High-Momentum-Transfer Electron Scattering from ^{6}Li in a Deformed Cluster Model

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A simplified version of the α-d cluster formalism developed by Neudatchin, Smirnov, and their collaborators gives good agreement with the most recent elastic and inelastic (2.189-MeV) electron scattering data of Li *et al.* for momentum transfer $q^2 \lesssim 6.5$ fm^{-2}. Inclusion of lowest-order effects of dynamical volume deformations of the intercluster density extends the agreement to $q^2 \lesssim 10.5$ fm^{-2}. The contribution of the latter to the elastic form factor may be interpreted as due to an effective spherically symmetric "modulating charge" distribution superimposed upon the smoothly varying distribution responsible for the scattering at lower momentum transfer. This modulating charge is calculated explicitly and exhibits an oscillatory behavior. A similar analysis is carried out in the inelastic channel.

I. INTRODUCTION

In the past several years, electron scattering data at very high values of momentum transfer have been obtained from a number of nuclei by the Stanford group.[1] These data are of special theoretical interest, because one is generally unable to explain them using the phenomenological charge distributions giving an adequate fit at lower energies. Typically, it is necessary to modify the latter, such that the goodness of fit at smaller angles remains unchanged, but in such a way as to produce a good fit at the larger angles. This type of modification corresponds to introducing a phenomenological modification in the Fermi distributions with which the data at lower q are usually fitted, which has an oscillatory character.

In the case of the Ca isotopes, a number of widely differing theoretical approaches have been used to explain this effect, including short-range correlations,[2] and the use of configuration mixing generated in a nonlocal potential.[3] It appears to be very difficult to distinguish between these approaches experimentally; indeed, Negele[4] has carried out a Brueckner-Hartree-Fock calculation in the local density approximation in which these effects, taken together, appear to largely cancel one another. In yet a third approach, which departs from spherical symmetry, Raphael and Rosen[5] have developed a theory of electron scattering incorporating the effects of generalized dynamical deformations. This theory, applied to the Ca isotopes,[6] not only produces the oscillatory modification of the charge distribution required to reproduce the high momentum transfer data, but also explains the anomalous behavior of the rms charge radii of the Ca isotopes.

It is interesting to observe that the effect being discussed also occurs in one of the simplest nuclei to exhibit shell structure, namely ^{6}Li. While Li *et al.*[7] found that a three-parameter Gaussian distribution gave a very good fit to their data below $q^2 = 6$ fm^{-2}, it did not reproduce the diffraction feature at $q^2 \sim 7.7$ fm^{-2}, or the higher q^2 behavior up to 13 fm^{-2}. However, this entire range of data could be reproduced by introducing an oscillatory modification in the charge distribution, in much the same way as for the Ca isotopes. Several authors[8-10] have pointed out that the shell model with short-range correlations is capable of reproducing the diffraction feature. However this can also be done with an α-d cluster model, without correlations.[11] None of the theoretical explanations thus far proposed fit the data beyond the diffraction minimum.[12] One approach might be to incorporate short-range correlations in the framework of the α-d cluster description. A second, which we choose to pursue here, is to examine the effects of generalized volume deformations on the intercluster density distribution, along the lines which proved successful[6] in the case of the Ca isotopes. One might hope that ^{6}Li, because of its simplicity and well-established cluster structure, may afford a means of distinguishing between these widely differing theories, which is obscured in the heavier nuclei.

In what follows, we shall first develop a simplified version of the α-d cluster formalism capable of giving a good account of the elastic and inelastic (2.189-MeV level) Coulomb electron scattering form factors for $q^2 \lesssim 6.5$ fm^{-2} (Sec. II). In a manner similar to our earlier formalism, we shall then introduce in Sec. III the deformation as an operator in the Hilbert space of the intercluster density, the internal densities of each cluster being assumed c numbers. While the transformation properties of these operators under rotations are prescribed, their dynamical character, e.g., commutation relations, is left unspecified. The result is to modify the undeformed form factor by

a series of additional terms which influence the cross section at progressively higher values of momentum transfer, and which converge rapidly for not-too-large deformations. Each term is multiplied by the reduced matrix element of tensor products of the deformation operator, which is regarded as a free parameter in fitting the data. It will be shown that a single such term suffices to enlarge the region of fit to $q^2 \lesssim 10.5$ fm^{-2} in both the elastic and inelastic channels. An inversion procedure is then used to find an equivalent spherically symmetric intercluster density capable of giving rise to the deformation contribution to the form factor. This modulating density exhibits an oscillatory behavior in both channels.

While the proposed deformation mechanism thus appears capable of yielding the correct high momentum transfer behavior of the form factors, it would be premature to draw any but the most tentative conclusions from this fact. Our calculation neglects effects of nucleon exchange between clusters. While this is justified at small ($q^2 < 4$ fm^{-2}) momentum transfer due to the small degree of cluster overlap, there are indications[11] that exchange effects may play an increasingly important role at higher momentum transfer. Furthermore, other effects begin to appear at high q^2 which serve to further complicate the picture, such as scattering from the neutron charge distribution and magnetic moment.[13] We nevertheless feel it worthwhile to call attention to the potentialities of this very general model for the interpretation of high momentum transfer effects, which seem to be insufficiently appreciated. For so light a nucleus, it suffices to work within the context of the Born approximation.

II. ELECTRON SCATTERING IN THE CLUSTER FORMALISM

Nucleon clustering in p-shell nuclei became of interest more than 30 years ago, when the extraordinary stability of α-particle nuclei was first recognized. This stability is attributable to the predominance of Majorana forces in the effective interaction, which leads to an LS coupling situation in which the orbital Young partition (λ, μ), the orbital angular momentum L and spin S, as well as J and T are good quantum numbers. Under these circumstances, the nucleus may be divided into clusters, each of which corresponds to a row in a Young tableau, and the wave function constructed by clusters. In the case of ^{6}Li, this takes the familiar resonating group form[14]

$$\psi_{^6\mathrm{Li}} = \mathcal{A}\left[\varphi_\alpha(1,2,3,4)\varphi_d(5,6) \times \chi_{\alpha-d}(\vec{r}_\alpha - \vec{r}_d)\xi_{\sigma\tau}(1,2,3,4;5,6)\right] \quad (1)$$

in which φ_α, φ_d are the internal α- and d-cluster functions, $\chi_{\alpha-d}$ is relative function depending on the separation $\vec{r} = \vec{r}_\alpha - \vec{r}_d$ of the cluster centers, and $\xi_{\sigma\tau}$ is an appropriate spin-charge function. Throughout the ensuing discussion as a matter of convenience we use the term "d cluster" or "deuteron cluster" for the $S=1$ two-nucleon association represented by φ_d in Eq. (1). It must not be thought, however, that we attribute to this association any of the detailed properties of the free deuteron, unless explicitly stated. A similar remark holds for the "α cluster." Galilean invariance of $\psi_{^6\mathrm{Li}}$ can be preserved by using Gaussian forms for the internal cluster wave functions. It has been shown that the ^{6}Li form factors, in particular those for electron scattering, show little sensitivity[15] to the asymptotic form of φ_d.

A more interesting question relates to the choice of the intercluster wave function, and the influence of antisymmetrization on the various form factors. In order to choose an appropriate function, physical nucleon-clustering effects can be regarded as a kind of perturbation of the independent-particle harmonic-oscillator shell model, in the following sense. If the size parameters b_α, b_d characterizing the clusters were the same as that (=b) for the nucleus as a whole, i.e., complete overlap, then the two models would simply be related by a unitary transformation, with no new physical content. In that case, one can easily show[16] that the intercluster wave function is

$$\chi_{\alpha-d}(\vec{r}) = N r^2 e^{-(2/3)r^2} Y_{LM}(\hat{r}), \quad (2)$$

with N a normalization factor and $\vec{r}$ in units of b. One then assumes this form remains valid as the cluster sizes are allowed to become different from the nuclear size. The justification for maintaining this connection with the shell model is that the latter describes rather well certain properties, such as level structure and magnetic moments.[17] While more elaborate assumptions can be made within the context of a variational approach,[18] their utility is rendered dubious by the uncertainties in the residual interaction, as Neudatchin and Smirnov[15] have pointed out.

A considerable number of form factors using Eq. (2) both with and without antisymmetrization between cluster nucleons were calculated in Refs. 11 and 15. It was found that for most of the form factors, antisymmetrization effects at least for $q^2 < 4$ fm^{-2}, were not significant,[19] due to the fact that the cluster centers, using this form of $\chi_{\alpha-d}$, are well separated. The fits could all be characterized by two ratios[20] of the three lengths b_α, b_d,

and b:

$$X \equiv b_\alpha^2/b^2 \approx 0.4, \tag{3a}$$

$$Y \equiv b_\alpha^2/b_\alpha^2 \approx 0.4 \tag{3b}$$

with sensitivity principally to X. The authors thereby obtain good simultaneous fits to the elastic and inelastic (3^+ level) scattering data for $q^2 < 4$ fm^{-2}.

We may make use of these results to recast the cluster claculation of the electron scattering Coulomb form factors into a particularly simple form. From the viewpoint of physical input, the chief difference between our calculation and that of Refs. 11 and 15 consists in the use of the experimental ^{6}Li rms radius as a given parameter, which will enable us to express both the elastic and inelastic form factors in terms of the single parameter X. From the viewpoint of methodology, we will neglect antisymmetrization from the beginning, and assume (as they do) Gaussian forms for the internal cluster wave functions. We will further consider that the $T=0$ levels of ^{6}Li, which include the ground state, correspond to rotational excitations of the intercluster wave function Eq. (2) (though for the higher levels this excitation is probably not pure). Accordingly, the ^{6}Li charge density operator may be written as a sum of convolutions:

$$\hat{\rho}_{^6\mathrm{Li}}(\vec{r}) = \frac{2}{3}\int \hat{\rho}_{\alpha-d}(3\vec{r}_\alpha)\rho_\alpha(\vec{r}-\vec{r}_\alpha)d\vec{r}_\alpha + \frac{1}{3}\int \hat{\rho}_{\alpha-d}(\tfrac{3}{2}\vec{r}_d)\rho_d(\vec{r}-\vec{r}_d)d\vec{r}_d \tag{4}$$

in an obvious notation; and its Fourier transform, whose reduced matrix element gives the required form factor, is then given by

$$\hat{F}_{^6\mathrm{Li}}(q) = \tfrac{2}{3}\hat{F}_{\alpha-d}(q/3)F_\alpha(q) + \tfrac{1}{3}\hat{F}_{\alpha-d}(2q/3)F_d(q). \tag{5}$$

Internal cluster excitations are here ignored. When appropriate account is taken of the center-of-mass constraint and the finite proton size, the internal form factors $F_\alpha(q)$, $F_d(q)$ are given by

$$F_\alpha(q) = \exp(-\tfrac{3}{16}q^2 b_\alpha^2)F_p(q), \tag{6a}$$

$$F_\alpha(q) = \exp(-\tfrac{1}{8}q^2 b_\alpha^2)F_p(q). \tag{6b}$$

Here the proton form factor is taken from Janssens *et al.*[21]:

$$F_p(q) = \frac{1.249}{1+q^2/15.6\ \mathrm{fm}^{-2}} + \frac{0.7982}{1+q^2/26.6\ \mathrm{fm}^{-2}} + \frac{0.5819}{1+q^2/8.19\ \mathrm{fm}^{-2}} - 0.0326, \tag{7}$$

and is a slowly varying function of q. Finally, the Fourier transform of the intercluster density $\rho_{\alpha-d} = |\chi_{\alpha-d}|^2$ for $L=0$ is

$$F^{(0)}_{\alpha-d}(x) = \tfrac{1}{15}(4x^4 - 20x^2 + 15)e^{-x^2}, \tag{8}$$

with $x^2 = \frac{3}{16}q^2 b^2$.

A low-q expansion of Eq. (5) takes the form

$$b^2 = \tfrac{12}{7}[\langle r^2\rangle_{^6\mathrm{Li}} - \tfrac{2}{3}\langle r^2\rangle_\alpha - \tfrac{1}{3}\langle r^2\rangle_d]; \tag{9}$$

while the assumed form Eq. (6a) for the α-cluster amplitude gives

$$b^2 = (8/9X)[\langle r^2\rangle_\alpha - \langle r^2\rangle_p] \tag{10}$$

in terms of the basic parameter X of Eq. (3a). From Eq. (7), one finds $\langle r^2\rangle_p = 0.726$ fm^2. One may then solve for the rms d-cluster radius:

$$\langle r^2\rangle_d = 3\langle r^2\rangle_{^6\mathrm{Li}} - [2 + (\tfrac{14}{9}X)]\langle r^2\rangle_\alpha + (\tfrac{14}{9}X)\langle r^2\rangle_p. \tag{11}$$

Now the free α particle is a tightly bound structure, and we therefore represent the rms α-cluster radius by that of the free α particle. The free deuteron on the other hand is loosely bound and compressible, and therefore we use Eq. (11) to fix the rms d-cluster radius in terms of the known rms radius[22] of ^{6}Li, $\langle r^2\rangle_{^6\mathrm{Li}}^{1/2} = 2.54 \pm 0.05$ fm. Thus the parameter X is the single free parameter to be used in fitting the data at higher q. It has been pointed out[23] that $\langle r^2\rangle_{^6\mathrm{Li}}$ is sensitive to exchange effects in the cluster model. The use of the experimental value of $\langle r^2\rangle_{^6\mathrm{Li}}$ largely incorporates these effects.

The squared elastic form factor, calculated with the aid of the foregoing expressions is shown by the dashed curve in Fig. 1, together with the most recent data of Li *et al.*[7] There is very good agreement out to the diffraction minimum, $q^2 \sim 7.7$ fm^{-2}, which is of course a zero in the Born approximation. The fit is roughly comparable to that shown by Kudeyarov *et al.*,[11] using the more elaborate form of the cluster model. It corresponds to a somewhat smaller value of X,

$$X = 0.31 \pm 0.02. \tag{12a}$$

The value of Y, which in our case is fixed by X and the experimental rms ^{6}Li radius, is

$$Y = 0.33 \pm 0.02. \tag{12b}$$

The relation between the rms radii of ^{6}Li and its component clusters corresponding to the values of Eqs. (12a) and (12b) is shown in Fig. 2, together with the average separation of cluster centers,

$$\bar{r} = [4\pi\int_0^\infty dr\, r^4 \rho_{\alpha-d}(r)]^{1/2} = 3.752\ \mathrm{fm}. \tag{13}$$

In terms of this parameter, the mean square radius of ^{6}Li is

$$\langle r^2\rangle_{^6\mathrm{Li}} = \tfrac{1}{3}\langle r^2\rangle_d + \tfrac{2}{3}\langle r^2\rangle_\alpha + \tfrac{2}{9}\bar{r}^2 . \quad (14)$$

Observe that there is on the average no cluster overlap, so that cluster exchange effects in the elastic channel, consistent with our initial assumptions, play at most a small role. The rms deuteron cluster radius as determined from Eqs. (11) and (12a) comes out 2.09± 0.15 fm, which is within the fitting error, the same as the free deuteron value[24] of 2.17± 0.05 fm.

Quadrupole elastic scattering from the ^{6}Li nucleus is of course possible, but is rendered unobservable in view of the small value of the ^{6}Li quadrupole moment ($Q_{^6\mathrm{Li}} = -0.08$ fm^2. This may be made plausible by the following argument. The complete Coulomb elastic squared form factor, including the quadrupole contribution, is

$$[F_{^6\mathrm{Li}}(q)]^2 = [F_{^6\mathrm{Li}}(q)]_{L=0}{}^2 + \frac{q^4 Q_{^6\mathrm{Li}}{}^2}{162}[F_{^6\mathrm{Li}}(q)]_{L=2}{}^2 . \quad (15)$$

Now with the α cluster in its ground state, the $L=2$ contribution can arise only from the second term of Eq. (5),

$$[F_{^6\mathrm{Li}}(q)]_{L=2}{}^2 = \tfrac{1}{9}[F_{\alpha-d}(\tfrac{2}{3}q)]^2[F_d(q)]_{L=2}{}^2 . \quad (16)$$

If we assume (only for the purposes of this argument) that the d cluster is in fact identical to the free deuteron, we may make the identification

$$[G_b(q)]^2 = \frac{q^4 Q d^2}{18}[F_d(q)]_{L=2}{}^2 , \quad (17)$$

where $G_b{}^2$ is the charge quadrupole part of the squared elastic electron-deuteron form factor. Hence Eq. (15) becomes

$$[F_{^6\mathrm{Li}}(q)]^2 = [F_{^6\mathrm{Li}}(q)]_{L=0}{}^2 + \frac{1}{81}\left(\frac{Q_{^6\mathrm{Li}}}{Q_\alpha}\right)^2 \times [F_{\alpha-d}(\tfrac{2}{3}q)]^2[G_b(q)]^2 . \quad (18)$$

The quantity $G_b{}^2$ has been calculated by McIntyre and Dhar[25] and tabulated in Ref. 24 for a number of potentials. At 500 MeV it is a slowly varying function of q^2, having a value typically of the order of 0.004, and not deviating from this value by more than a factor of 2 over the range $0<q^2<16$ fm^{-2}. The most favorable place to observe quadrupole effects is near the diffraction minimum ($q^2\approx 7.5$ fm^{-2}). Using the ratio $(Q_{^6\mathrm{Li}}/Q_d)^2 \approx 0.08$ and Eq. (8), the second term of Eq. (18) is roughly 10^{-9}; while at $q^2 = 13$ fm^{-2}, it is roughly 10^{-11}. This is several orders of magnitude below the experimental squared ^{6}Li form factor, and hence may be safely neglected even if the d-cluster quadrupole moment were in fact considerably different from that of the free deuteron. As to the magnetic elastic contribution, it was pointed out in Ref. 7 that this contribution, as obtained *experimentally* by measuring the form factors at different angles and constant momentum transfer, was found to be completely negligible except in the vicinity of the diffraction minimum.

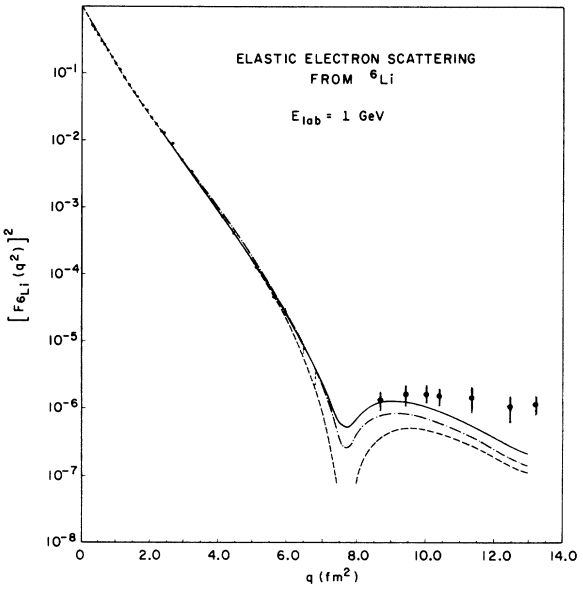

FIG. 1. Monopole elastic electron scattering from ^{6}Li as a function of squared momentum transfer. The data are from Li *et al.* (Ref. 7). The dashed curve shows the fit obtained with the cluster model in the absence of deformations. The inclusion of deformation effects to the first nonvanishing order leads to the dot-dashed curve when the renormalization procedure is carried out on the intercluster form factor only; and to the solid curve, when carried out for the complete form factor.

It should be noted that for $L=2$ elastic scattering, the low-q limit of Eq. (5) provides a relationship between the ^{6}Li and d-cluster quadrupole moments: $Q_{^6\mathrm{Li}} = \frac{1}{3}Q_d$. If the d cluster has the same D-state admixture as the free deuteron, then $Q_d = 0.27$ fm^2 and the relation gives the correct order of magnitude for $Q_{^6\mathrm{Li}}$, but the wrong sign. However, it is by no means clear that this assumption is correct. It is also probable that, as in the case of the ^{6}Li rms radius, exchange effects may play a significant role in determining the quadrupole moment. Elsewhere[26] the author has pointed out that high-energy proton-^{6}Li scattering experiments, which will soon be possible at the Los Alamos Meson Physics Facility (LAMPF), should be sensitive to the D-state admixture in the deuteron cluster via the real part of the p-d cluster amplitude.

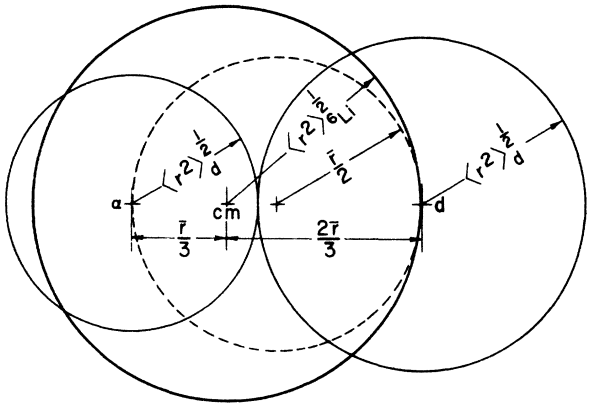

FIG. 2. A schematic representation to scale of the cluster structure of the ^{6}Li nucleus, in which $\langle r^2\rangle_{^6\mathrm{Li}}$ is given its experimental value, $\langle r^2\rangle_\alpha^{1/2}$ is assumed to be that of the free α particle, =1.63 fm, and $\langle r^2\rangle_d^{1/2}=2.16$ fm and $\bar{r}=3.75$ fm are fixed from the constraint Eq. (11) and the value of $x=0.31$ Eq. (12a) required to fit the elastic scattering data below the diffraction minimum (dashed curve of Fig. 1).

Exactly analogous calculations may be carried out for the inelastic scattering of electrons from the $T=0$, $J^\pi=3^+$ (2.189-MeV) excited state of ^{6}Li, which is here regarded as an $L=2$ excitation of the intercluster wave function Eq. (2). While a more complicated polynomial could be used for the radial dependence, the simple assumption of a purely rotational excitation enabled a good fit to the inelastic data for $q^2<4$ fm^{-2} to be obtained in Refs. 11 and 14, and is for that reason retained here. That this assumption is not entirely adequate is reflected in the necessity of using a somewhat different size parameter for ^{6}Li in the inelastic channel, as will appear immediately below. The intercluster form factor in this case is found to be

$$F_{\alpha\text{-}d}^{(2)}(x_i)=\tfrac{6}{5}x_i^2(1-\tfrac{2}{7}x_i^2)e^{-x_i^2} \tag{19}$$

by using the same normalization as for the ground state. The size b_i characterizing the intercluster transition density need not be the same as b, and therefore Eq. (19) is expressed in units of $x_i=\frac{3}{16}q^2b_i^2$. In the limit of small q, one then finds a $B(E2)$ value for ^{6}Li of 26.5×10^{-52} e^2 cm^4, in satisfactory agreement with the experimental value[27] of $30\pm3\times10^{-52}$ e^2 cm^4. The other ingredients entering into the form-factor calculation at higher momentum transfer are unchanged from the elastic case. The result is shown by the dashed curve in Fig. 3 together with the data of Li *et al.*[7] The range of quantitative agreement is approximately the same as in the elastic case, and comparable with the fit obtained in Refs. 11 and 15 to older data[28] over the more limited range $q^2<4$ fm^{-2}. The fit limits the value of the parameter X in the inelastic channel to

$$X_{\text{inelastic}}\equiv b_\alpha^2/b_i^2 = 0.25\pm0.01, \tag{20}$$

a value somewhat smaller than that from elastic scattering. The inelastic size parameter $b_i=2.62$ fm, compared to the elastic value $b=2.35$ fm. This difference may reflect the presence of centrifugal "stretching" effects in the intercluster wave function for the excited state.

The simplified version of the α-d cluster model developed in this section is already able to give a good account of electron scattering data in both the elastic and inelastic channels for $q^2<6.5$ fm^{-2}. It is remarkable that only one adjustable parameter, X, is required in each channel. Moreover, the model parameters have values reasonably close to those found in Refs. 11 and 15 to fit a number of additional form factors. We again point out that these fits, as well as those obtained in Refs. 11 and 15 are much superior to those obtained with the shell model.[8-10] In the following section it will be shown that the range of agreement can be greatly extended by including the effects of dynamical volume deformations into the intercluster density. In carrying out this development, it will be assumed that exchange effects continue to be insignificant over the augmented range of momentum transfer to be considered.

III. EFFECT OF GENERALIZED VOLUME DEFORMATIONS

The intercluster density operator for a given rotational state λ is a scalar, and can be written in general

$$\hat{\rho}_\lambda(r,\theta,\phi)=(4\pi)^{1/2}\sum_\mu \hat{\rho}_{\lambda\mu}(r)Y^*_{\lambda\mu}(\theta,\phi) \tag{21}$$

in which r is measured in units of b, and the angles θ, ϕ refer to a spherical coordinate system with the nuclear center of mass as origin. The coulomb multipole operator for this state is then

$$\hat{M}_{LM}^{(\lambda)}(q)=\int d^{(3)}r\,\hat{\rho}_\lambda(r,\theta,\phi)j_L(qr)Y_{LM}(\theta,\phi)$$

$$=(4\pi)^{1/2}\sum_\mu\int_0^\infty dr\,r^2\hat{\rho}_{\lambda\mu}(r)j_L(qr)\delta(L,\lambda)\delta(M,\mu); \tag{22}$$

and the L-pole Coulomb form factor for transitions between initial and final rotational states (L_i, L_f) is, in general

$$F_{\alpha-d}^{(L_i,\lambda)}(q) \equiv (4\pi)^{1/2}\langle L_f \| \hat{M}_L^{(\lambda)}(q) \| L_i \rangle$$
$$= (4\pi)\int_0^\infty dr\, r^2 \langle L_f \| \hat{\rho}_\lambda(r) \| L_i \rangle j_L(qr)\delta(L,\lambda) . \tag{23}$$

The distinction between L and λ, while superfluous here, becomes meaningful in what follows. The analysis of the last section suggests that the radial dependence of $\hat{\rho}_\lambda$ be taken as state independent, and equal to $\rho_{\alpha-d}(r)$:

$$\rho_{\alpha-d}(r) = \langle L_f \| \hat{\rho}_\lambda(r) \| L_i \rangle \Delta(L_f \lambda L_i)$$
$$= N^2 r^4 e^{-(4/3)r^2} , \tag{24}$$

in accordance with Eq. (2). Here Δ expresses the triangle relationship satisfied by the vectors $\vec{L}_f, \vec{\lambda}, \vec{L}_i$.

The operator $\hat{\rho}_\lambda(r, \theta, \phi)$ is now deformed by the prescription

$$\hat{\rho}_\lambda(r, \theta, \phi) \to \hat{\rho}_\lambda[r(1+\hat{D}), \theta, \phi] , \tag{25}$$

where the deformation operator D is given by

$$\hat{D} = (4\pi)^{1/2} \sum_{lm} \hat{a}_{lm} Y_{lm}^*(\theta, \phi\rangle . \tag{26}$$

The $\hat{a}_{lm}$ are irreducible tensor operators in the nuclear Hilbert space transforming as $Y_{lm}(\theta, \phi)$. For convenience, we treat the $l=0$ term in Eq. (26) in a special way. We may write

$$\hat{a}_{00} = (\hat{a}_{00} - \langle L_i | \hat{a}_{00} | L_i \rangle) + \langle L_i | \hat{a}_{00} | L_i \rangle .$$

The second term on the right can be absorbed into the size parameter b, and the remaining operators similarly redefined. In this way the ground-state expectation value of the leading term of Eq. (26) can be taken to vanish. In what follows, it will not be necessary to impose dynamical restrictions on the $\hat{a}_{lm}$,

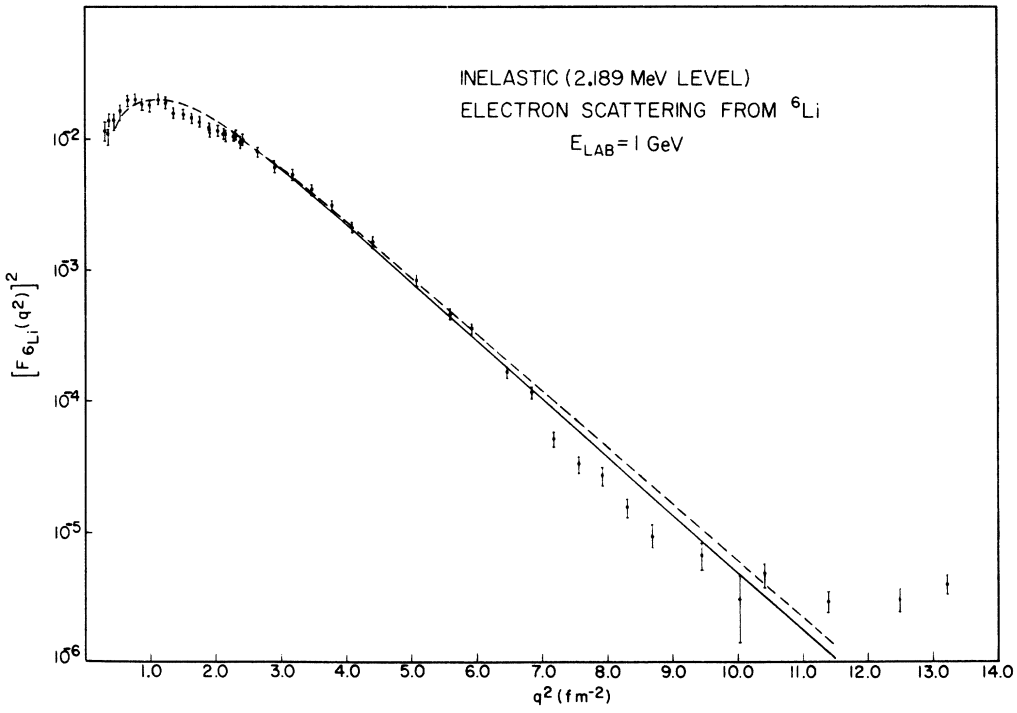

FIG. 3. Inelastic Coulomb scattering from the $J^\pi = 3^+$ ($L=2$ excitation of the intercluster wave function) level of ^{6}Li as a function of squared momentum transfer. The data are from Li *et al*. The dashed curve shows the fit obtained by the cluster model in the absence of deformations, corresponding to $X_{\text{inelastic}} = 0.25$. The solid curve includes the effects of deformation, with renormalization of the complete ^{6}Li form factor.

although this can of course be done via suitable commutation relations in the investigation of specific models.

It will be noted that we do not allow the internal cluster densities to become deformed, consistently with our original assumption that the clusters themselves remain in their ground states, and also with requirements of simplicity of description.[29] Nevertheless, it will be seen that deformation of the intercluster density requires a renormalization of the individual cluster sizes.

A Taylor expansion of the deformed density yields

$$\hat{\rho}_\lambda[r(1+\hat{D}),\theta,\phi]=\hat{\rho}_\lambda(r,\theta,\phi)+\sum_n \frac{r^n}{n!}\left\{\hat{\rho}_\lambda^{(n)}(r,\theta,\phi)\sum_{lm}\hat{a}_{lm}^{(n)}\right\}Y_{lm}^*(\theta,\phi), \tag{27}$$

in which $\hat{\rho}_\lambda^{(n)}(r,\theta,\phi)=(d^n\,|\,dr^n)\hat{\rho}_\lambda(r,\theta,\phi)$, the braces indicate appropriate symmetrization of the operator products, and the $\hat{a}_{lm}^{(n)}$ are n-fold tensor products of the operator $\hat{a}_{lm}$:

$$\hat{a}_{lm}^{(1)}=\hat{a}_{lm}, \tag{28a}$$

$$\hat{a}_{lm}^{(2)}=\sum_{l'l''}\hat{l}'\hat{l}''\begin{pmatrix}l' & l'' & l\\ 0 & 0 & 0\end{pmatrix}[\hat{a}_{l'}\otimes a_{l''}]_l^m, \tag{28b}$$

$$\hat{a}_{lm}^{(3)}=\sum_{l'l''l'''l^{iv}}\hat{l}'\hat{l}''\hat{l}'''\hat{l}^{iv}\begin{pmatrix}l'' & l''' & l^{iv}\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l & l' & l^{iv}\\ 0 & 0 & 0\end{pmatrix}\{\hat{a}_{l'}\otimes[\hat{a}_{l''}\otimes\hat{a}_{l'''}]_{l^{iv}}\}_l^m. \tag{28c}$$

Here and in what follows, we make use of the standard abbreviation $\hat{L}=(2L+1)^{1/2}$, and define $3j$ and $6j$ symbols according to Edmonds.[30] Equation (27) thus provides explicit expressions for the deformed $\hat{\rho}_{\lambda\mu}(r)$, using Eq. (21). After a straightforward calculation, we find for the Coulomb form factor

$$F_{\alpha-d}^{(L,\lambda)}(q)=4\pi\int_0^\infty dr\,r^2\rho_{\alpha-d}(r)j_L(qr)\delta(L,\lambda)$$
$$+(4\pi)^{1/2}\sum_n\frac{1}{n!}\sum_l(-)^{l-\lambda}\hat{\lambda}\hat{l}\hat{L}^2\begin{pmatrix}\lambda & l & L\\ 0 & 0 & 0\end{pmatrix}\int_0^\infty dr\,r^{2+n}\langle L_f\|\hat{T}_{l\lambda}^{(L)}\|L_i\rangle j_L(qr), \tag{29}$$

in which

$$\langle L_f\|\hat{T}_{l\lambda}^{(L)}\|L_i\rangle=\langle L_f\|\{\hat{\rho}_\lambda^{(n)}\otimes\hat{a}_l^{(n)}\}_L\|L_i\rangle$$
$$=(-)^{L_f+L_i+L}\hat{L}\sum_{L'}\begin{Bmatrix}\lambda & l & L\\ L_i & L_f & L'\end{Bmatrix}\langle L_f\|\hat{\rho}_\lambda^{(n)}\|L'\rangle\langle L'\|\hat{a}_l^{(n)}\|L_i\rangle. \tag{30}$$

Now by assumption

$$\langle L_f\|\hat{\rho}_\lambda^{(n)}(r)\|L'\rangle=\rho_{\alpha-d}^{(n)}(r)\Delta(L_f\lambda L'), \tag{31}$$

so that finally we obtain

$$F_{\alpha-d}^{(L,\lambda)}(q)=4\pi\int_0^\infty dr\,r^2\rho_{\alpha-d}(r)j_L(qr)\delta(L,\lambda)+4\pi\sum_n\frac{1}{n!}A_{L,\lambda}^{(n)}\int_0^\infty dr\,r^{2+n}\rho_{\alpha-d}^{(n)}(r)j_L(qr), \tag{32}$$

where the parameters

$$A_{L,\lambda}^{(n)}=(-)^L\hat{\lambda}\hat{L}^2(4\pi)^{-1/2}\sum_l\begin{pmatrix}\lambda & l & L\\ 0 & 0 & 0\end{pmatrix}\langle l\|\hat{a}_l^{(n)}\|0\rangle \tag{33}$$

emerge as the natural ones for the problem, and we have specialized to the case at hand, $L_i=0$. Contributions to the scattering in a given multipole order L can now in general arise from any rotational state λ, in view of the coupling of the latter to the multipole deformation l; however, in view of the assumption Eq. (31), such effects are all contained in the parameters $A_{L\lambda}^{(n)}$, which are to be fixed from the data. Hence it makes no difference to the ensuing argument whether or not the sum over λ is carried out. For not-too-large deformations, the sum in Eq. (32) may be presumed to converge rapidly, so that, depending on the range of q for which the scattering is measured, only the first few terms need be retained. Progressively higher terms in the sum have an appreciable effect on the scattering only at progressively higher values of momentum transfer. We now evaluate Eq. (32) separately in the cases of elastic and inelastic scattering.

A. Elastic Scattering

In the case $L=0$, $\lambda=0$, Eq. (32) may be evaluated straightforwardly. In view of the argument following Eq. (26), and Eq. (28a), the $n=1$ term does not contribute. In the limit $q=0$, we find the normalization

$$N^2 = 2^9 3^{-9/2} 5^{-1} \pi^{-3/2}(1 - 6A_{00}^{(2)} + 10A_{00}^{(3)}), \tag{34}$$

correct to third order. The monopole intercluster form factor is then

$$F_{\alpha-d}(x) = F_{\alpha-d}^{(0)}(x) + A_{00}^{(2)} F_{\alpha-d}^{(2)}(x) + A_{00}^{(3)} F_{\alpha-d}^{(3)}(x) \tag{35}$$

in which the leading term is given by Eq. (8), and

$$F_{\alpha-d}^{(2)}(x) = \tfrac{1}{15} x^2(8x^6 - 108x^4 + 378x^2 - 315)e^{-x^2}, \tag{36a}$$

$$F_{\alpha-d}^{(3)}(x) = \tfrac{1}{45} x^2(16x^8 - 320x^6 + 1992x^4 - 4368x^2 + 2625)e^{-x^2}, \tag{36b}$$

The factor x^2 in front of the correction terms already shows that the deformation effects set in at high momentum transfer. However, as we have pointed out in our earlier papers dealing with this general subject,[5,6] these modifications are in part unobservable. This is because the analysis of elastic scattering is usually done in terms of a spherical charge distribution with parameters chosen to reproduce the data. Hence, as much as possible, deformation effects are already incorporated into these parameters. For this reason, it is necessary to renormalize the size parameters of the problem by demanding that the first few terms of a power series expansion of the form factor with deformation effects included be identical to those of the form factor in the absence of deformations. This will be done first for the intercluster form factor.

We therefore set $b^2 \to b^{*2} = b^2(1+\lambda)$, with $\lambda = \lambda_2 A_{00}^{(2)} + \lambda_3 A_{00}^{(3)}$, and adjust the λ_j such that the first two terms of a low-q expansion of Eq. (35) agree with those of a similar expansion of Eq. (8). We find $\lambda_2 = -9$, $\lambda_3 = 25$. We thus obtain the renormalized intercluster form factor, expressed however in terms of b:

$$F_{\alpha-d}^{*}(x) = F_{\alpha-d}^{(0)*}(x) + A_{00}^{(2)} F_{\alpha-d}^{(2)*}(x) + A_{00}^{(3)} F_{\alpha-d}^{(3)*}(x), \tag{37}$$

where $F_{\alpha-d}^{(0)*}(x)$ again is given by Eq. (8), but now

$$F_{\alpha-d}^{(2)*}(x) = \tfrac{1}{15} x^4(8x^4 - 72x^2 + 126)e^{-x^2} \tag{38a}$$

and

$$F_{\alpha-d}^{(3)*}(x) = \tfrac{1}{45} x^4(16x^6 - 320x^4 + 1692x^2 - 2268)e^{-x^2}. \tag{38b}$$

Observe that the deformation effects are now operative at yet higher values of momentum transfer.

The lowest-order deformation contribution to the complete ^{6}Li form factor can now be evaluated by inserting the first two terms of Eq. (32) into Eq. (5). The parameter $A_{00}^{(2)}$ is adjusted to obtain agreement with the data at the lowest value of q^2 ($=6.5$ fm^{-2}) for which the unmodified form factor lies outside the experimental error. The result, obtained for $A_{00}^{(2)} = 0.01$, is given by the dot-dashed curve in Fig. 1, and already shows a marked improvement in the fit to the data beyond the diffraction minimum. The deformation also fills in the minimum to a considerable extent.

Before carrying out the renormalization for the complete ^{6}Li form factor, it is of interest to find the equivalent spherically symmetric density which gives rise to the form factor $F_{\alpha-d}^{(2)*}(x)$, i.e., the density $\rho_{\alpha-d}^{(\mathrm{mod})}(y)$ in the expression

$$(4\pi)^{1/2} \int_0^\infty dy\, y^2 \rho_{\alpha-d}^{(\mathrm{mod})}(y) j_0(xy) = F_{\alpha-d}^{(2)*}(x) \tag{39}$$

in which $y = (4/\sqrt{3})r$. This density must clearly oscillate, since $F_{\alpha-d}^{(2)*}(x)$ vanishes in the limit $x=0$. Also, its second moment vanishes. To find this "modulating density" explicitly, we make the *ansatz*

$$y\rho_{\alpha-d}^{(\mathrm{mod})}(y) = e^{-y^2/4} \sum_{n=0}^{\infty} a_{2n+1} H_{2n+1}(y/2) \tag{40}$$

a series in odd Hermitian polynomials. The integration in Eq. (39) may then be done exactly, and both sides expanded in a power series in x to determine the coefficients a_{2n+1}. We find

$$\rho_{\alpha-d}^{(\mathrm{mod})}(r) = (8/1215\pi) r^4(64r^4 - 432r^2 + 567)e^{-4r^2/3}. \tag{41}$$

This result, together with the unmodified intercluster density Eq. (24), is plotted in Fig. 4. The modulat-

ing density has two zeros, of which only the first is less than the average intercluster separation $\bar{r}$.

It is, of course, more meaningful to carry out the renormalization procedure for the complete ^{6}Li form factor. There are now two additional size parameters available, namely $\langle r^2\rangle_\alpha$, $\langle r^2\rangle_d$, subject to the constraint Eq. (9). The renormalization can then be carried out to one higher term in the power series expansion of the modified and unmodified ^{6}Li form factors. Excluding the proton form factor, which is unaffected by these manipulations, the expression to be renormalized is

$$\tfrac{2}{3}F_{\alpha-d}(\omega^2)e^{-8x_\alpha\omega^2}+\tfrac{1}{3}F_{\alpha-d}(4\omega^2)e^{-8x_d\omega^2} \tag{42}$$

in which $x_\alpha=\langle r^2\rangle_\alpha/b^2$, $x_d=\langle r^2\rangle_d/b^2$, $\omega^2=x^2/9$, and $F_{\alpha-d}$ is given by Eq. (35). We now set $b\to b^*$, as before, and in addition

$$\langle r^2\rangle_\alpha\to\langle r^2\rangle_\alpha^*=\langle r^2\rangle_\alpha(1+\sigma)\,, \tag{43}$$

with $\sigma=\sigma_2A_{00}^{(2)}+\sigma_3A_{00}^{(3)}$. The constraint Eq. (9) then implies for x_d^*:

$$x_d^*=(1+\lambda)^{-1}\{x_d-[\tfrac{7}{4}(\lambda_2+9)+2\sigma_2x_\alpha/x_d]A_{00}^{(2)}-[\tfrac{7}{4}(\lambda_3-25)+2\sigma_3x_\alpha/x_d]A_{00}^{(3)}\} \tag{44}$$

Inserting Eq. (43) and Eq. (44) into Eq. (42) and proceeding as before, we again find $\lambda_2=-9$, $\lambda_3=25$, so that the expression Eq. (37) for the renormalized intercluster form factor remains unchanged. In addition, however,

$$\sigma_2=-(189/20)x_\alpha^{-1}[8(x_\alpha-x_d)-7]^{-1}\,, \tag{45a}$$

$$\sigma_3=-6\sigma_2\,. \tag{45b}$$

Reexpressing Eq. (42) in terms of renormalized quantities, we obtain the complete renormalized ^{6}Li form factor:

$$F_{^6\mathrm{Li}}^*(\omega)=[F_{^6\mathrm{Li}}^{(0)*}(\omega)+A_{00}^{(2)*}F_{^6\mathrm{Li}}^{(2)*}(\omega)+A_{00}^{(3)*}(\omega)]e^{8x_p\omega^2}F_p(\omega) \tag{46}$$

with $x_p=\langle r^2\rangle_p/b^2$, and $F_{^6\mathrm{Li}}^{(0)*}(\omega)$ is given by

$$F_{^6\mathrm{Li}}^{(0)*}(\omega)=\tfrac{1}{45}\{[8\omega^4-40\omega^2+30]\exp[-(1+8x_\alpha)\omega^2]+[64\omega^4-80\omega^2+15]\exp[-4(1+2x_d)\omega^2]\}\,. \tag{47}$$

The second- and third-order deformation corrections are given by

$$F_{^6\mathrm{Li}}^{(2)*}(\omega)=\tfrac{4}{45}\omega^2\{[4\omega^6-4(9+4x_\alpha\sigma_2)\omega^4+(63+80x_\alpha\sigma_2)\omega^2-60x_\alpha\sigma_2]\exp[-(1+8x_\alpha)\omega^2]$$
$$+4[128\omega^6+32(2x_\alpha\sigma_2-9)\omega^4+2(63-40x_\alpha\sigma_2)\omega^2+5x_\alpha\sigma_2]\exp[-4(1+2x_d)\omega^2]\}\,; \tag{48a}$$

$$F_{^6\mathrm{Li}}^{(3)*}(\omega)=\tfrac{8}{135}\omega^2\{[4\omega^8-80\omega^6+3(48x_\alpha\sigma_2+141)\omega^4-9(80x_\alpha\sigma_2+63)\omega^2+540x_\alpha\sigma_2]\exp[-(1+8x_\alpha)\omega^2]$$
$$+4[512\omega^8-2560\omega^6+24(141-24x_\alpha\sigma_2)\omega^4+18(40x_\alpha\sigma_2+63)\omega^2+135x_\alpha\sigma_2]\exp[-4(1+2x_d)\omega^2]\}\,. \tag{48b}$$

An expansion for small ω of these expressions begins with ω^6. The ^{6}Li elastic form factor, with the lowest-order deformation Eq. (48a) included, is shown by the solid curve in Fig. 1. The value of $A_{00}^{(2)}$ remains 0.01 as previously, but renormalization of the complete form factor has resulted in a marked improvement, without any change in the number of parameters involved in the fit. We now have excellent agreement with the data for $q^2\lesssim10.5$ fm^{-2}, and inclusion of the third-order deformation correction would undoubtedly push this agreement out to the limit of the experimental range, but at the cost of one additional parameter. For this reason, and in order to stress the remarkable fit already obtained by the theory to this order, we have not carried out this further extension.

Again we may calculate the modulating density $\rho_{^6\mathrm{Li}}^{(\mathrm{mod})}$ giving rise to Eq. (48a), which must have vanishing zeroth, second and fourth moments indicating a more rapid oscillation than $\rho_{\alpha-d}^{(\mathrm{mod})}$. The procedure is the same as previously except that the *ansatz* now takes the form

$$\rho_{^6\mathrm{Li}}^{(\mathrm{mod})}(z)=\rho_1(z)+\rho_2(z) \tag{49}$$

with

$$z\rho_1(z)=\exp(-z^2/4A^2)\sum_{n=0}^{\infty}a_{2n+1}H_{2n+1}(z/2A)\,, \tag{50a}$$

$$z\rho_2(z)=\exp(-z^2/4D^2)\sum_{n=0}^{\infty}d_{2n+1}H_{2n+1}(z/2D)\,, \tag{50b}$$

where $A=(1+8x_\alpha)^{1/2}$, $D=2(1+2x_d)^{1/2}$, and $z=(4/\sqrt{3})r$. The unmodified density giving rise to the elastic form factor in the absence of deformation effects can be calculated in the same way. The results are plotted in Fig. 5. The unmodified density is substantially in agreement with the empirical three-parameter density used by Li *et al.* to fit their data below 6 fm^{-2}, except that the maximum proton density is somewhat larger than theirs. The modulating density Eq. (49) oscillates as rapidly within the well as does the empirical modification of Li *et al.* (involving three additional parameters) which they require to fit the data beyond the diffraction minimum. However, it has the opposite sign from theirs, and is also numerically much smaller. (The plotted curve does not include the factor $A_{00}^{(2)}$.)

B. Inelastic Scattering

We now take $L=2$, $\lambda=2$ in Eq. (32), and for the unmodified density use Eq. (19), with the value of $X_{\text{inelastic}}$ given by Eq. (20). To second order, the series in Eq. (32) may now contain both a linear and a quadratic term; however, it turns out that the complete form factor to this order may be characterized by the single parameter $B=A_{22}^{(2)}-(A_{22}^{(1)})^2$. Also, the intercluster form factor can no longer be renormalized separately from the complete form factor. We therefore give only the renormalized inelastic ^{6}Li form factor, correct to second order in the deformation:

$$F_6^{(2)}(\omega_i^2)=\tfrac{8}{35}\omega_i^2\Big(\{\tfrac{7}{2}-\omega_i^2+B\omega_i^2[-\tfrac{14}{3}\sigma_2'+(11+\tfrac{34}{27}\sigma_2')\omega_i^2-2\omega_i^4]\}\exp[-(1+8x_{\alpha i})\omega_i^2]$$
$$+2\{\tfrac{7}{2}-4\omega_i^2+B\omega_i^2[\tfrac{7}{3}\sigma_2'+(176-\tfrac{104}{27}\sigma_2')\omega_i^2-128\omega_i^4]\}\exp[-4(1+2x_{di})\omega_i^2]\Big)\exp(8x_{pi}\omega_i^2)F_p(\omega_i)\,, \tag{51}$$

where the subscript i means all lengths are expressed in units of b_i, $\omega_i^2=x_i^2/9$, and

$$\sigma_2'=6534[368-672(x_{\alpha i}-x_{di})]^{-1}. \tag{52}$$

The value of B as determined from the mismatch between the data and the unmodified transition form factor at $q^2=6.5$ fm^{-2} is 0.0025. This result is shown by the solid curve in Fig. 3. While there is some improvement over the agreement in the absence of deformation, the change is not nearly so marked as in the elastic channel. Beyond $q^2=7$ fm^{-2}, the experimental points exhibit a definite change of curvature which on a shell-model picture might perhaps be attributable to $L=4$ waves.[31] Our curve is to remarkable accuracy, a straight line over this region. Reexamining our assumptions, the two most likely sources of error are (1) exchange effects which may become significant at these very high values of q^2, and (2) detailed differences between the intercluster wave function in the elastic and inelastic channels. That these differences exist is shown by the somewhat different values of the intercluster separation parameter X required to fit the data below $q^2=7$ fm^{-2}. At larger values of momentum transfer, these stretching effects probably cannot be represented in so simple a manner. In spite of these shortcomings, the over-all goodness of fit in the range $q^2\lesssim 10.5$ fm^{-2} is impressive. To the author's knowledge, no calculations have been published at the time of writing which attempt to fit the newer data at $q^2>4$ fm^{-2}.

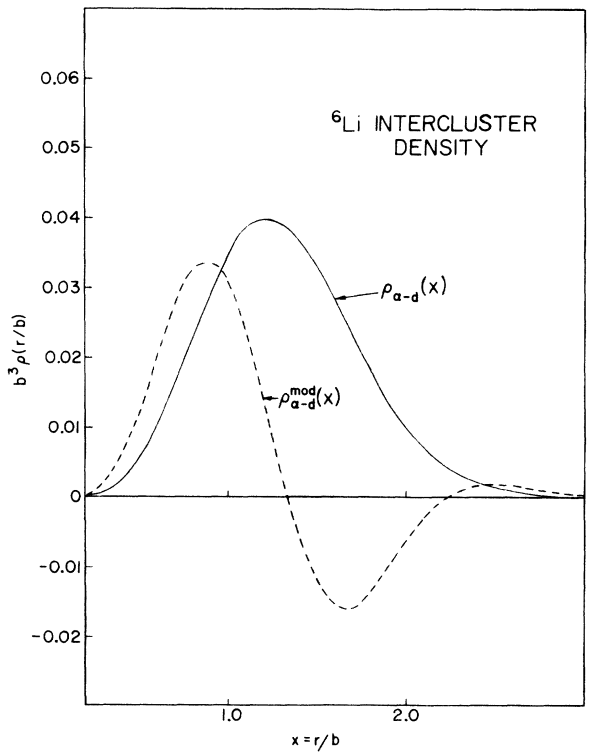

FIG. 4. The solid curve shows the intercluster density $\rho_{\alpha-d}$, Eq. (124), normalized to unity, as a function of $x=r/b$. The dashed curve shows the modulating density, Eq. (41), exclusive of the factor $A_{00}^{(2)}=0.01$, required to reproduce the lowest-order effect of deformations on the intercluster form factor.

It is again of interest to find the modulating density $\rho_{^6\mathrm{Li}}^{(\mathrm{mod,tr})}$ responsible for the form factor Eq. (51), according to

$$4\pi\int_0^\infty dz_i\, z_i^2\, \rho_{^6\mathrm{Li}}^{(\mathrm{mod,tr})}(z_i) j_2(z_i\,\omega_i) = F_{^6\mathrm{Li}}^{(2)}(\omega_i)\,, \quad (53)$$

and compare it with the transition density $\rho_{^6\mathrm{Li}}^{\mathrm{tr}}$ in the absence of deformations. A power series expansion of $F_{^6\mathrm{Li}}(\omega_i)$ reveals that the second, fourth and sixth moments of $\rho_{^6\mathrm{Li}}^{(\mathrm{mod,tr})}$ all vanish. Here it is convenient to express the *ansatz* in terms of associated Laguerre polynomials:

$$\rho_{^6\mathrm{Li}}^{(\mathrm{mod,tr})}(z_i) = \rho_1^{\mathrm{tr}}(z_i) + \rho_2^{\mathrm{tr}}(z_i) \quad (54)$$

with

$$\rho_1(z_i) = z_i^2\, e^{-z_i^2/4A^2} \sum_{n=0}^{\infty} a_n L_n^{(5/2)}(z_i^2)\,, \quad (55a)$$

$$\rho_2(z_i) = z_i^2\, e^{-z_i^2/4D^2} \sum_{n=0}^{\infty} d_n L_n^{(5/2)}(z_i^2) \quad (55b)$$

with A, D, and z_i as previously defined. The unmodified transition density can be found in a similar manner. The results are displayed in Fig. 6. The modulation, which is numerically quite small, acts to shift some of the transition charge density to larger distances.

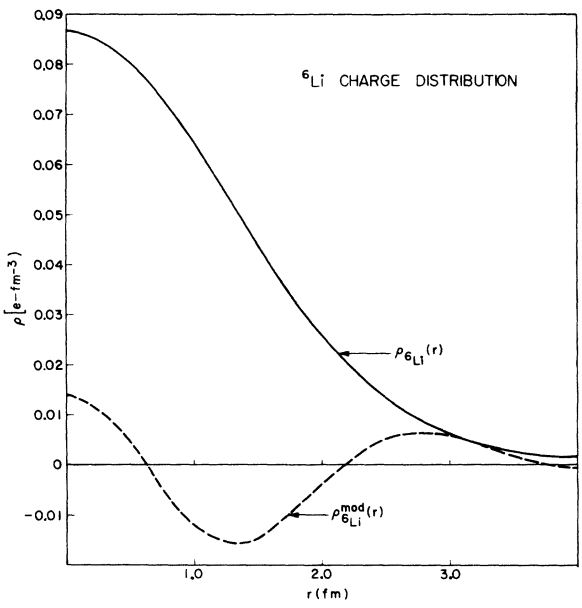

FIG. 5. The solid curve shows the complete ^{6}Li density in protons fm^{-3}, as calculated on the cluster model in the absence of deformations. The dashed curve shows the modulating density, exclusive of the factor $A_{00}^{(2)} = 0.01$, required to reproduce the lowest-order effect of deformations on the complete ^{6}Li elastic form factor.

IV. SUMMARY

The α-d cluster model of ^{6}Li, as developed by Neudatchin, Smirnov, and their collaborators, has proven remarkably effective in giving a unified account of a wide variety of form-factor data, including those for the reactions $(p, 2p)$, (p, pd), π capture as well as the older Coulomb and magnetic electron scattering data. For this reason it is of interest to apply the cluster theory to the most recent very-high-momentum-transfer elastic and inelastic electron scattering data from Stanford. In making this application, we have made use of two results of the detailed calculations of Refs. 11 and 15 to simplify the theory: (1) that, except at very low momentum transfer, exchange effects between cluster nucleons are not significant; and (2) that the ^{6}Li form factors do not depend sensitively on the asymptotic form of the internal wave functions of the clusters. We have further taken into account phenomenologically the known sensitivity of the ^{6}Li rms radius to exchange effects by using the experimental rms radius as a constraint. The resultant form factors, in both elastic and inelastic channels give excellent agreement with the data for momentum transfers $q^2 < 6.5$ fm^{-2}, and in particular give a diffraction minimum in the elastic channel at 7.7 fm^{-2}, as observed. A single free parameter, the intercluster separation parameter, was required in each channel, whose value as fixed by the data

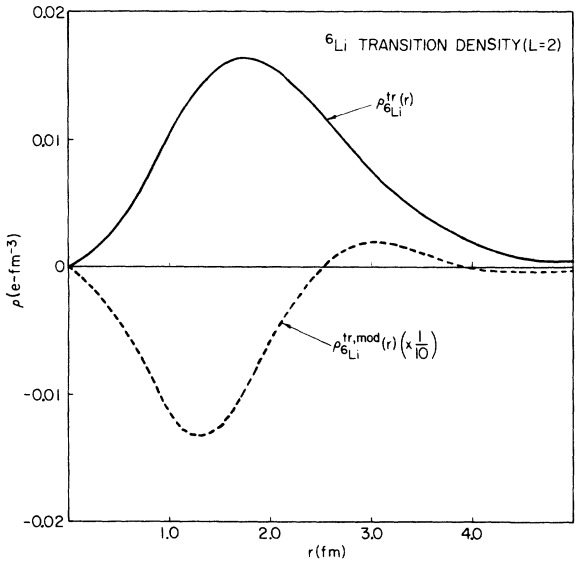

FIG. 6. The solid curve shows the transition ^{6}Li density in protons fm^{-3} for the 2.189-MeV level, in the absence of deformation: The dashed curve shows modulating density, exclusive of the factor $B = 0.0025$, required to reproduce the lowest-order effect of deformations on the ^{6}Li inelastic form factor.

was found to be reasonably close to the values attained in Refs. 11 and 15. Our fit in the elastic channel appears roughly comparable to that reported by Kudeyarov *et al.*[11]; in the inelastic channel, it appears to fit the data over a considerably larger range of momentum transfer.

With these results established, we examined the data at very high momentum transfer, which have not yet been treated theoretically. In common with a number of other nuclei, the high momentum transfer data requires an oscillatory modification of the empirical charge density needed to fit the data at lower q. In previous papers, we have shown that this modification may be attributed to dynamical deformations of the charge density. Applying this idea to the cluster formalism, we allowed the intercluster density to undergo dynamical volume deformations. The deformations are characterized by certain operators in the Hilbert space of intercluster rotational states whose reduced matrix elements enter into the form factor as parameters to be fitted to the data. The individual clusters are assumed to remain in their ground state.[29] After carrying through a renormalization procedure to incorporate as much as possible of the deformation into the parameters of the spherically symmetric base distribution, we obtained, particularly in the elastic channel, a striking improvement in the fits, which are now good over the range $0<q^2<10.5$ fm^{-2}. A single additional parameter was required to obtain agreement over this range of momentum transfer, making two parameters in all, for each channel.

By an inversion procedure, we then found the equivalent spherically symmetric "modulating densities," whose Fourier transforms produce the additional terms in the form factors due to the deformations. These distributions were found to oscillate rapidly in sign, in a manner similar to the empirical modifications mentioned above. From this point of view, these oscillations are not real, but are the consequence of viewing the deformation within the framework of spherical symmetry.

On the whole, we see these results as providing a strong confirmation of the α-d cluster model of ^{6}Li, as well as evidence in favor of long-range correlations as an explanation of the very high momentum transfer data. However, exchange effects at the higher values of momentum transfer, if significant, could alter our estimate of the magnitude of the deformation contribution to the form factors, and even its sign, so that to this extent our results remain tentative. Furthermore, as mentioned in the introduction, there is no dearth of other explanations available. In particular, the use of short-range correlations in conjunction with the cluster model should be studied. It is obviously essential that the correct theory provide a unified description of the totality of available data, including widths and excitation energies. This more general problem is currently under investigation, in collaboration with M. Rosen. The relative simplicity of ^{6}Li, the abundance of detailed measurements of a wide variety of form factors, and the already very effective cluster treatment of this nucleus, suggests that ^{6}Li be used as a test nucleus for the theoretical investigation of high momentum transfer phenomena.

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intermediate-coupling shell model (albeit at too low momentum transfer to fit the present data), could eliminate this supposedly undesirable feature only at the cost of making the ^{6}Li quadrupole moment a factor of 10 larger than the experimental value. The introduction of short-range correlations mitigated this situation somewhat, but not sufficiently.

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