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K^--Meson Absorption in Nuclei with Below-Threshold Effects

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Energy level widths and level shifts of K^--mesonic atomic levels corresponding to terminal atomic transitions preceding nuclear capture are calculated for ^{10}B, ^{11}B, ^{12}C, ^{31}P, ^{32}S, and ^{35}Cl. In this calculation we have considered the effects on the $\bar{K}N$ system arising from the presence of the $Y_0^*(1405)$ quasibound state below threshold as well as the momentum distribution of the nucleons in the nucleus. The mass difference between the $\bar{K}^0n$ and K^-p channels is also taken into account. The Klein-Gordon equation for the K^- meson is solved by an iterative procedure in the presence of the nuclear optical potential and the Coulomb potential. Results are seen to compare quite favorably with experimental values given by Backenstoss *et al.*

I. INTRODUCTION

Experiments by Wiegand and Mack[1] on x-ray yields in K^--mesonic atoms have given rise to considerable interest in the nuclear capture of K^- mesons. Since the capture takes place predominantly in the low-density surface region of the nucleus, calculations of capture rates based on first-order perturbation theory were made by various workers[2-4] principally with a view to studying the structure of the surface. However, Krell[5] has shown that the nuclear absorption is so strong that a perturbation theory is inapplicable even in the surface region. Another difficulty in the theoretical analysis of Wiegand's work[1] is that nuclear capture rates have to be inferred from x-ray intensities in a way which is closely tied to details of the atomic cascade process, which is rather complicated.[6] Thus comparison of theoretical results with these experiments may not be the best way of looking at the problem.

Recently, measurements have been made by Backenstoss *et al.*[7,8] on the energy shifts and linewidths of kaonic x rays from ^{10}B, ^{11}B, ^{12}C, ^{31}P, ^{32}S, and ^{35}Cl. These measurements are quite independent of the cascade process and nuclear capture rates may be obtained directly from the Lorentzian linewidths corresponding to the terminal atomic transition immediately preceding such capture. For example, in ^{32}S,

$$P_{\text{cap}} = \frac{\Gamma_{3d}}{\hbar}\,, \qquad (1)$$

where Γ_{3d} is the Lorentzian linewidth for the transition $4f \to 3d$, which is the last atomic transition observed in this case. At the same time, the level shift gives a measure of the distortion effects caused by the strong interaction potential of the nucleus on the lower kaonic orbits. Both these aspects of the problem can be treated theoretically in a unified manner by solving the Klein-Gordon equation for the K^- meson in the presence of the nuclear potential. Such an approach, which is independent of perturbation theory, has been used by Ericson and Scheck,[9] by Krell,[5] by Rook and Wycech,[10] and by Bardeen and Torigoe.[11] The nuclear potential experienced by the K^- meson is constructed in the Born approximation from the K^--nucleon s-wave scattering amplitude, the latter being expressed in terms of the $\bar{K}N$ s-wave scattering lengths in the appropriate isospin channel. Using this approach, Krell[5] has made a detailed analysis of the strong interaction effects on the dynamics of the K^- meson. He finds that agreement with the sulphur data can be obtained by using a K^--meson–nucleon scattering length of which the absorptive part is the same as in the case of the free nucleon, but the real part is attractive rather than repulsive. His analysis points up the strong energy dependence of the scattering lengths for the bound nucleon. Actually, one must remember that one is working below the $\bar{K}N$ threshold, so that it is necessary to give greater weight to experimental data in this region. By making a phenomenological analysis of the relative yields of the decay products, namely the Σ hyperons and pions, Burhop[6] has shown that better agreement with experiment may be secured by using a variant of Kim's scattering-length solution with a reduced imaginary part for the $T=1$ channel. Such a choice appears to correspond more closely to the data in the energy region below threshold.

One could thus see the necessity of constructing a K^--nucleus potential which is appropriate to the region below the $\bar{K}N$ threshold. This is rendered all the more imperative by the presence of the $Y_0^*(1405)$ resonance in the K^--proton channel at an energy of about -27 MeV with respect to the threshold. The Y_0^* has a width of about 50 MeV, so that it can strongly influence the absorption process at the energies actually available to the K^-–bound-proton system. One would expect a relative enhancement of the absorption rates on protons as compared to neutrons, because the K^--n system belongs to the $T=1$ channel which has no such quasibound state in this region. This would qualitatively explain the result obtained by Burhop.

Another element in the energy dependence of the nuclear interaction of K^- mesons arises from the momentum distribution of the nucleons within the nucleus. Bardeen and Torigoe[11] treated this problem by devising an energy-averaged scattering amplitude $\bar{f}_{KN}$ which is obtained by weighting the $\bar{K}N$ amplitude with the probability $P(W)$ of finding the K^--nucleon system with total center-of-mass energy W. The important point to note, however, is that one is actually dealing with a K^-–bound-nucleon system below threshold, so that the free $\bar{K}N$ amplitude is not quite appropriate to the situation. One must devise here some way to handle the bound system.

The object of the present calculation is to treat the basic features of the problem delineated above within the framework of a simple nonperturbative calculation. Our approach broadly agrees with other recent work[5,9-11] in that we solve the Klein-Gordon equation for the K^- meson in the presence of an optical potential of the Woods-Saxon type. However, our central motivation is to consider the energy dependence of the $\bar{K}N$ interaction by including the below-threshold effects arising from the proximity of the $Y_0^*(1405)$ and the effect of the momentum distribution of nucleons within the nucleus. In fact these two effects are closely connected, because the effect of the Y_0^* hump on the $\bar{K}N$ system would depend strongly on the momentum. The existence of a momentum (or energy) distribution among the nucleons implies a kind of averaging procedure for the scattering amplitude, but this average amplitude should also relate to the energy dependence of the $\bar{K}N$ interaction below threshold. In other words, one must think in terms of the overlap of the below-threshold resonant amplitude and the momentum distribution of the nucleons in this region. A simple phenomenological choice of energy corresponding to either the peak of the Y_0^* resonance or any other mean energy would miss the point, because even though a sufficient number of protons might possibly possess this energy so as to make it a favored parameter for the interaction, the detailed features of the overlap would inevitably be lost in such a treatment. Also, we have to go far enough below the $\bar{K}N$ threshold so as to adequately cover the interplay between these effects.

II. CALCULATION

The optical potential seen by a K^- meson in the presence of a nuclear density distribution $\rho(r)$ (number of nucleons per unit volume) is given by

$$V(r)+iW(r)=-\frac{2\pi}{\mu}\hbar^2\left(1+\frac{m_K}{m_N}\right)\rho(r)\bar{A}\,, \tag{2}$$

where $\bar{A}$ is the momentum-averaged scattering length for the basic K^-–bound-nucleon interaction,

$$\bar{A}=\int\rho(k)A(k)d\boldsymbol{k}\,. \tag{3}$$

Here $\rho(k)$ is the density distribution function of the nucleons in momentum space (integrated over the solid angle Ω_k in this space) and $A(k)$ is the $\bar{K}N$ scattering length corresponding to momentum k in the region below threshold. We have chosen

$$\rho(k)=\frac{4}{\pi^{1/2}\kappa^3}k^2e^{-k^2/\kappa^2}\,, \tag{4}$$

the factor $4/\pi^{1/2}\kappa^3$ being required by normalization, with $\kappa=168$ MeV/c. $A(k)$ is given by

$$A(k)=\frac{1}{N+Z}\left[Nf_1^n+\tfrac{1}{2}Z\left(f_1^p+f_0\right)\right], \tag{5}$$

where f_0 is the below-threshold $\bar{K}N$ scattering amplitude for $T=0$ and f_1^p, f_1^n the corresponding amplitudes for the proton and the neutron, respectively, in the $T=1$ channel. For free $\bar{K}N$ scattering, these amplitudes are given by

$$f_0=\frac{A_0(1-ik_0A_1)}{D}\,, \tag{6a}$$

$$f_1^p=\frac{A_1(1-ik_0A_0)}{D}\,, \tag{6b}$$

$$f_1^n=\frac{A_1}{1-ikA_1}\,, \tag{6c}$$

with

$$A_T=a_T+ib_T\quad(T=0,1)\,, \tag{7a}$$

$$D=1-\tfrac{1}{2}i(A_1+A_0)(k+k_0)-kk_0A_0A_1\,, \tag{7b}$$

k_0 and k being the channel momenta for the $\bar{K}^0n$ and K^-p channels. For the region below threshold, we make the analytic continuation $k\to i|k|$ in Eqs. (6) and (7). With this mapping we have checked that the correct parameters for the $Y_0^*(1405)$ resonant amplitude are reproduced in the $T=0$ channel. Substituting the resulting amplitudes in Eq. (5), one gets a reasonable momentum-dependent scattering length for the bound $\bar{K}N$ system.

From Eqs. (6) and (7) it is clear that we have taken into account the mass difference between the $\bar{K}^0n$ and K^-p channels. The corresponding channel momenta k_0 and k are related in the region below threshold by

$$k_0^2=k^2+2\mu\Delta\,, \tag{8}$$

where Δ is the mass difference.

Equation (3), which expresses the momentum-averaged scattering length $\bar{A}$ of the K^-–bound-nucleon system as the overlap integral of the nucleon momentum distribution $\rho(k)$ and the below-threshold $\bar{K}N$ scattering length $A(k)$, embodies the basic elements of our treatment. Because $A(k)$ incorporates the $Y_0^*(1405)$ resonance, our momentum-averaging automatically takes into account the effect of this resonance over the entire range of integration.

We now substitute this $\bar{A}$ into Eq. (2) to obtain the K^--nucleus optical potential. For the configuration-space density distribution $\rho(r)$ we use the Woods-Saxon form

$$\rho(r)=\frac{\rho(0)}{1+e^{r-R/a}}\,, \tag{9}$$

where $\rho(0)$, R, and a are the usual parameters. $\rho(0)$ is normalized to give the correct mass number and R and a are obtained from electron-scattering experiments.[12]

With this optical potential, we solve the Klein-Gordon equation

$$\nabla^2\Psi(\vec{r})=\left(\frac{[E+i\Gamma-V'(r)]^2-\mu_k{}^2c^4}{-\hbar^2c^2}\right)\Psi(\vec{r})\,. \tag{10}$$

Here

$$V'(r)=-Ze^2/r+V(r)+iW(r) \tag{11}$$

and

$$\Psi(\vec{r})=\Psi(r,\theta,\varphi)=R(r)Y(\theta,\varphi)\,. \tag{12}$$

E and Γ are the real and imaginary parts of the energy which are suitably chosen to start the calculation. μ_K is the reduced mass of the K^--nuclear system. Remembering that the radial function $R(r)$ is complex within the range of the optical potential, we write $R(r)=u(r)+iv(r)$. For large values of r, $v(r)\to 0$. We get from Eq. (10) the coupled radial equations

$$u''(r)=-(2/r)u'(r)+A(r)u(r)+B(r)v(r)\,, \tag{13a}$$

$$v''(r)=-(2/r)v'(r)+A(r)v(r)-B(r)u(r)\,, \tag{13b}$$

where

$$A(r)=\frac{1}{\hbar^2c^2}\left[\mu_k{}^2c^4-E^2+\Gamma^2-\frac{Z^2e^4}{r^2}-V^2+W^2+\frac{2Ze^2V(r)}{r}+2EV(r)-\frac{2ZEe^2}{r}-2\Gamma W(r)\right] \tag{14a}$$

and

$$B(r)=\frac{1}{\hbar^2c^2}\left[\frac{2Ze^2W(r)}{r}-2V(r)W(r)-2E\Gamma+2EW(r)+2\Gamma V(r)-\frac{2\Gamma Ze^2}{r}\right]. \tag{14b}$$

Equations (13) are integrated with appropriate boundary conditions. Actually, we start out by taking $v=0$ and u equal to the Coulomb function at a point sufficiently outside the nucleus and integrate inwards by using Adam's method. Close to the origin our wave function goes to zero faster than r^{l+1} because of the attractive nuclear potential.

Having obtained u and v for different values of r we now proceed to find a second set of values for E and Γ by a process of integration. From Eqs. (13) we get after a little manipulation

$$\int_0^\infty u(r)\Theta(r)u(r)r^2\,dr+\int_0^\infty v(r)\Theta(r)v(r)r^2\,dr\equiv X=\mu_K{}^2c^4-E^2+\Gamma^2 \tag{15a}$$

and

$$\int_0^\infty v(r)\Theta(r)u(r)r^2\,dr-\int_0^\infty u(r)\Theta(r)v(r)r^2\,dr+\int_0^\infty(u^2+v^2)[B(r)-2E\Gamma]r^2\,dr\equiv Y=2E\Gamma\,, \tag{15b}$$

where

$$\Theta(r)=\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-A(r)+\mu_K{}^2c^4-E^2+\Gamma^2\,. \tag{16}$$

In this operator, we use for E and Γ the values used in Eq. (10). We have also used the normalization

$$\int_0^\infty(u^2+v^2)r^2dr=1\,. \tag{17}$$

The new values of E and Γ are obtained by solving Eqs. (15).

The entire calculation depends on an iterative procedure with E and Γ as iteration parameters. The output of each cycle is fed as input into the next cycle till convergence is obtained. To start the calculation, we choose $E=\mu_Kc^2/[1+(\alpha Z/N)^2]^{1/2}$, the Coulomb energy for principal quantum number N, and a small value for Γ. If these values diverge from one cycle of the iteration to the next,

TABLE I. Values of level width Γ and level shift ΔE.

Nucleus	Meson state n	l	Nuclear size parameters (present calculation) R (fm)	a (fm)	Theoretical results (Other workers): Ericson and Scheck (Ref. 9) ΔE (keV)	Γ (keV)	Rook and Wycech (Ref. 10) ΔE (keV)	Γ (keV)	Bardeen and Torigoe (Ref. 11) ΔE (keV)	Γ (keV)	Experimental results (Ref. 8) ΔE (keV)	Γ (keV)	Present calculation ΔE (keV)	Γ (keV)
^{10}B	2	1	2.00	0.455	···	···	···	···	−0.30	0.65	−0.208 ±0.035	0.81 ±0.10	−0.210	0.799
^{11}B	2	1	2.00	0.455	···	···	···	···	−0.30	0.64	−0.167 ±0.035	0.70 ±0.08	−0.168	0.6999
^{12}C	2	1	2.30	0.42	···	···	···	···	−0.80	1.44	−0.59 ±0.08	1.73 ±0.15	−0.540	1.7115
^{31}P	3	2	3.34	0.557	···	···	−0.36	2.01	−0.52	1.65	−0.33 ±0.08	1.44 ±0.12	−0.314	1.399
^{32}S	3	2	3.33	0.591	···	2.87	−0.61	3.06	−0.88	2.73	−0.55 ±0.06	2.33 ±0.06	−0.487	2.1997
^{35}Cl	3	2	3.42	0.591	···	5.09	−1.00	4.68	−1.44	4.06	−0.77 ±0.40	3.80 ±1.0	−0.801	3.80

we vary Γ so as to get better convergence. Eventually, we find that convergence occurs for only a rather narrow range of choice of E and Γ. In other words, our iterative procedure is quite sensitive to these parameters.

III. RESULTS

Our values of level width Γ and level shift ΔE corresponding to the terminal atomic states in ^{10}B, ^{11}B, ^{12}C, ^{31}P, ^{32}S, and ^{35}Cl are shown in Table I. These values are based on Kim's scattering lengths. ΔE represents the strong interaction shift as compared with the Coulomb binding energy. In this table we have also exhibited the experimental values given by Backenstoss *et al.*[8] and the theoretical results obtained by other workers.[9-11]

It will be seen from this table that our results are quite close to the experimental values for most of the atoms considered. This is particularly interesting because as we have noted in Sec. II, our values of E and Γ represent to within a rather narrow range the only choices for which the iterative cycles would converge. It should be noted that we have not considered the modification of the K^- propagator within the body of the nucleus, i.e., we have ignored three-body effects. Comparison with the results obtained by Rook using Wycech's potential seems to indicate that these effects may not be as appreciable as the features considered in this paper.

TABLE II. Values of Re$\bar{A}$ and Im$\bar{A}$.

Nuclei	Re$\bar{A}$ (fm)	Im$\bar{A}$ (fm)
^{10}B	0.286	0.595
^{11}B	0.274	0.569
^{12}C	0.281	0.594
^{31}P	0.282	0.587
^{32}S	0.287	0.595
^{35}Cl	0.282	0.587

It is interesting to compare our result for the momentum-averaged scattering length $\bar{A}$ with the numerical value obtained by data fitting by Koch and Sternheim.[13] Evidently, assuming $N \simeq Z$, $\bar{A}$ should have the same value for all nuclei. Using the actual values of N and Z for the nuclei considered in this paper, we get the values of Re$\bar{A}$ and Im$\bar{A}$ shown in Table II. The average value of $\bar{A}$ for these nuclei is

$$\bar{A} = (0.28 + 0.59i)\ \text{fm}\,.$$

This is in qualitative agreement with the value given by Koch and Sternheim:

$$\bar{a}(\text{fitted}) = [0.44 \pm 0.04 + (0.83 \pm 0.07)i]\ \text{fm}\,.$$

Calculations on some other nuclei capable of being measured in the near future, as well as Zn which has already been measured, are now in progress and will be communicated shortly.

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