_{5/2-5/2}$	$\overline{W}^{T=0}_{5/2-5/2}$
−0.13	−0.97	+0.21	−0.94	−0.13	−1.3

TABLE VI. Calculated and experimental energy centroids (with respect to ground state) of ^{91,93,95,97}Nb states excited via transfer of a proton to $1g_{9/2}$ and $2d_{5/2}$ orbits.

	$\overline{E}_{T_<}$	
Residual nucleus (Target + ith proton)	Calc. (MeV)	Expt. (Ref. 8) (MeV)
^{90}Zr + $1g_{9/2}(p)$	0.01	0.00
^{92}Zr + $1g_{9/2}(p)$	0.04	0.05
^{94}Zr + $1g_{9/2}(p)$	0.08	0.00
^{96}Zr + $1g_{9/2}(p)$	0.01	0.00
^{90}Zr + $2d_{5/2}(p)$	3.29	3.29
^{92}Zr + $2d_{5/2}(p)$	2.57	1.81
^{94}Zr + $2d_{5/2}(p)$	1.92	⋯
^{96}Zr + $2d_{5/2}(p)$	1.93	1.99

culating the energy of $\overline{E}^{(+)}_{5/2}$ centroid ($T=\frac{5}{2}$) of ^{55}Mn, excited in the ^{54}Cr(^{3}He, d)^{55}Mn reaction, the proton being added to the $2p_{3/2}$ orbit. Equations (1) and (2) when applied to this case give (here the active orbits are $1f_{7/2}$ and $2p_{3/2}$):

$$\begin{aligned}\overline{E}^{(+)}_{5/2}[^{54}\mathrm{Cr}+2p_{3/2}(\mathrm{proton})] &= \Big\{\Big[p^{(+)}_{5/2}(\tfrac{3}{2}-\tfrac{7}{2})+N_{3/2}q^{(+)}_{5/2}(\tfrac{3}{2}-\tfrac{7}{2})\overline{W}^{T=1}_{(3/2-7/2)}+N_{3/2}r^{(+)}_{5/2}(\tfrac{3}{2}-\tfrac{7}{2})\overline{W}^{T=0}_{(3/2-7/2)}+p^{+}_{5/2}(\tfrac{3}{2}-\tfrac{3}{2})\\ &\quad+(N_{3/2}-1)q^{(+)}_{5/2}(\tfrac{3}{2}-\tfrac{3}{2})\overline{W}^{T=1}_{(3/2-3/2)}+(N_{3/2}+1)r^{(+)}_{5/2}(\tfrac{3}{2}-\tfrac{3}{2})\overline{W}^{T=0}_{(3/2-3/2)}\Big]\Big/\Big[\sum_J(2J+1)S_{J\,5/2}\Big]\Big\}+E^{(+)}(\mathrm{riz})\,. \end{aligned} \tag{3}$$

Remembering that the numerical values of $n_{7/2}$, $n_{3/2}$, $T_{0\,7/2}$, $T_{0\,3/2}$, T_0, and $f_{5/2}$ are 12, 2, 2, 1, 3, and −4, respectively, the coefficients $p^{(+)}_{5/2}$ $(\frac{3}{2}-\frac{7}{2})$, $q^{(+)}_{5/2}$ $(\frac{3}{2}-\frac{7}{2})$, $r^{(+)}_{5/2}$ $(\frac{3}{2}-\frac{7}{2})$, $p^{(+)}_{5/2}$ $(\frac{3}{2}-\frac{3}{2})$, $q^{(+)}_{5/2}$ $(\frac{3}{2}-\frac{3}{2})$, and $r^{(+)}_{5/2}$ $(\frac{3}{2}-\frac{3}{2})$, calculated with the help of the prescription given in Eq. (2), are, −0.506, 7.67, 4.33, −0.593, 0.833, and 1.17, respectively. We further have $N_{3/2}=4$ and the force parameters $\overline{W}^{T=1}_{(3/2-7/2)}$, $\overline{W}^{T=0}_{(3/2-7/2)}$, $\overline{W}^{T=1}_{(3/2-3/2)}$, and $\overline{W}^{T=0}_{(3/2-3/2)}$ as given by Table I are 0.16, −1.7, −0.3, and −1.3 MeV, respectively. The denominator $\sum_J(2J+1)S_{J\,5/2}=4.33$. All this, when substituted in Eq. (3), gives

$$\begin{aligned} &E^{(+)}_{5/2}(^{54}\mathrm{Cr}+2p_{3/2}(\mathrm{proton}))-E^{(+)}(\mathrm{riz})=-7.84\ \mathrm{MeV}\,,\\ &E^{(+)}_{5/2}-E^{(+)}_{\mathrm{g.s.}}=2.27\ \mathrm{MeV}\,. \end{aligned} \tag{4}$$

Tables I–VI give, respectively, the values of the two-body force parameters and the calculated values of $T_<$ centroids against the measured[4-8] values, in the calcium, nickel, and zirconium regions of the Periodic Table. The agreement between the theoretically calculated and experimentally measured values of centroids is reasonably good.

The sum-rule equations expressing the relationship between the energy centroids and the residual two-body-force parameters, given earlier and applied in this paper, in reasonable generality, seem to be doing quite a good job of predicting the centroids.

These equations are pretty powerful and significant in the sense that they furnish us with the knowledge of centroids without going into the laborious shell-model calculation. To the physicists interested in transfer reactions, these should prove very useful.

Further investigations of the theoretical study of meaningful averages in nuclear spectra and transition rates, along with further applications of the present type of sum rules are in progress.

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