Isobaric Mass Quartets in the Mass-21 and Mass-37 Nuclei*

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The $^{40}Ca(^3He, ^6He)^{37}Ca$ reaction has been used to measure the mass of ^{37}Ca in its ground and first excited states. The mass of ^{37}Ar and ^{37}K in their lowest $T=\frac{3}{2}$ states was also measured. The $^{24}Mg(^3He, ^6He)^{21}Mg$ reaction was used as a calibration, and consequently the energies of several excited states of ^{21}Mg were measured. The mass-37 measurements give new precision values for the coefficients of the isobaric multiplet mass equation, and the excited states in ^{21}Mg are members of quartets in the mass-21 nuclei.

I. INTRODUCTION

In this paper we discuss quartets of $T=\frac{3}{2}$ levels in the $A=21$ and 37 nuclei. The primary aim of the measurements was to reduce the errors of the masses of the quartet which includes the ^{37}Cl and ^{37}Ca ground states and their analogs in ^{37}Ar and ^{37}K. In the course of measuring the ^{37}Ca mass, several previously unobserved levels in ^{21}Mg were measured, and these are also discussed.

The dependence of the masses of the members of an isobaric quartet on its charge state is described by the isobaric multiplet mass equation (IMME). This equation, which is given by

$$M(T_z)=a+bT_z+cT_z^2$$

fits all the previously measured mass quartets remarkably well. The coefficients differ with the A and J^π of the levels in the quartet and are of interest for testing nuclear structure theories, since they measure both the Coulomb energy and its rate of change between isobars. In two of the completed quartets there is an indication that a small positive coefficient, d, for a cubic term is needed. These are the $A=9$ ground state[1,2] and the $A=25$ first excited state multiplet.[3]

II. EXPERIMENTAL

A. Nucleus of ^{37}Ca

The mass determination for ^{37}Ca was performed by comparing the rigidity of ^6He particles from the $^{40}Ca(^3He, ^6He)^{37}Ca$ reaction to the rigidity of ^6He's produced in the previously measured $^{58}Ni(^3He, ^6He)^{55}Ni$[4] and the $^{24}Mg(^3He, ^6He)^{21}Mg$[1] reactions. The experiment was performed in a split-pole spectrograph with 70.7-MeV ^3He particles from the Michigan State University sector-focused cyclotron. The position on the focal plane was determined by a resistive readout wire proportional counter. Particle identification was performed by time of flight relative to the cyclotron beam structure. A plastic scintillator provided time-of-flight and total energy information which was essential to the identification of the ^6He particles. The experimental arrangement is described in more detail in a paper on the $^{13}C(^3He, ^6He)^{10}C$ reaction.[5]

The targets which were used in the present experiment were natural Ca, one self-supporting 2.7 mg/cm² and the other 1.0 mg/cm² on a 50-mg/cm² carbon backing. Relatively thick targets are required because of the cross section (~ 0.2 μb/sr) of the reaction, which is small even when compared to other (^3He, ^6He) reactions. The calibration reactions on ^{58}Ni and ^{21}Mg, for example, have 5 to 10 times greater cross sections. The thickness of the targets used was determined both before and after the measurements by observing the energy loss of ^{241}Am α particles in the targets. Spectra from ^{58}Ni and ^{24}Mg and the thicker of the two Ca targets are shown in Fig. 1. The three spectra were taken with magnetic fields calculated to place the ground-state ^6He particles at the same location (channel number) on the focal plane. The proton NMR resonant frequencies given on the figure correspond to these three fields calculated from previous measurements of the ^{55}Ni, ^{37}Ca, and ^{21}Mg masses. A peak corresponding to the previously observed[6] first excited state of ^{37}Ca is seen, as is the broadening and shifting of the ^{37}Ca ground-state peak compared to the other two ground-state peaks. The broadening is due to the thicker target, but most of the shift is an indication of the 90-keV difference between the present and previous measurements. A comparison of the present to the previous measurement of ^{37}Ca is given in Table I. The error of the ^{37}Ca mass measurement is dominated by the uncertainties in the Q values of the calibration reactions and by the uncertainty in thickness corrections. Errors in the determination of the magnetic fields needed to bend the highly rigid ^6He particles also contributed to the error.

B. $T=\frac{3}{2}$ Levels in ^{37}Ar and ^{37}K

^{37}Ar and ^{37}K were studied by means of the ^{39}K(p, t) and ^{39}K(p, ^{3}He) reactions. The target consisted of 50-μg/cm^2 natural K deposited on a 20-μg/cm^2 carbon backing. The ^{12}C and ^{16}O present in the target provided the calibration reactions. Runs were taken with 40.1- and 42.2-MeV protons. The focal plane detector was a wire counter setup like the one previously described or a silicon position-sensitive detector. The silicon detector gave better resolution but only covered a small region of excitation. The spectra shown in Figs. 2–4 were taken with the wire counter, in which a resolution of 30–40 keV was obtained. The energy

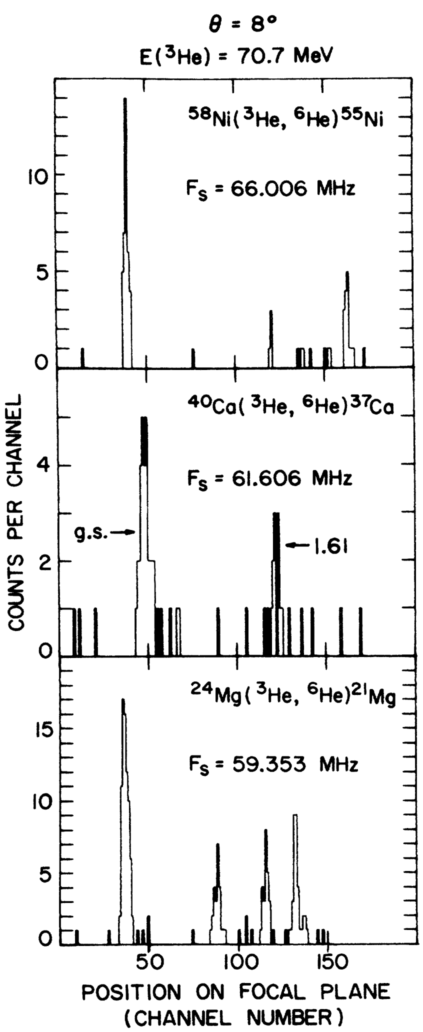

FIG. 1. Spectra from the (^{3}He, ^{6}He) reaction on ^{24}Mg, ^{40}Ca, and ^{58}Ni at 8°. F_s is the NMR frequency for the magnetic field of the spectrometer. Excitation energy increases with channel number at 22 keV per channel.

TABLE I. Present and previously reported measurements of the $T=\frac{3}{2}$ levels in $A=37$ nuclei. All energies are in MeV with errors in parenthesis in keV.

Nucleus	T_z	J	Excitation energy	Mass excess	Reference
^{37}Ca	$-\frac{3}{2}$	$\frac{3}{2}^+$	g.s.	−13.144(25)	Present
				−13.230(5)	a
			1.61(20)		Present
			1.62(30)		b
^{37}K	$-\frac{1}{2}$	$\frac{3}{2}^+$	5.045(4)	−19.753(3)	Present
			5.048(3)		c
			5.035(25)		d
		($\frac{1}{2}^+$)	6.67(20)		Present (Tentative)
^{37}Ar	$+\frac{1}{2}$	$\frac{3}{2}^+$	4.993(6)	−25.954(6)	Present
			4.999(12)		e
			4.979(20)		f
		$\frac{1}{2}^+$	6.67(20)		Present
		$\frac{1}{2}^+$	6.652(20)		f
^{37}Cl	$+\frac{3}{2}$	$\frac{3}{2}^+$	g.s.	−31.7615(0.4)	g
		$\frac{1}{2}^+$	1.726(3)		e

[a] R. Mendelson *et al.*, Phys. Rev. Letters 25, 533 (1970).

[b] Reference 6.

[c] D. R. Goosman and R. W. Kavanagh, Phys. Rev. 161, 1156 (1967).

[d] A. M. Poskanzer *et al.*, Phys. Rev. 152, 999 (1966).

[e] P. M. Endt and C. Van der Leun, Nucl. Phys. A105, 1 (1967).

[f] P. Parker, private communication.

[g] Reference 7.

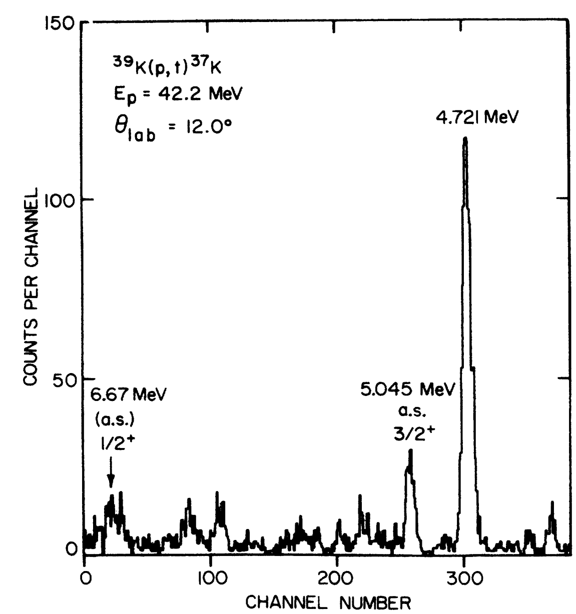

FIG. 2. A portion of the ^{39}K(p, t)^{37}K spectrum at E_p =42.2 MeV and θ =12°.

scale in these spectra is 7 keV per channel. The silicon detector was used for the actual mass measurements whereas the wire counter data were used for a search for the analogs of the ^{37}Ca and ^{37}Cl $\frac{1}{2}^+$ first excited states. Possible candidates for these levels are indicated in the spectra. These levels are so weakly excited that they could not be positively identified as either single states or as $T=\frac{3}{2}$ levels. However, they do fall at the energy expected from the IMME as will be discussed in a later section.

The Q value of the calibration reaction ^{16}O(p, ^{3}He) is so close to the Q value of ^{39}K(p, ^{3}He)^{37}Ar to the lowest analog state that it was necessary to go to a more backward angle than was used in the other measurements. In this case angles between 18 and 30° were employed, and it was necessary to measure the angle more accurately than is required at forward angles where kinematic shifts are small. The angle measurement was performed by observing the rigidity of protons elastically scattered from the hydrogen in the target. This method was accurate to better than 0.1°, which is adequate for the present purposes.

Differences in the effective target thickness for primary and calibration reactions were determined by rotating the target through 180°. They were only important for the (p, ^{3}He) reaction and account for the difference in quoted error for the (p, ^{3}He) and (p, t) experiments (6 and 3 keV, respectively). Other sources of error include uncertainties in centroid, magnetic field, beam energy, beam spot location, and angle measurement. The experiments were repeated on different days with various changes in detector, beam energy, angle, etc. and found to be consistent to well within the quoted errors.

Table I lists the results of the present measurements and of the previous measurements of the same level. The measurements determine the Q value and therefore the mass excess of the level directly, independent of the ground-state mass excess. The excitation energies quoted for the present experiment are the difference between the measured mass excesses and those of the ground states given in the 1971 Mass Table.[7] Since the ground-state masses are accurately known, the error of the excitation energy is not appreciably different from that of the mass excess. It can be noted from Table I that the agreement between the present measurement and previous ones is quite good. The ^{37}Ca mass agreement is only fair, but it is not improbable that such a difference should appear occasionally for two completely independent measurements. The excitation energy of the first excited state is in excellent agreement. One difference which can be noted, however, is that the yield to the first excited state relative to

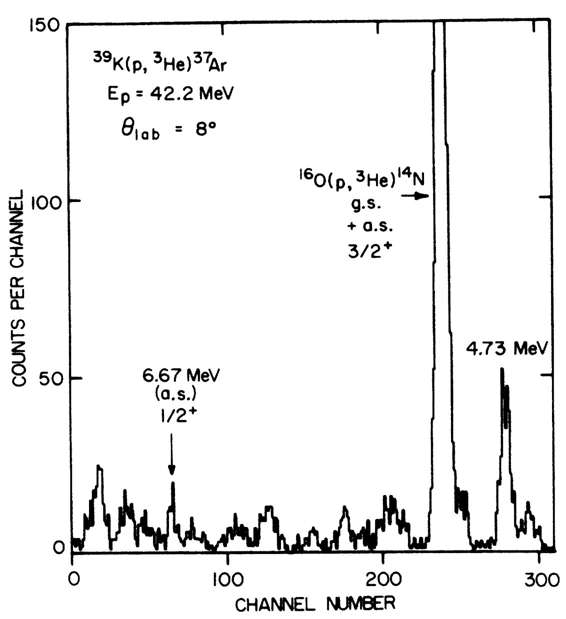

FIG. 3. A portion of the ^{39}K(p, ^{3}He)^{37}Ar spectrum at E_p = 42.2 MeV and θ = 8°.

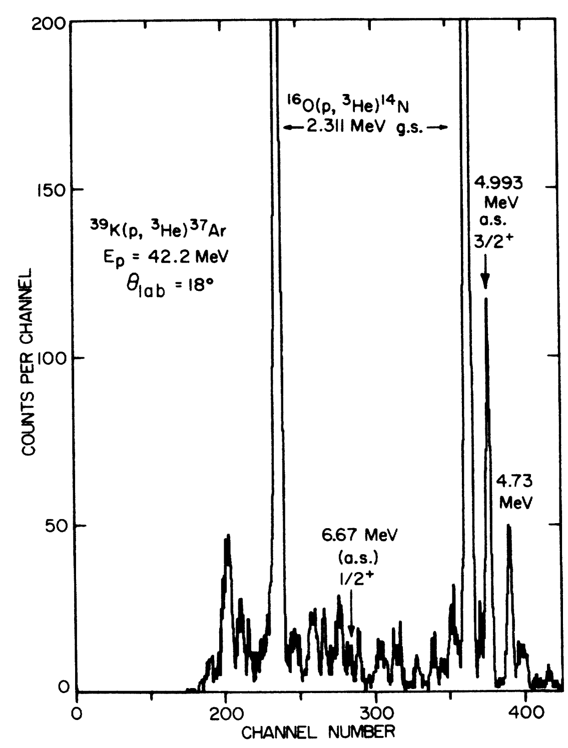

FIG. 4. A portion of the ^{39}K(p, ^{3}He)^{37}Ar spectrum at E_p = 42.2 MeV and θ = 18°.

the ground state is much greater in the 55.9-MeV data of Ref. 6 than in the present 70.7-MeV data.

C. Nucleus ^{21}Mg

The ^{21}Mg calibration spectrum, which is shown in Fig. 1, includes the ground state and a number of excited states. This is just one of the numerous ^{21}Mg spectra which were recorded in the course of the experiment. The energy levels deduced from analyzing these data are summarized in Table II and compared to other $T=\frac{3}{2}$ levels in $A=21$ nuclei. The first excited state at 0.210 MeV is only visible as a few counts in Fig. 1 and in fact was equally weakly excited at all the angles employed, 8, 10, 12, and 15°. The weak excitation of this state is in strong contrast to the data of Ref. 6, in which the first excited state is comparable in strength to the ground state. Another difference is the appearance of peaks in the Ref. 6 data corresponding to 1.25–1.30 MeV excitation in ^{21}Mg. There was no evidence for this state in the present experiment, and there exists no analogous level in the comparatively well studied nucleus ^{21}F. Since the mass excess determined for ^{21}Mg in Ref. 6 differed considerably from later more accurate measurements, it is possible that there may have been an undetected shift in beam energy. Another possibility, which is certainly worth exploring, is that the cross sections for the (^{3}He, ^{6}He) reactions are extremely energy-dependent. This could explain at least the problem of the relative yields of the ground and first excited states in both ^{21}Mg and ^{37}Ca.

TABLE II. $T=\frac{3}{2}$ levels in $A=21$ nuclei. For ^{21}Ne and ^{21}Na the energy is from the lowest $T=\frac{3}{2}$ level (MeV).

J^π	^{21}F [a]	^{21}Ne [b]	^{21}Na [c]	^{21}Mg [d]
$\frac{5}{2}^+$	···	$E_x=8.856$	$E_x=8.973$	···
$\frac{1}{2}^+$	0.280	0.282	0.244	0.210±10
$(\frac{1}{2},\frac{3}{2})^-$	1.101	1.106	(1.13) [e]	1.08 ±10
$(\frac{3}{2},\frac{5}{2})^+$	1.730	1.748		1.64 ±20
	1.755			
	2.040			2.01 ±20
	2.071			

[a] E. K. Warburton and S. W. Olness, Phys. Rev. C 2, 2235 (1970).
[b] D. C. Hensley, Phys. Letters 27B, 644 (1968).
[c] A. B. McDonald, J. R. Patterson, and H. Winkler, Nucl. Phys. A137, 545 (1959); R. C. Bearse, J. C. Legg, G. C. Morrison, and R. E. Segel, Phys. Rev. C 1, 608 (1970).
[d] Present work, errors in keV.
[e] From Ref. 6, not identified as $T=\frac{3}{2}$.

III. DISCUSSION

Accurate mass measurements away from the line of β stability can be used to extend and improve the precision of nuclear mass relationships. A Garvey-Kelson[8] charge symmetry relation which is indicated schematically in part (a) of Fig. 5,

$$^{37}\mathrm{Ca} = {}^{37}\mathrm{Cl} + {}^{35}\mathrm{Ar} - {}^{35}\mathrm{Cl} + {}^{37}\mathrm{K} - {}^{37}\mathrm{Ar} + {}^{39}\mathrm{Ca} - {}^{39}\mathrm{K}$$

predicts a mass excess = − 13 127 ± 6 keV. This result agrees very well with the present measurement − 13 144 ± 25 keV.

With an accurate mass for ^{37}Ca, one is in a good position to predict the binding energy of ^{36}Ca. A relation of type (b) in Fig. 5 can be used to find the ^{36}Ca − ^{35}K mass difference:

$$^{36}\mathrm{Ca} - {}^{35}\mathrm{K} = {}^{36}\mathrm{Ar} - {}^{35}\mathrm{Ar} + {}^{37}\mathrm{Ca} - {}^{37}\mathrm{K} = -5473 \pm 7\ \mathrm{keV}.$$

This implies that ^{36}Ca is proton bound by 2.82 MeV. In order to estimate whether ^{36}Ca is stable to two-proton decay one needs to estimate the mass of ^{35}K by a type (a) mass relationship.

$$^{35}\mathrm{K} = {}^{35}\mathrm{S} + {}^{33}\mathrm{Cl} - {}^{33}\mathrm{S} + {}^{35}\mathrm{Ar} - {}^{35}\mathrm{Cl} + {}^{37}\mathrm{K} - {}^{37}\mathrm{Ar}$$
$$= 11\,149 \pm 4\ \mathrm{keV}.$$

This result shows that ^{36}Ca is also stable to two-proton decay by about the same amount as one-proton decay. The nuclei ^{36}Ca and ^{35}K are accessible by (^{4}He, ^{8}He) and (p, ^{6}He) reactions on ^{40}Ca and will be very useful for extending mass relationships further into the $Z>N$ region.

The isobaric multiplet mass equation (IMME) can be used with the mass excess of ^{37}Cl and the lowest $T=\frac{3}{2}$ states in ^{37}Ar and ^{37}K to predict the mass excess of ^{37}Ca. The result, 13 139 ± 15 keV, is in very good agreement with the measured value. Another way of measuring the applicability of the IMME is to cite a d coefficient for a cubic term

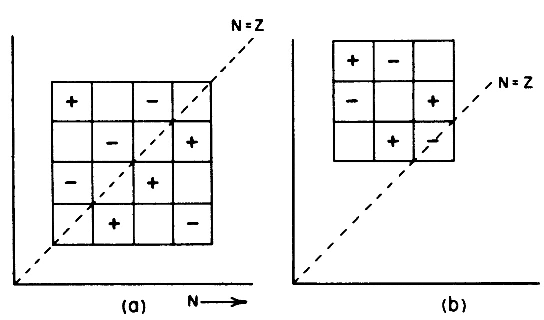

FIG. 5. (a), (b) Schematic representation of the mass relationships which are discussed in the text. Note that relations of the (b) type are only valid if the $N=Z$ nucleus is even–even.

TABLE III. Comparison of experimentally determined coefficients of the IMME to various theoretical models. All values in keV with errors in parentheses.

A	J^π	Experimental b	Experimental c	Experimental d	Δ_d	McGrory-Wildenthal b	McGrory-Wildenthal c	McGrory-Wildenthal Δ_d	Lev, Auerbach, and Kashy b	Lev, Auerbach, and Kashy c	Lev, Auerbach, and Kashy Δ_d	De Meijer, Van Royen, and Brussaard b	De Meijer, Van Royen, and Brussaard c	De Meijer, Van Royen, and Brussaard Δ_d
21	$\frac{5}{2}^+$	−3667(14)	237(5)	6.3(6.9)	3975	−3570	208	3936	−3660	239	3964	−3626	222	3945
	$\frac{1}{2}^+$	−3628(14)	228(6)	−2.4(7.0)	3954	−3528	208	3894				−3650	221	3997
37	$\frac{3}{2}^+$	−6200(8)	200(5)	−2.4(4.9)	6582	−6390	184	6804	−6180	199	6564	−6278	214	6658
	$\frac{1}{2}^+$	−6172(32)	203(10)	2.4(15.0)	6548	−6251	197	6639				−6254	196	6648

added to the equation. This is done in Table III for the four quartets under discussion. The equation works well in all cases, and the coefficients provide a convenient parametrization for comparisons with Coulomb energy shifts calculated on the basis of theoretical models.

One should note that $\frac{1}{2}^+$ quartet for mass 37 is only tentatively considered to be complete. The weak yield to the $T_z = -\frac{1}{2}$ member prohibited any positive identification. The angular dependence is consistent with the expected $L=2$, however, and the mass excesses are fitted well by the IMME. The quartet is included in spite of its tentative nature because of the interest in $\frac{1}{2}^+$ states for testing the shell-model calculations. Protons in the $s_{1/2}$ orbital in the proton-rich member of the quartet should reduce the Coulomb displacement energy considerably. This effect, which is due to the absence of a centrifugal barrier, is very evident in the Coulomb energies between the mirror nuclei ^{17}F-^{17}O. The Coulomb displacement energy, Δ_d, between $T_z = +\frac{3}{2}$ and $+\frac{1}{2}$ members is related to the b and c coefficients by the relation

$$\Delta_d = -b - 2c + \Delta_{np},$$

where Δ_{np} is the neutron-proton mass difference.

There have been very few attempts to calculate Coulomb displacement energies in mass quartets. The three calculations summarized in Table III were performed using the shell model as the basic tool. In the calculations of McGrory and Wildenthal,[9] Coulomb effects were added to the shell model by (a) including Coulomb two-body matrix elements calculated in a harmonic-oscillator basis and (b) using single-particle energies taken from experiment for protons and neutrons separately. Since the single-particle energies were taken directly from experiment (^{17}O and ^{17}F for neutrons and protons, respectively), most electromagnetic effects are automatically incorporated into the calculations which were performed with the Oak Ridge Rochester code. In the calculations of Lev, Auerbach, and Kashy[10] and De Meijer, Van Royen, and Brussaard[11] the electromagnetic effects were added to the Hamiltonian and calculated in first-order perturbation theory. Lev, Auerbach, and Kashy used the lowest-order shell-model wave functions and adjusted the radius to agree with the existing measured b coefficient. The c coefficient calculated with the same wave function agrees well with experiment. De Meijer, Van Royen, and Brussaard used more realistic wave functions and predict both b and c coefficients. Their results give good agreement as well.

IV. CONCLUSIONS

The shell model, the isobaric multiplet mass equation, and Garvey-Kelson mass relationships all appear to fit quite well the mass quartet information in the sd shell. Except for mass 17 and 33, all of the $4n+1=A$ ground-state quartets are now known up to mass 37. Experiments to measure the mass of ^{17}Ne and ^{33}Ar are being planned to fill these gaps. In all the mass systems, better information on the $T=\frac{3}{2}$ levels in the $T_z=\pm\frac{1}{2}$ nuclei would be very useful.

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