PHYSICAL REVIEW C VOLUME 8, NUMBER 6 DECEMBER 1973

Comments

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Exclusion Principle Effects in Pion-Deuteron Scattering near the 3,3 Resonance*

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(Received 27 August 1973)

The role of the Pauli exclusion principle in inelastic πd scattering in the 3,3 resonance region is studied using dispersion relations. Estimates are based on the deuteron S state. The 3,3 resonance energy is raised by antisymmetrization in one partial wave ($J=1$) by 7.5 MeV but only by 1.3 MeV in the total cross section.

[NUCLEAR SCATTERING $d(\pi,\pi)$, $E\sim$150–250 MeV; calculated reaction cross section. Role of Pauli principle, dispersion theory.]

Recently Bethe[1] and others[2] observed that for pion-nucleus scattering in the neighborhood of a πN resonance such as the dominant $\Delta_{33}(1236)$, the fundamental pion-nucleon scattering event may be modified by the exclusion principle as a consequence of blocking of intermediate nucleon states. The Pauli principle can also restrict the states available for nucleon recoil between successive pion scattering events in a dispersion theoretical treatment or in a multiple scattering approach. Antisymmetrization is ignored in earlier work by De Alfaro and Stroffolini,[3] who estimate higher-order effects which arise in the Watson multiple scattering formalism, and by Ioffe, Pomeranchuk, and Rudik[3] who use dispersion relations. There are crude estimates[2] of the main qualitative effects of the exclusion principle on the 3, 3 resonance which are based on modifications of Chew-Low theory in nuclear matter. They are found to be similar to those involved in the quenching of magentic moments[4] in nuclei and consist in narrowing the width of the 3, 3 resonance and shifting the resonance energy upwards.

We expect these effects, albeit reduced in size, to occur already in the deuteron. On the other hand, the relatively simple structure of the deuteron allows us to examine (by means of dispersion relations) possible modifications of the elementary pion-nucleon scattering event while avoiding many-body complications which usually necessitate invoking assumptions of dubious validity.

In the following we calculate the inelastic pion-deuteron (πd) cross section σ_R from the imaginary part of the forward πd scattering amplitude. Since the Born diagrams which involve πN scattering are real, they can be ignored here. For the absorptive part $\mathrm{Im}F_{\pi d}$, we use unitarity taking account of the major inelastic channels only, viz., intermediate πNN states. Our emphasis is less on the construction of a model which can be compared with experiments than on estimating effects related to proper antisymmetrization. Thus we ignore crossing symmetry amongst other possible refinements, and work in the static limit (using $m_d, m \gg \mu =$ pion mass) keeping only the S state of the deuteron.

We choose the usual kinematic variables (see Fig. 1),

$$s=(q+d)^2 \approx m_d{}^2+2m_d\omega \tag{1}$$

for the total c.m. energy, and

$$s_1=(q'+p_1)^2 \approx m^2+2m\omega_1 \tag{2}$$

for the πN_1 subchannel. Equations (1) and (2) define the nonrelativistic energies ω and ω_1.

In each channel of total angular momentum J and πN_1 subchannel of angular momentum j, unitarity restricted to πNN intermediate states yields

$$\mathrm{Im}F_{\pi d}(\omega)=\int_\mu^{\omega-B} d\omega_1 \rho(\omega_1,\omega)\,|F_{23}(\omega_1,\omega)|^2, \tag{3}$$

where $\rho(\omega_1,\omega)\sim(\omega-\omega_1-B)^{1/2}\rho(\omega_1)$ is the πNN phase space, $\rho(\omega_1)$ the πN_1 phase space, and F_{23} the (J,j) partial-wave amplitude for the reaction

$\pi d \to \pi NN$. For F_{23}, we use the s_1 subchannel dispersion relation

$$F_{23}(\omega_1, \omega) = B_{23}(\omega_1, \omega) + \frac{1}{\pi} \int_{\mu}^{\infty} \frac{d\omega_1'}{\omega_1' - \omega_1 - i\epsilon} F_{23}(\omega_1', \omega)\rho(\omega_1')F(\omega_1'), \tag{4}$$

where F denotes the πN Chew-Low amplitude. The full antisymmetrized Born amplitude B_{23} consists of the direct graph (a) and two exchange diagrams (b) and (c) of Fig. 1, which result from (a) upon interchanging the intermediate nucleon lines N_1 and N_2, and then, in (b), the nucleon lines N_1 and N_2, respectively.

In the nonrelativistic limit, the pole structure of these amplitudes is readily seen to be given by the nucleon propagators involved; e.g.,

$$-2m/[(p_1 - q)^2 - m^2] \approx 1/\tilde{q}_0 \approx 1/\omega_1, \tag{5a}$$

where $\tilde{q}_0$ is the energy of the incoming pion in the c.m. frame of nucleon N_1 of momentum p_1 and the final pion; $\tilde{q}_0$ is evaluated assuming $d - p_2$ to be on the mass shell. Similarly

$$-2m/[(d - p_2)^2 - m^2] \approx 1/(\omega - \omega_1) \tag{5b}$$

results in the c.m. frame of q and d taking account of the binding energy B of the deuteron and the recoil of nucleon N_2. Equation (5b) represents the

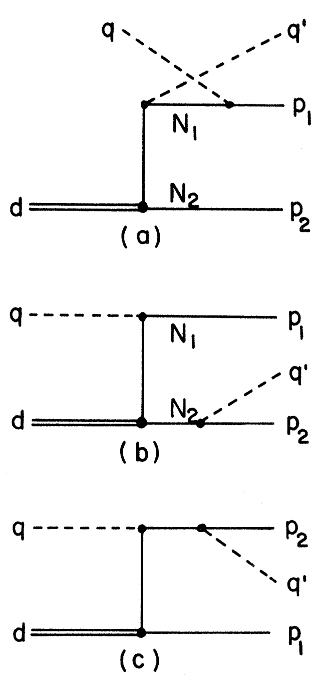

FIG. 1. The direct Born graph (a) and two exchange diagrams (b) and (c) for the process $\pi d \to \pi NN$.

S wave Hulthén wave function of the deuteron. Next, we obtain

$$-2m/[(p_1 - q)^2 - m^2] \approx 1/\omega, \tag{5c}$$

assuming the nucleon N_2 in Fig. 1(b) of momentum $p_2 + q'$ to be on the mass shell, while

$$-2m/[(p_2 + q')^2 - m^2] \approx -1/\omega_1 \tag{5d}$$

and

$$-2m/[(d - p_1)^2 - m^2] \approx 1/\omega \tag{5e}$$

are needed for Figs. 1(b) and (c).

The isospin matrix elements are obtained from $\tau_{1,i}\, \tau_{1,f}\, (1 - \vec{\tau}_1 \cdot \vec{\tau}_2)/4$ for the direct graph and analogously for the others; thus the relative phase of the exchange graphs (b) and (c) is negative compared to (a) for both πN_1 subchannel isospins $t = \frac{1}{2}$ and $\frac{3}{2}$. The angular momentum dependence of (a) is

$$S_a = \vec{\sigma}_1 \cdot \hat{q}\, \vec{\sigma}_1 \cdot \hat{q}'(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/4, \tag{6}$$

with obvious modifications for diagrams (b) and (c). This implies the partial-wave projections

$$\begin{aligned} S_a &= 2P^2_{3/2} + (2/\sqrt{3})\, P^1_{3/2} - \sqrt{\tfrac{2}{3}}\, P^1_{1/2} - P^0_{1/2}, \\ S_b &= (4/\sqrt{3})\, P^1_{3/2} + \sqrt{\tfrac{2}{3}}\, P^1_{1/2} - 3P^0_{1/2}, \\ S_c &= (4/\sqrt{3})\, P^1_{3/2} + \sqrt{\tfrac{2}{3}}\, P^1_{1/2} + 3P^0_{1/2}, \end{aligned} \tag{7}$$

where

$$P^J_j = \sum_M |(\hat{q}'\, \tfrac{1}{2})j, \tfrac{1}{2}; JM\rangle \langle \hat{q}, \tfrac{1}{2}\, \tfrac{1}{2})1; JM| \tag{8}$$

is the channel projection operator. A common and constant normalization factor is ignored throughout. Upon including the energy and isospin dependence, the partial amplitude for $J = 1$ and $j = \frac{3}{2}$ becomes

$$B^J_j = \left(1 + 4\, \frac{\omega - \omega_1}{\omega}\right) \Big/ (\omega - \omega_1)\omega_1, \tag{9}$$

while the second term in Eq. (9) from the exchange graphs vanishes for $J = 2$, with the direct contribution being $\sqrt{3}/(\omega - \omega_1)\omega_1$. As a result, the exchange contributions yield an enhancement in the Born amplitude in the physical region where $\mu \leq \omega_1 \leq \omega - B$ in the $J = 1, j = \frac{3}{2}$ channel, while for $j = \frac{1}{2}$ there is sometimes suppression. The relative magnitude of the Born amplitudes in the $J = 1$ and $J = 2$ states of $1 : \sqrt{3}$ in conjunction with the statistical weight factor $2J + 1$ will reduce the Pauli effects for inelastic πd scattering to $\frac{1}{6}$ of the size in the $J = 1$ channel.

Next we solve the dispersion relation (4) using a πN Chew-Low amplitude of the form

$$F(\omega_1)=N(\omega_1)/D(\omega_1)\,,$$

$$N(\omega_1)=\lambda/\omega_1\,,$$

$$D(\omega_1)=1-\frac{\lambda\omega_1}{\pi}\int_\mu^\infty \frac{d\omega_1'}{\omega_1'^2}\,\frac{\rho(\omega_1')}{\omega_1'-\omega_1-i\epsilon}\,, \tag{10}$$

$$\mathrm{Im}F(\omega_1)=\rho(\omega_1)\,|F(\omega_1)|^2$$

$$\sim\frac{\rho(\omega_1)/\rho(\omega_r)}{(2(\omega_1-\omega_r)/\Gamma)^2+(\rho(\omega_1)/\rho(\omega_r))^2}\,,$$

where $\rho(\omega_1)=(\omega_1^2-\mu^2)^{3/2}$, the resonance energy is taken to be $\omega_r=2.4\mu$ and its width $\Gamma=0.8\mu$.

Since the singularity $(\omega_1-\omega)^{-1}$ of B_{23} lies in the domain of integration in Eq. (4), we factor out this pole. Then

$$\bar{F}_{23}(\omega_1,\omega)=(\omega_1-\omega)F_{23}(\omega_1,\omega)\,, \tag{11}$$

along with the corresponding $\bar{B}_{23}$, satisfies a once-subtracted form of the dispersion relation (6). By writing

$$\bar{F}_{23}(\omega_1,\omega)=\bar{N}_{23}(\omega_1,\omega)/D(\omega_1)\,, \tag{12}$$

with the Chew-Low denominator D of Eq. (10), and requiring no unitarity cut in $\bar{N}_{23}$, one gets for the simple solution[5] of the once-subtracted dispersion relation for $\bar{N}_{23}$

$$\bar{N}_{23}(\omega_1,\omega)=\bar{N}_{23}(\omega_0,\omega)+5\lambda_{23}\left(\frac{1}{\omega_1}-\frac{1}{\omega_0}\right)=\bar{B}_{23}(\omega_1,\omega)\,, \tag{13}$$

the subtraction being made at $\omega_1=\omega_0$, and λ_{23} being the foregoing over-all normalization constant. It is important that the subtraction constant disappears from the final solution.

The solution of Eq. (11) implies the inelastic πd cross section

$$\sigma_R\sim(\omega^2-\mu^2)^{1/2}\int_\mu^{\omega-B}d\omega_1\frac{(\omega-\omega_1-B)^{1/2}}{(\omega-\omega_1)^2}\,\frac{\sigma_{33}(\omega_1)}{(\omega_1^2-\mu^2)^{1/2}}\times\left(1+4\frac{\omega-\omega_1}{\omega}\right)^2, \tag{14}$$

for $J=1$, $j=\frac{3}{2}$, where σ_{33} is the total pion-nucleon scattering cross section. Upon omitting the exchange diagrams, Eq. (14) reduces to the impulse approximation. In this context, we note that the S-state deuteron wave function is

$$\psi(\vec{p}_1)\sim1/(\vec{p}_1^{\,2}+Bm)\sim1/(\omega-\omega_1) \tag{15}$$

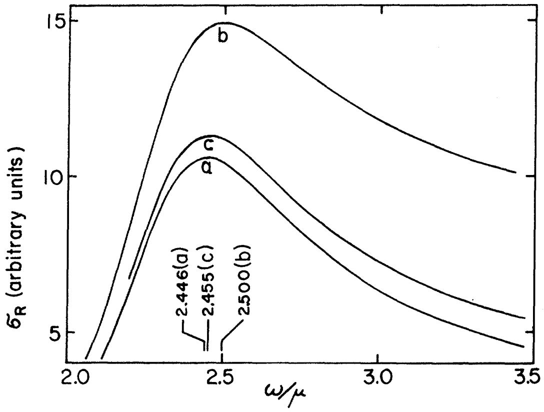

FIG. 2. Inelastic pion-deuteron scattering cross sections versus total pion energy in units of the pion mass. Curve (a) represents the total cross section $\sigma_R=\sigma_{J=1}+\sigma_{J=2}$, as well as $6\sigma_{J=1}$ and $\frac{6}{5}\sigma_{J=2}$, when the exclusion principle is ignored. Curves (b) and (c) denote, respectively, $6\sigma_{J=1}$ and σ_R when all three diagrams of Fig. 1 are included, while the $J=2$ partial cross section is unchanged by the exclusion principle. The peak energies are shown on the energy scale.

and $p_1\approx[m(\omega-\omega_1-B)]^{1/2}$.

The results of Fig. 2 show a slight upward shift of ~7.5 MeV for the maximum in the $J=1$, $j=\frac{3}{2}$ channel and ~1.3 MeV for the total inelastic pion-deuteron cross section. This upward energy shift is indirect inasmuch as it arises from the ω_1 integration in Eq. (14), while the elementary πN resonance position, i.e., the zero of ReD in Eq. (10), remains unchanged.

Thus we conclude that exchange effects in πd inelastic scattering yield corrections similar to those expected from the quenching of magnetic moments. While these results can be qualitatively reproduced in pion-nucleus scattering by Pauli blocking through a weight factor in the driving Born diagram and the dispersion term of Chew-Low theory adapted to the case of nuclear matter,[2] our detailed partial wave decomposition for πd scattering shows that actually suppression and enhancement of the Born amplitudes occur in different partial waves.

We acknowledge an interesting discussion with Dr. R. R. Silbar.

*Supported in part by the National Science Foundation.

[1] H. A. Bethe, Phys. Rev. Lett. 30, 105 (1973).
[2] C. B. Dover and R. H. Lemmer, Phys. Rev. C 7, 2312 (1973). J. M. Eisenberg and H. J. Weber, Phys. Lett. 45B, 110 (1973); in Proceedings of the International Summer School on Pion-Nucleus Multiple Scattering, Los Alamos, 1973 (unpublished); J. B. Cammarata and M. K. Banerjee, *ibid.*; M. Johnson, *ibid.*;

R. H. Landau and M. McMillan, to be published.
[3]V. De Alfaro and R. Stroffolini, Nuovo Cim. 11, 447 (1959); B. L. Ioffe, I. Ia. Pomeranchuk, and A. P. Rudik, Zh. Eksp. Teor. Fiz. 31, 712 (1956) [transl.: Sov. Phys.–JETP 4, 588 (1957)].
[4]S. D. Drell and J. D. Walecka, Phys. Rev. 120, 1069 (1960).
[5]The additional unitarity integrals which occur in the dispersion relation of $\overline{N}_{23}$ cancel each other because of the simple form of $\overline{B}_{23}=(\lambda_{23}/\omega_1)\,[1+4(\omega-\omega_1)/\omega]$. We also require that $\overline{N}_{23}(\omega_1,\omega)\rightarrow\lambda_{23}/\omega_1$ as $\omega_1\rightarrow\omega$.