

Study of ^{18}O Levels with $E_x < 8$ MeV Using the $^{19}\text{F}(t, \alpha\gamma)^{18}\text{O}$ Reaction*

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Levels in ^{18}O were populated with the $^{19}\text{F}(t, \alpha)^{18}\text{O}$ reaction. Extensive data were collected with good α -particle energy resolution (40 keV) at $E_t = 2.4$ MeV, using a computer based data acquisition system. Reaction-produced α particles were detected near 180° in a solid-state annular counter and γ radiation was detected in a 10.2×10.2 -cm NaI(Tl) crystal at detection angles in the interval $0^\circ \leq \theta_\gamma \leq 90^\circ$. Multiparameter data collection techniques were used to store the matrix of particle- γ pulse-height pairs for coincident events. Angular correlations of γ rays from levels with $E_x < 8$ MeV were extracted from these data from which level spins, together with γ -ray branching ratios and γ -ray multipole mixing ratios, were deduced. Results include the spin assignments $J=1$ for the 6.19-MeV level, $J=0(1)$ for the 6.86-MeV level, $1 \leq J \leq 4$ for the 7.75-MeV level, and $1 \leq J \leq 5$ for the 7.96-MeV level. Some new γ -ray decay modes were found. The 7.96-MeV level decays 67% to the 1.98-MeV level, 12% to the 5.09-MeV level, and 21% to the 5.37-MeV level. The 7.75-MeV level decays 50% to the 1.98-MeV level, 11% to the 4.45-MeV level, and 39% to the 5.09-MeV level. The 6.86-MeV level decays 100% to the 4.45-MeV level. The 6.19-MeV level decays 88% to the ground state, 6% to the 4.45-MeV level, and 6% to either the 5.25- or 5.33-MeV level. The 6.34-MeV level decays 35% to the 1.98-MeV level, 53% to the 3.91-MeV level, and 11% to the 4.45-MeV level. A number of γ -ray multipole mixing ratios are also reported. For the 7.10-MeV level, $\Gamma_\gamma/\Gamma_\alpha$ was found to be 0.9 ± 0.1 . These results are compared with predictions of the model due to Ellis and Engeland.

I. INTRODUCTION

Nuclei with a few valence nucleons in the nuclear $2s$ - $1d$ shell are continuing to be the subject of intensive theoretical and experimental work. In all but the most recent model calculations it was supposed that the positive-parity levels could be treated as though the ^{16}O core is inert, and that the valence nucleons occupy the orbitals $2s_{1/2}$ and $1d_{5/2}$. For instance, Arima, Cohen, Lawson, and MacFarlane¹ found that with few exceptions the static and dynamic properties of low-lying levels were well described by such a model. However, when levels with higher excitation energies are considered, it is clear that one needs to include explicitly orbits arising from core excitation as well, since the model with an inert ^{16}O core does not provide enough states; e.g., at 5 MeV excitation energy in ^{18}O there are $J^\pi = 0^+$ and 2^+ levels which the model cannot account for.² Further, at this excitation energy negative-parity levels are observed, which clearly demand core excitation. Several calculations now exist which include core excitation and which use different approaches to limit the model space. Detailed calculations have been done by Ellis and Engeland using a weak coupling approach.³ Millener, on the other hand, has used an SU_3 approach to provide a limited basis for calculation.⁴ Zucker, Buck, and McGrory^{5,6} have done a calculation in which valence nucleons are allowed in the

$p_{1/2}$, $d_{5/2}$, and $s_{1/2}$ orbitals. Ellis and Engeland,³ in particular, have made extensive comparisons between the predictions of their model and experimental results, including γ -ray transitions and spectroscopic factors.⁷

Among these nuclei, ^{18}O is of special interest. Not only have many of the shell-model calculations been restricted to the oxygen isotopes, but with only two neutrons beyond the ^{16}O core, the nucleus is a natural test case for these calculations. Experimentally, the spectroscopy of ^{18}O continues to be pursued, especially in the region of 5–6 MeV. At the time first reports of this work were given,^{8–10} the spectroscopy of these states was uncertain, although all the states included here had been located through high-resolution magnetic analysis of charged-particle reaction products. In this laboratory, we were in a good position to investigate this nucleus at high excitation energies, even though limited in beam energy to 3 MeV. This is because we could use the $^{19}\text{F}(t, \alpha\gamma)^{18}\text{O}$ reaction to populate these states, and take advantage of both the high reaction Q value (11.82 MeV), and the high penetrability of the triton. Secondly, a system incorporating the method II geometry of Litherland and Ferguson,¹¹ a convenient way to study the nucleus, was in the process of development.

Accordingly, measurements were undertaken to extend the spectroscopy of the ^{18}O nucleus, using α - γ correlation techniques. Particular emphasis

was placed on ascertaining the γ -ray decay modes of ^{18}O levels and extracting spins and γ -ray multipole mixing ratios from the α - γ correlations.

II. EXPERIMENT

A. Arrangement

The ^{18}O levels were populated in the $^{19}\text{F}(t, \alpha)^{18}\text{O}$ reaction. Tritons were accelerated by the Lockheed Palo Alto Research Laboratory Van de Graaff accelerator. Several incident energies near 2.5 MeV were used. After momentum analysis, the incident beam was collimated to a target spot size $\approx 1 \times 1$ mm. The targets were prepared by evaporation of CaF_2 onto C backing foils $10 \mu\text{g}/\text{cm}^2$ thick. Target thickness was chosen so that the α -particle spectra were collected with an energy resolution of about 40–50 keV. Reaction-produced α particles were detected in an annular surface-barrier detector 3 cm from the target and centered at 180° with respect to the incident beam direction. Detector size and collimation limited the α -particle detection angle to $166^\circ \leq \theta_\alpha \leq 176^\circ$. The γ -ray detector was a 10.2×10.2 -cm $\text{NaI}(\text{Tl})$ crystal, located with

its front face 9.5 cm from the reaction site. To measure the γ -ray angular correlations, coincident γ -ray yields were measured with the γ -ray detector at various angles in the interval $0^\circ \leq \theta_\gamma \leq 90^\circ$. Details of the target chamber and shielding have been given previously.¹²

Detector events coincident in time were detected with a coincidence resolving time of ≈ 50 nsec, and the corresponding energy-analog signals were digitized and stored using multiparameter data acquisition systems. In early experiments, data were stored using a TMC two-parameter analyzer with 4096 words of core storage. The data described here were collected at $E_t = 2.4$ MeV, using the computer based data acquisition system described by Chalmers.¹³ In this system, all coincident events are stored on magnetic tape; a limited area of computer memory (~ 2000 words) is available for monitoring data on line, and for subsequently analyzing the data stored on magnetic tape. The data taken consisted of measurements of the γ -ray yields in coincidence with the reaction-produced α particles, for γ -ray detection angles $0^\circ \leq \theta_\gamma \leq 90^\circ$. In total, data were collected at 18 angles.

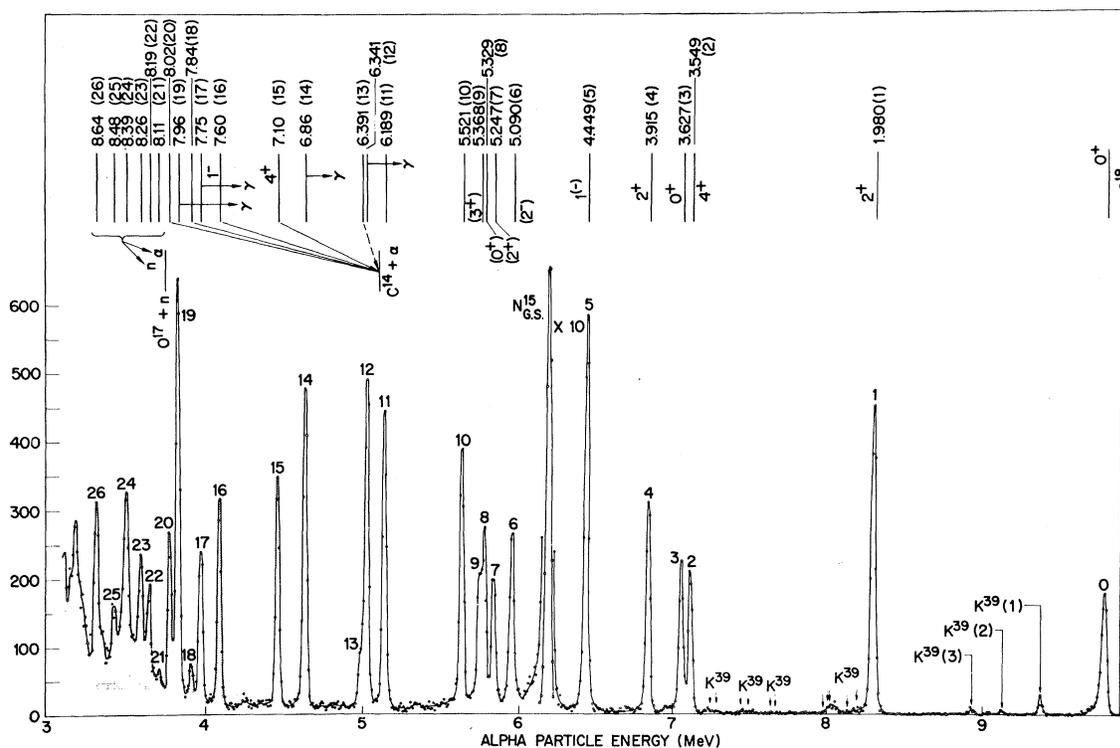


FIG. 1. A thin target singles α -particle spectrum produced in the $^{19}\text{F}(t, \alpha)^{18}\text{O}$ reaction, measured at $\bar{\theta}_\alpha = 171^\circ$. Above the α -particle spectrum is a partial level scheme for ^{18}O , with energies in MeV. Peaks are labeled according to their level number, given in the inset above. Contaminant peaks are labeled according to the residual nucleus. The crosshatched portion of the spectrum illustrates portions of the spectrum which do not appear in the (α, γ) coincidence spectrum.

B. Data Reduction and Analysis

The α -particle spectrum was collected with good energy resolution, so that individual α -particle groups representing the population of different levels in ^{18}O are readily apparent in the spectrum. Thus, the γ -ray spectrum associated with a particular level in ^{18}O could be readily extracted by integration over the line shape of the α -particle group, without making background subtractions for contributions due to nearby α -particle groups. An α -particle spectrum collected without coincidence requirements is illustrated in Fig. 1. It is clear that the resolution is quite good—the α -particle groups to the corresponding levels at 3.55 and 3.63 MeV

are well separated—and that most of the known ^{18}O levels in this energy range are excited. The particle groups which are crosshatched in Fig. 1 indicate ^{18}O states which decay into $^{14}\text{C} + \alpha$. Measurements were generally made with a slightly thicker target, due to the low yield of α - γ coincidences. For each of the 18 runs the γ -ray spectrum associated with a given level was extracted by integrating the two-parameter matrix over the corresponding individual α -particle group. These spectra were corrected for background by subtraction of the random coincidence spectrum. The real coincidence γ -ray spectra due to each of the 18 runs were then added together to improve the statistical accuracy so that the spectral features

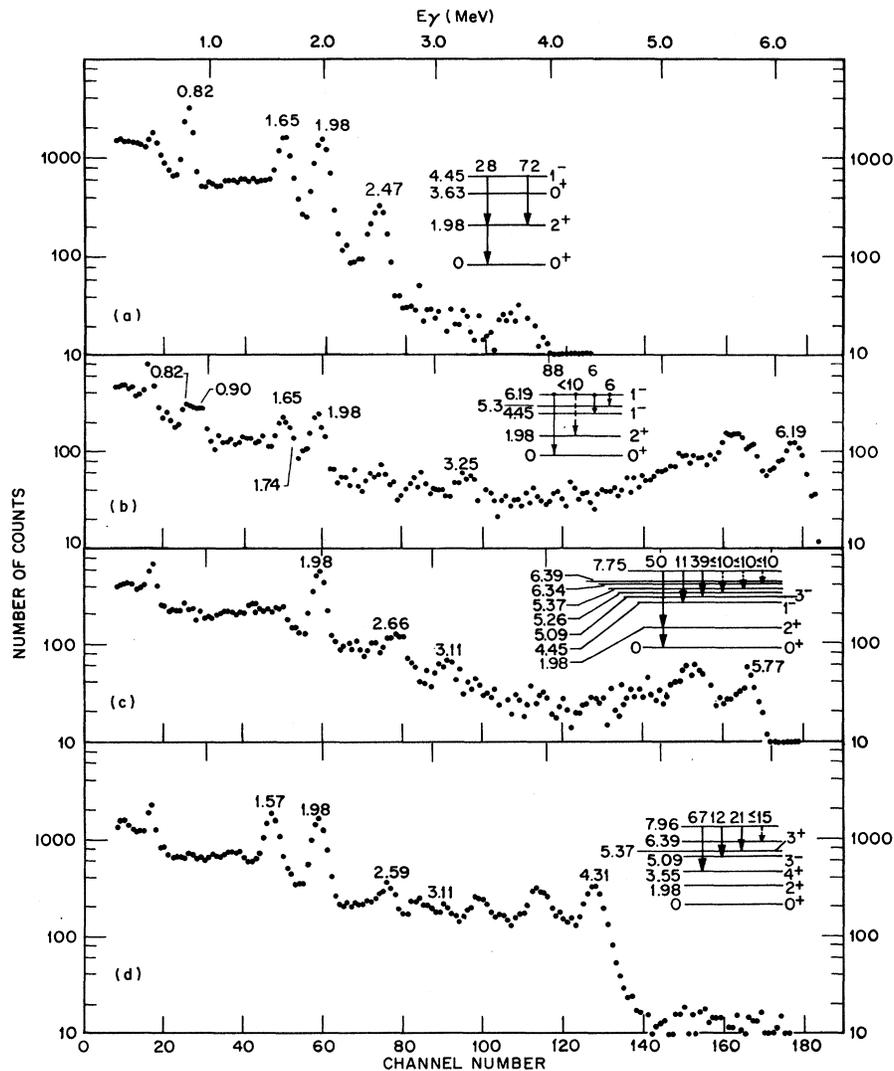


FIG. 2. The γ -ray pulse-height distributions coincident with individual α -particle groups corresponding to population of (a) the 4.45-MeV level, (b) the 6.19-MeV level, (c) the 7.75-MeV level, and (d) the 7.96-MeV level. The γ rays are labeled by their energy in MeV, while the inset shows the decay scheme associated with each spectrum.

could be better understood. These spectra are of good quality; a few, for which a detailed discussion is given, are shown in Fig. 2.

C. γ -Ray Decay Schemes and Branching Ratios

The γ -ray spectra were first examined for obvious full-absorption peaks, which were assigned γ -ray energies using a calibration based on the energy of the annihilation quantum, 0.511 MeV, and the first excited state in ^{18}O with $E_x = 1.98$ MeV. A more detailed attempt to establish precise γ -ray energies and the corresponding level energies was not felt to be worthwhile once it was clear that our results were consistent with earlier work. Next, using the known location of the energy levels in ^{18}O , the observed γ rays were assigned to transitions in ^{18}O , and level decay schemes were inferred. Branching ratios were deduced from the full-absorption peak yields, corrected for the detector efficiency. There is one case of an unassigned γ ray: Figure 2(a) illustrates the γ -ray spectrum associated with the decay of the 4.45-MeV level. In addition to the γ rays with energy E_γ (MeV) = 0.82, 1.65, and 2.47, readily associated with the decay of the 4.45-MeV level, the counts in the spectrum near channel 106 suggest a peak corresponding to $E_\gamma = 3.71$ MeV.

Interpretation of the γ -ray spectra associated with levels of excitation less than 4.45 MeV was straightforward. Having already discussed the γ -ray decay of the 4.45-MeV level, we discuss next the γ -ray decay of levels with $E_x > 5$ MeV, in order of increasing excitation energy. It should be remembered that the γ -ray spectra associated with the levels with $E_x > 5$ MeV are rather complicated and it is possible that low-energy γ -ray transitions with small branching ratios have been overlooked. The α -particle groups to the triplet of levels at $E_x = 5.3$ MeV were not resolved. Of these levels we report data for the 5.25-MeV level only, obtained from examination of an appropriate region of the α -particle spectrum. These are the branches 5.25 \rightarrow 0 (32%) and 5.25 \rightarrow 1.98 (68%). Figure 2(b) illustrates the decay of the 6.19-MeV level, and it is indeed an interesting one. It is one of the few levels to show a transition to the ground state, the dominant feature of the spectrum. There are also present in the spectrum γ rays attributable to the decay of the 4.45-MeV level, *viz.*, the 0.82- and the 1.65-MeV γ rays as well as a γ ray with energy $E_\gamma = 1.74$ MeV representing the 6.19 \rightarrow 4.45 transition. However, the peak representing the 0.82-MeV γ ray is at least a doublet, the other part corresponding to a γ ray of about 0.90-MeV energy. This could represent a transition(s) to either or both of the levels at 5.25 and 5.33 MeV.

There is also evidence in the spectrum for a peak at about 3.25 MeV which could represent the decay of the 5.25- and the 5.33-MeV states to the 1.98-MeV level. Since the 5.33-MeV level has a 27% branch to the 4.45-MeV level, it is difficult to ascertain the exact branching of the 6.19-MeV level. The 0.82- and 0.90-MeV transitions have equal intensity: Thus, if the 0.90-MeV γ ray represents entirely transitions to the 5.33-MeV level, then the 6.19-MeV level branches 7% to the 5.33-MeV level and 5% to the 4.45-MeV level, while if the 0.90-MeV γ ray represents branches entirely to the 5.25-MeV level, then it is a 6% branch, with a 6% branch to the 4.45-MeV level. We list the branching as 6% to the 4.45-MeV level, 6% to the doublet at 5.25 and 5.33 MeV. This represents a change from the results presented in Refs. 9 and 10. We also note the possibility of the transition 6.19 \rightarrow 1.98, and place it at 10% or less. This transition is suggested by the line shape of the 6.19-MeV γ ray in the region of about 4.2 MeV, which does not fall off quite as rapidly as expected. The γ -ray spectrum in coincidence with the unresolved α -particle groups populating the 6.34- and 6.39-MeV levels was attributed entirely to the decay of the 6.34-MeV level, since (see Fig. 1) the shape of the α -particle peak suggests that these levels are populated in the ratio 10:1.

The γ -ray spectra associated with the decay of the 6.86-MeV level exhibited a high-energy tail of unknown origin; otherwise the spectrum is consistent with the decay of the 6.86-MeV level 100% to the 4.45-MeV level. The decay scheme deduced for the 7.10-MeV level is in accord with earlier work. There is some weak evidence for the transitions 7.10 \rightarrow 4.45 and 7.10 \rightarrow 5.09, with 15% upper limits each. Next, we consider the decay of the 7.75-MeV level, see Fig. 2(c); the deduction of the decay scheme from this spectrum is uncertain, due in part to the poor statistical accuracy. To begin with, there is clearly a transition to the 1.98-MeV level, 7.75 \rightarrow 1.98; the two γ -ray peaks labeled 2.66 and 3.11 MeV are believed to be complex, with contributions due to the transitions 7.75 \rightarrow 5.09 and 7.75 \rightarrow 4.45. We place an upper limit of 10% for a possible transition to the doublet at 6.34 MeV. In Fig. 2(d), we have the γ -ray spectrum associated with the 7.96-MeV level. Here again the major decay of this state is obvious, 7.96 \rightarrow 3.55; the γ rays with energy 1.57, 1.98, and 4.32 MeV all belong to this branch. However, the γ -ray yield in the neighborhood of channels 60 \rightarrow 100 is not consistent with that due to a single γ ray with energy 4.32 MeV. Detailed analysis suggests that there are contributions from γ rays of energy $E = 2.59$ and 3.11 MeV, indicating the decay modes 7.96 \rightarrow 5.37 and 7.96 \rightarrow 5.09, respectively. Another

possible branch to the doublet at 6.34 MeV is put at $\leq 15\%$.

The branching ratios deduced in this manner are summarized in Table I, where we have also included branches reported by several other groups¹⁴⁻²⁰ for comparison. Except for the 7.75- and 7.96-MeV levels, errors in this work are typically $\pm 2\%$ in the branching ratio. On the whole, agreement is quite good for the states of lower energy. For the 5.25-MeV level, however, our report of 32% for the ground-state branch and 68% for the branch to the 1.98-MeV level is somewhat different from earlier reports^{14, 16, 18} which result in 40% for the ground-state branch, and 60% for the branch to the first excited state. For the 5.52-MeV level, we

find 48% for the branch to the 1.98-MeV level, smaller than the measurement of Lopes *et al.*¹⁸ of 65%. For the 6.19-MeV level, we report two new decays, a 6% branch to the 4.45-MeV level, and a 6% branch to the 5.25-5.33-MeV doublet. Berant *et al.*¹⁵ report only the 6.19 \rightarrow 0 transition. For the 7.10-MeV level, our results agree better with those of Lee, Krone, and Prosser¹⁹ than the earlier results of Gove and Litherland.²⁰ The weak γ -ray branching ratios for ^{18}O states must be viewed with some caution. Even though the α -particle spectrum (Fig. 1) is well understood and the α -particle groups were collected with good resolution, there were still several unidentified features of our γ -ray spectra. For the decay of the 4.45-

TABLE I. Electromagnetic branching ratios of ^{18}O (in %).

State (MeV)	Decay to											
	0	1.98	3.55	3.63	3.91	4.45	5.09	5.25	5.33	5.37	6.34	6.39
1.98	100											
3.55		100										
3.63		100										
3.91	a	17	83									
	b	15 \pm 2	85 \pm 2									
	c	12	88									
	d	15 \pm 5	85 \pm 5									
4.45	a		28		72							
	b		36 \pm 5		64 \pm 5							
	c		32		68							
5.09	a		78	6		16						
	b	<10	>90									
	e		81	5		14						
5.25	a	32	68									
	b	40 \pm 8	60 \pm 8									
	f	41	59									
	e	40 \pm 5	60 \pm 5									
5.33	e		63		10	27						
	g		50			50						
5.37	e		85		15							
	c		85		15							
	f		88		12							
5.52	a		48		23	29						
	e		65		10	25						
6.19	a	88	≤ 10			6				{ 6 }		
	c	100										
6.34	a		34		53	11						
6.39	c		90		10							
6.84	a					100						
7.10	a		29	71		≤ 15	≤ 15					
	g		26	70	4							
	h		44	56								
7.75	a		50			11	39	≤ 10		≤ 10	≤ 10	≤ 10
7.96	a			67			12			21		≤ 15

^a This experiment. Errors are $\pm 2\%$.

^b Reference 14.

^c Reference 15.

^d Reference 17.

^e Reference 18.

^f Reference 16.

^g Reference 19.

^h Reference 20.

MeV level, the high-energy portion of the spectrum is best described as due to a γ ray with $E_\gamma = 3.71$ MeV. For the decay of the 6.19-MeV level, we pointed out the presence of a high-energy tail in the spectrum, again of unknown origin. If we ascribe these features to some weak contaminants, unidentified in the spectrum illustrated in Fig. 1, then we may well expect some misidentifications on the γ -ray branchings, either for weak branches or for higher excited states, where the level density is greater.

The branching ratios we have quoted were obtained from full-absorption-peak yields using relative full-absorption efficiencies determined from particle- γ coincidence measurements made in the same geometry as used for the $^{19}\text{F}(t, \alpha\gamma)^{18}\text{O}$ experiment. γ -ray line shapes and angular distributions were obtained for γ rays with energies 0.87, 2.22, and 3.85 MeV produced in the $^{19}\text{F}(d, \alpha)^{17}\text{O}$ reaction, the 3.37-MeV γ ray produced in the $^9\text{Be}(d, p)^{10}\text{Be}$ reaction, and the 6.13-MeV γ ray produced in the $^{19}\text{F}(p, \alpha)^{16}\text{O}$ reaction. To obtain the relative γ -ray detection efficiency, the coincident charged-particle yield was compared with the singles charged-particle spectrum, which was collected simultaneously in these measurements. (It was, of course, necessary to extrapolate the measured line shapes to zero pulse height.) The shape of the full-absorption-peak efficiency curve determined in this way made a smooth overlay with the shape of the relative γ -ray full-absorption-peak efficiency curve determined using radioactive sources.

D. γ -Ray Angular Correlations

Yields of the various γ rays as a function of detector angle were deduced from the individual spectra collected at the different detection angles. Generally, full-absorption-peak yields were extracted with background subtraction due to either extrapolated or measured line shapes. For example, it was possible to use the line shape of the 1.98-MeV γ ray obtained in coincidence with the α particles populating the 1.98-MeV level to deduce "pure" 1.57- and 1.65-MeV γ -ray line shapes (and full-absorption-peak yields) from the spectra obtained in coincidence with the α -particle groups populating the 3.55- and 3.63-MeV levels, and so on. In other cases, a smooth extrapolation based on measured γ -ray line shapes was more than adequate.

For the doublet at 3.55 and 3.63 MeV, a slightly more complicated procedure was followed to extract the yields, since the α -particle groups are unresolved in the charged-particle spectrum. Here, α -particle spectra associated with appropriate intervals of γ -ray pulse height were produced. With the aid of a Gaussian fitting program, the α -

particle peaks due to the 3.55-3.63-MeV doublet were analyzed into two components. γ -ray energy intervals were chosen so that the coincident yields proportional to the yields of the 1.98-MeV γ ray together with either the 1.57- or 1.65-MeV γ rays could be deduced.

Yields deduced from the separate runs were normalized both to a fixed value of integrated beam current and to the yield of γ rays from the 6.86-MeV level, which is isotropic. The two methods agreed to within 2%. Two examples of these data are illustrated in Ref. 10. Angular correlations obtained in this fashion were parametrized by a Legendre polynomial expansion

$$W(\theta) = 1 + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta). \quad (1)$$

The expansion coefficients A_2 and A_4 for the γ -ray correlations are listed in Table II.

The analysis of angular correlations such as these in terms of level spin and γ -ray multipole mixing ratios is quite standard. We follow the particular procedure used by Poletti and Warburton.²¹ The phase used for multipole mixing is due to Rose and Brink.²² The results are given below and compared with previous information, where relevant

TABLE II. Legendre polynomial coefficients. Angular correlations are normalized to the yield of γ rays from the 6.86-MeV level.

		$W(\theta) = 1 + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta)$	
E_x (MeV)	E_γ (MeV)	A_2^a	A_4^a
1.98	1.98	0.491 ± 0.017	-0.503 ± 0.021
3.55	1.98	0.447 ± 0.051	-0.108 ± 0.059
	1.57	0.282 ± 0.072	-0.123 ± 0.085
3.63	1-2	-0.055 ± 0.023	0.012 ± 0.025
	1.98	-0.028 ± 0.048	0.021 ± 0.050
	1.65	0.000 ± 0.076	-0.052 ± 0.086
3.91	3.91	0.302 ± 0.082	-0.452 ± 0.088
	{1.93} {1.98}	0.132 ± 0.031	-0.220 ± 0.035
4.45	2.47	-0.052 ± 0.067	0.016 ± 0.072
	0.82	-0.312 ± 0.025	-0.003 ± 0.024
	1.98	0.026 ± 0.027	-0.017 ± 0.028
5.09	3.11	-0.252 ± 0.042	+0.026 ± 0.039
	{1.98} {1.94}	0.362 ± 0.054	-0.054 ± 0.056
5.25	5.25	0.449 ± 0.070	0.206 ± 0.086
5.52	3.54	0.343 ± 0.087	-0.147 ± 0.091
6.19	6.19	0.206 ± 0.028	-0.029 ± 0.030
6.34	4.36	0.082 ± 0.070	-0.009 ± 0.076
6.86	>0.6	0.004 ± 0.007	-0.018 ± 0.008
7.10	3.55	0.381 ± 0.027	-0.071 ± 0.031
	1.57	0.333 ± 0.040	-0.067 ± 0.047
	5.12	0.495 ± 0.052	-0.151 ± 0.062
7.75	5.77	0.294 ± 0.054	-0.133 ± 0.061
7.96	4.41	0.437 ± 0.032	-0.085 ± 0.035

^a Attenuation coefficients for $E_\gamma = 2$ MeV: $Q_2 = 0.87$, $Q_4 = 0.61$.

or informative. Levels are discussed in order of increasing excitation energy, generally.

The levels at 1.98, 3.55, 3.63, 3.91, and 4.45 MeV have spin and parity $J^\pi = 2^+, 4^+, 0^+, 2^+$, and 1^- , respectively, established in earlier experiments.² We are able to independently verify the spin assignments except for the 3.63-MeV level, where the solution $J=1$ is also permitted. The mixing ratio for the 3.91–1.98 transition, a $J=2 \rightarrow 2$ transition, is of some interest. A simultaneous fit of the correlations in the 3.91–0 transition and in the 3.91–1.98–0 cascade transition (with the 1.98- and 1.93-MeV γ rays unresolved) results in $\delta(E2/M1) = +0.19 \pm 0.08$ for the 1.93-MeV γ ray, in agreement with earlier results. Ollerhead, Lopes, Poletti, Thomas, and Warburton¹⁴ give $\delta = 0.18 \pm 0.05$.¹⁵ In this group of levels, there is only one other mixing ratio to be determined, that of the 4.45–1.98 transition. From a simultaneous fit of the correlations of the 2.47 (4.45–1.98) and 0.82 (4.45–3.63) transitions, we obtain $\delta(E2/M1) \leq -0.17$ or alternatively ≥ 5.7 . This overlaps the results of Ollerhead *et al.*,¹⁴ who give the alternative values $\delta = -0.09 \pm 0.36$ or $\delta = 5.7 \pm 3.9$. In addition, Ollerhead *et al.*¹⁴ noted that, although the experimental evidence is generally consistent with $\pi(4.45) = -1$, there is a conflict between the differential cross section measurements of the $^{17}\text{O}(d, p)^{18}\text{O}$ reaction due to Yagi *et al.*²³ and Hewka, Middleton, and Wiza.²⁴ Yagi *et al.* report a strong $l_n = 1$ transition to the 4.45-MeV level, while Hewka *et al.* find that this state is only weakly populated. At $E_t = 10.0$ MeV, Middleton and Pullen²⁵ report a small differential cross section for this state in the $^{16}\text{O}(t, p)^{18}\text{O}$ reaction, and are unable to make a definite l value (and, hence parity) assignment. The results of Berant *et al.*,¹⁵ however, confirm the negative-parity assignment since they find the state strongly excited in the $^{18}\text{O}(\alpha, \alpha')^{18}\text{O}$ reaction at $\theta_\alpha = 180^\circ$.

From the early work of Ollerhead *et al.* the assignment $J^\pi = 2^-$ was suggested for the 5.09-MeV level based on (1) the γ -ray decay of this level, thought to be >90% to the 1.98-MeV level, and (2) a comparison with the known properties of the $T=1$ states in ^{18}F . More recent work results in a $J=3$ assignment, according to both Lopes *et al.*¹⁸ and subsequently Berant *et al.*¹⁵ Lopes *et al.*¹⁸ measured the angular correlations of the 3.11- and 1.98-MeV γ -ray decays of the 5.09-MeV level in a collinear geometry. Analysis of the correlation of the 5.09–1.98 transition restricts $J(5.09)$ to either $J=1$ or 3; $J=2$ is specifically excluded. The correlation of the 1.98-MeV γ ray required a $P_4(\cos\theta)$ term in the Legendre polynomial expansion, and thus $J=3$. In this experiment we find $1 \leq J(5.09) \leq 3$, with corresponding mixing ratios

summarized in Table III. Lopes *et al.* find $\delta(3.11) = 0 \pm 0.02$; for $J=3$ we find $\delta = -0.02 \pm 0.08$ or alternatively $+4.02 \pm 0.68$. The parity of the level, suggested by the direct reaction results^{24–26} to be $\pi = -1$, is clearly $\pi = -1$.^{15, 27}

The α -particle groups corresponding to population of the triplet of levels at 5.25, 5.33, and 5.37 MeV were not separated clearly in the α -particle spectrum, cf., Fig. 1; the only angular correlation extracted from our data is that of the 5.25–0 transition, since the 5.25-MeV level is the only level in the group with a ground-state transition.¹⁸ The γ -ray decay branching ratios of this level could also be extracted by examining an appropriate region of the α -particle spectrum; we report the branches 5.25–0 and 5.25–1.98. The χ^2 analysis of the correlation of the 5.25–0 transition results in minimum χ^2 values of 2.3, 1.9, 2.7, 3.5, and 4.4 for $J=1, 2, \dots, 5$, respectively. These results certainly favor $J=2$. However, in view of the fact that the distribution has a χ^2 minimum of 1.9, the possible $J=1$ and 3 assignments should also be considered. Lopes *et al.*¹⁸ based on an analysis of the angular correlations of the 5.25–0 and the 5.25–1.98 transitions find $J=2$ or 3; they invoke the results of the direct reaction work to argue against the $J=3$ alternative. The recent work due to Berant *et al.*¹⁵ established $J=2$ uniquely. The state has positive parity.^{25, 15, 27} There is agreement among the measured mixing ratios of the 5.25–1.98 transition; Lopes *et al.*¹⁸ report $\delta = 0.2 \pm 0.1$ while Berant *et al.*¹⁵ find $\delta = 0.14 \pm 0.04$. The other states in this triplet, at $E_x = 5.33$ and 5.37 MeV, have $J^\pi = 0^+$ and 3^+ , respectively.²

For the 5.52-MeV level, analysis of the correlation in the 5.52–1.98 transition leads to no spin restriction; values of J in the interval $1 \leq J \leq 5$ are allowed with appropriate multipole mixing ratio restrictions associated with each spin. The alternatives $J=4$ and 5 may be ruled out by considering the lower limit for the radiative width for the 5.52–4.45 transition, with a branching ratio 29%, together with the upper limit for the lifetime of the levels obtained from the shape of the prompt coincidence curve measured here, $\tau_m \leq 20$ nsec. If we use the limit due to Olness, Warburton, and Becker,²⁸ $\tau_m \leq 25$ fsec, then $J=3$ may be firmly rejected. Thus, $J=1$; $-\infty \leq \delta \leq -0.27$ or $0.32 \leq \delta \leq 1.54$ or $J=2$, $\delta \leq +0.19$. The angular correlation of the combined 1.98+1.93-MeV γ rays was also extracted from the data and parametrized by a Legendre polynomial expansion. Examination of the expansion coefficients might, in principle, have permitted a choice between the alternative assignments, since, if $J=1$, $P_4(\cos\theta)$ terms are not allowed in the expansion. However, a P_4 term

was not required and thus both the $J=1$ or 2 alternatives remain. Previously, Lopes *et al.* have reported $J=1$ or 2 based on a study of the correlations of the 5.52-1.98, 5.52-4.45, and 5.52-3.63 transitions. The multipole mixing ratios for the 5.52-1.98 transition allowed in their work are $J=1$, $\delta = -10 \leq \delta \leq -4$; or $J=2$, $\delta = 0 \pm 0.02$. A comparison of the two experiments does not permit a choice between the alternative values, $J=1$ or 2.

The levels in the energy interval $6 \leq E_x$ (MeV) ≤ 7 are discussed next. For the 6.19-MeV level, analysis of the decay to the ground state leads directly to the unambiguous spin assignment $J=1$. The angular correlation of the 6.19-MeV γ ray has been illustrated in Ref. 10. Berant *et al.*¹⁵ on the basis

of their $^{18}\text{O}(\alpha, \alpha'\gamma)^{18}\text{O}$ studies also report $J=1$ and fix the parity as well, $\pi = -1$. Ollerhead *et al.*²⁷ also find natural parity. The angular correlation of the 6.19-4.45 transition was not extracted due to the complicated γ -ray spectrum. Previously, Lee, Krone, and Prosser¹⁹ reached a different conclusion for the spin of the 6.19-MeV level, i.e., $J=2$ or 3. Examination of their γ -ray spectrum, Ref. 19, Fig. 6, indicates that the yield of the 6.19-MeV γ ray is not only rather weak, as Lee, Krone, and Prosser point out, but also partially obscured by the line shape of the 7.63-MeV γ ray present in this spectrum. Extraction of a reliable angular correlation must have been rather difficult, and this could account for the discrepancy.

TABLE III. Electromagnetic transition multipole mixing ratios in ^{18}O .

$E_\gamma(E_i \rightarrow E_f)$ (MeV)	J^π_i	Present	$\delta(E2/M1)^a$		Theory ^b
			Other		
1.93(3.91 \rightarrow 1.98)	2 ⁺	0.19 \pm 0.08	0.08 \pm 0.05 ^c 0.18 \pm 0.10 ^d		0.09
0.87(4.45 \rightarrow 1.98)	1 ⁻	≤ -0.17 ; > 5.7	-0.09 \pm 0.36; 5.7 \pm 3.9 ^d		
3.11(5.09 \rightarrow 1.98)	3 ⁻	-0.04 \pm 0.05; 3.7 \pm 0.6	0 \pm 0.02 ^e		
3.27(5.25 \rightarrow 1.98)	2 ⁺	...	0.2 \pm 0.1 ^e 0.14 \pm 0.04 ^c		0.22
3.52(5.52 \rightarrow 1.98)	1	$-\infty \leq \delta \leq -0.27$; 0.32 $\leq \delta \leq 1.54$	$-10 \leq \delta \leq -0.4$ ^e		
1.07(5.52 \rightarrow 4.45)	2	$-\infty \leq \delta \leq +0.19$	0 \pm 0.02 ^e		
	1	...	$ \delta \leq 0.2$ ^e		
4.36(6.34 \rightarrow 1.98)	2	...	0 \pm 0.04 ^e		
	0	...			
5.12(7.10 \rightarrow 1.98)	1	$-\infty \leq \delta \leq +\infty$			
	2	+0.22 \pm 0.14; -(5.7 ^{+17.2} _{-2.5})			
	3	>11; 0.22 \pm 0.10			
3.55(7.10 \rightarrow 3.55)	4	-0.052 \pm 0.035 0.07 \pm 0.07; -0.90 \pm 0.13	0 0.04 $\leq \delta \leq 0.14$ ^f -0.035 \pm 0.035 ^g		-0.03
5.77(7.75 \rightarrow 1.98)	1	$-\infty \leq \delta \leq 0.22$; 0.27 $\leq \delta \leq 1.60$; >11.4			
	2	-4.0 $\leq \delta \leq +0.22$			
	3	-0.38 \pm 0.06; -7.6 ^{+8.8} _{-2.7}			
	4	0.07 \pm 0.05; +7.6 ^{+8.8} _{-2.7}			
4.41(7.96 \rightarrow 3.55)	1	≤ -1.33 ; ≥ 11 ; +0.34 \pm 0.10			
	2	0.19 $\leq \delta \leq 0.83$			
	3	0.45 $\leq \delta \leq 1.23$			
	4	-1.0 $\leq \delta \leq 0.1$			
	5	-0.44 \pm 0.04			

^a Reference 22.

^b Reference 3.

^c Reference 15.

^d Reference 14.

^e Reference 18.

^f Reference 20.

^g Reference 19.

The 6.86-MeV level decays 100% to the $J^\pi = 1^-$ level at $E_x = 4.45$ MeV. All the γ rays in coincidence with the particle group corresponding to the population of this level exhibit an isotropic angular correlation. This leads to the alternative and equally probably spin assignments $J=0$ or 1 for this level, hitherto unassigned. Since this level has not been observed to decay by α -particle emission, although energetically permitted to do so, the allowed spin and parity combinations are presumably $J=0^-$ or 1^+ . The γ -ray decay mode favors $J=0$. No information on multipole mixing ratios is deducible from these data.

The 6.34-MeV level does not have a spin and parity assignment. The lifetime limit $\tau_m < 0.035$ psec, due to Olness, Warburton, and Becker,²⁸ together with the 11% decay to the 4.45-MeV level, leads to $J \leq 3$, using arguments based on radiative widths. The angular correlation of the 6.34–1.98 transition is described by the Legendre polynomial expansion coefficient $A_2 = +0.08 \pm 0.07$. Analysis of this correlation does not significantly restrict the choice of spin and γ -ray multipole mixing ratios: $J=0$, $J=1$, $-\infty \leq \delta \leq \infty$; $J=2$, $\delta = +0.22 \pm 0.14$ or $-(5.7_{-2.5}^{+17.2})$; and $J=3$, $|\delta| > 11$ or -0.22 ± 0.10 . Unnatural parity is suggested for this state.²⁷ The other member of the doublet, the 6.39-MeV level, has $J^\pi = 3^-$.¹⁵

Gove and Litherland have studied the 7.10-MeV level using the $^{14}\text{C}(\alpha, \gamma)^{18}\text{O}$ reaction and γ - γ correlation techniques, and find $J^\pi = 4^+$. We can also show that the 7.10-MeV level has $J=4$; to do this, however, we must analyze both members of the 7.10–1.98–0 cascade. The intensity of the 1.98-MeV γ ray due to this cascade may be found from the spectra by subtracting straightforwardly from the total intensity of the 1.98-MeV γ ray the contribution from these other branches. As well as showing that the 7.10-MeV level has $J=4$, the above procedure gives for the 7.10–1.98 transition, $\delta = -(0.052 \pm 0.035)$ while for the transition 7.10–3.55, $\delta = 0.07 \pm 0.07$ or alternatively -0.90 ± 0.13 . Lee, Krone, and Prosser¹⁹ have also found $J^\pi = 4^+$. We have also deduced $\Gamma_\gamma/\Gamma_\alpha = 0.9 \pm 0.1$ for this level, from a comparison of peak areas in the α -particle spectrum taken with and without γ -ray coincidence requirements. (See below.)

Above the 7.10-MeV level, angular correlations were extracted for only two levels, the 7.75- and 7.96-MeV levels. Analysis of the angular correlations did not yield much information. The 7.75-MeV level does not have a spin-parity assignment. Ollerhead *et al.*²⁷ measured energy spectra for the reaction $^{18}\text{O}(\alpha, \alpha')^{18}\text{O}$ at θ_α near 180° for 18 different bombarding energies in the interval $20.0 \leq E$ (MeV) ≤ 23.4 . They find that this state is weakly populated at several of the higher bombarding energies. On the other hand, based on a comparison

of both singles and coincident α -particle spectra measured here, we find $\Gamma_\gamma > 9\Gamma_\alpha$ (see below), suggesting unnatural parity. Thus, though there is some doubt about the parity of this state, unnatural parity seems to be the best choice. Analysis of the angular correlation of the 5.77-MeV γ ray (7.75–1.98) leads to the spin restriction $1 \leq J \leq 5$. Arguments involving the partial lifetime of the transition to the 4.45-MeV level may be used to reject the $J=5$ possibility using the fact that $\tau_m < 20$ nsec. The multipole mixing ratio restrictions associated with the various spin alternatives are given in Table III. The 7.96-MeV level has also not received a firm spin and parity assignment. Analysis of the correlation in the transition 7.96–3.55 leads to the restriction $J > 0$, together with multipole mixing ratio restrictions associated with each spin alternative. An upper limit of $J=6$ to the possible spin values may be deduced from the usual consideration of electromagnetic transition probabilities for the 7.96–5.37 transition. Of course, the electromagnetic branching ratios suggests $J > 3$ for this level. Multipole mixing ratios associated with each spin are given in Table III. In our earlier reports,^{9,10} the 7.75- and 7.96-MeV levels were labeled (2^-) and $(3^+, 4^-)$, respectively; these represent only our best choices, and are not rigorous assignments.

E. States Above the $^{14}\text{C} + \alpha$ Threshold

Above 6.23-MeV excitation energy in ^{18}O , energetics permit the decay mode $^{18}\text{O} \rightarrow ^{14}\text{C} + \alpha$. Thus, a comparison of the α -particle spectrum measured in singles and coincidence permits identification of levels whose primary decay mode is via α -particle or γ -ray emission. Referring to Fig. 1, the α -particle groups indicated by crosshatching do not appear in the α - γ coincidence spectrum. These are the levels at excitation energy 7.60, 7.84, 8.02, 8.11, 8.19, 8.26, 8.39, 8.48, and 8.64 MeV. The 7.10-MeV level exhibits both α - and γ -ray decay and from a peak area comparison, we find $\Gamma_\gamma/\Gamma_\alpha = 0.9 \pm 0.1$. The 6.34-, 6.86-, 7.75-, and 7.96-MeV levels, however, appear to exhibit only γ -ray decay, and we estimate $\Gamma_\gamma > 9\Gamma_\alpha$ for these levels. These results were obtained without any correction for γ -ray detection efficiency and have an estimated error of 10%.

III. SUMMARY AND DISCUSSION

The results of this work include a reliable set of γ -ray branching ratios for ^{18}O levels with $E_x \leq 7.96$ MeV, γ -ray multipole mixing ratios, several new spin assignments and spin restrictions, and a measurement of $\Gamma_\gamma/\Gamma_\alpha$ for the 7.10-MeV level. This work, together with previous work, is convenient-

ly summarized in Fig. 3, an energy level diagram for ^{18}O . Only one state below 7.10 MeV does not have a definite spin and parity assignment, the 6.34-MeV level, and here Olness, Warburton, and Becker²⁸ limit J to be 1 or 2.

Due to the experimental arrangement, i.e., the simultaneous measurement of the γ -ray and α -particle spectrum, the data and results obtained are expected to be especially reliable and consistent. Indeed, the γ -ray spectra were obtained with good resolution and for the major components, exhibit good signal-to-noise ratio. The γ -ray angular correlations were also consistent over all 18 data points.

Table I contains a best set of γ -ray branching

ratios for levels with $E_x \leq 7.96$ MeV, including several reported for the first time. While the resolution of the γ -ray spectrometer is such that the γ -ray decays of the 7.75- and 7.96-MeV levels could not be identified well enough to allow good angular correlations to be extracted, they appear to merit further study; their observed properties suggest themselves, respectively, as candidates for the predicted 2^- and 4^- levels belonging to the three-particle-one-hole configuration.³ Turning now to the remaining angular correlation measurements, the spin assignments for four of the first five excited states were verified, and the spin of the other level restricted to one of two values; however, spin assignments for the higher excited

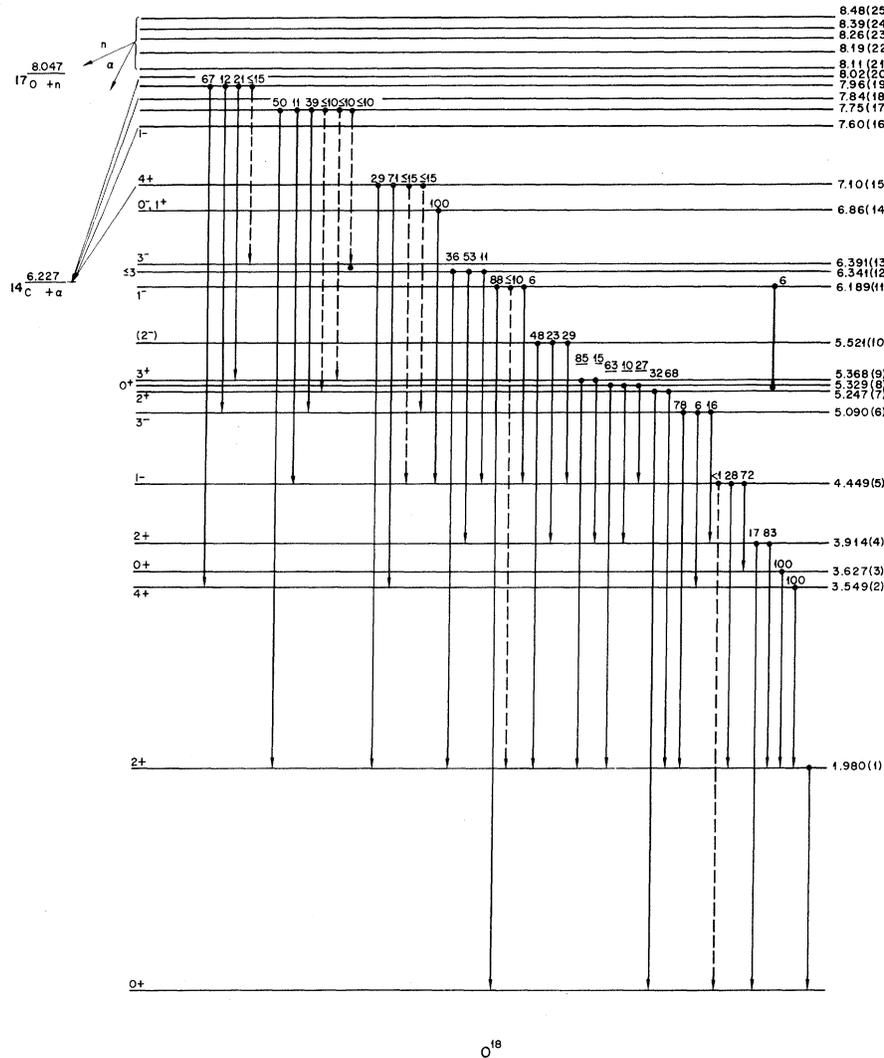


FIG. 3. The level scheme for ^{18}O . The spin assignments given in the figure represent a summary. Excitation energies are due to S. Hinds, H. Marchant, and R. Middleton, Nucl. Phys. 38, 81 (1962). Underlined γ -ray branching ratios are from the work of Refs. 14 and 18. For some transitions, the final state is uncertain; these decays are shown to terminate in dots located between the possible final states.

states were more elusive. This comes about because frequently the major decay of the levels is via cascade through the 1.98-MeV level. The correlation of the high-energy γ ray did not have a pronounced P_4 term in the Legendre polynomial expansion. Since the 1.98-MeV level has $J=2$, and there are the unknown multipole mixing ratios and the two magnetic substate populations to determine, ambiguous spin assignments were the rule.^{14, 21}

Furthermore this cascade together with any other γ -ray branches contribute to the yield of the 1.98-MeV γ ray, and so the correlation of the 1.98-MeV γ ray does not appreciably help in fixing level spins. The γ -ray spectrum is dominated by the γ -ray line shape of the transition $E_x \rightarrow 1.98$ and, attempting to extract other angular distributions, one starts off with a relatively high "background," and so it is difficult to extract reliable γ -ray components from these spectra. Thus, new spin assignments were limited to $J=1$ for the 6.19-MeV level, $J=0$ or 1 for the 6.86-MeV level, with $J=0$ preferred due to the γ -ray branching, and some loose spin restrictions for the 6.34-, 7.75-, and 7.96-MeV levels. Several γ -ray multipole mixing ratios were also obtained, and these are given in Table III. The predictions of Ellis and Engeland³ are given as well, for comparison. The mixing ratios and widths for the 3.92-, 5.25-, and 5.37-MeV levels have previously been discussed.^{3, 18}

The measurement of $\Gamma_\gamma/\Gamma_\alpha$ for the 7.10-MeV level combined with the measurement of $\Gamma_\alpha\Gamma_\gamma/\Gamma$ reported in the $^{14}\text{C}(\alpha, \gamma)^{18}\text{O}$ work permits the extraction of partial widths for the 7.10-MeV level. There are several measurements of $\Gamma_\alpha\Gamma_\gamma/\Gamma$ for the 7.10-MeV level; we take the value 4.2×10^{-2} eV, reported by Lee, Krone, and Prosser; this value relies on the relative values of Γ_γ for the 1.140- and 1.790-MeV resonances measured by Lee, Krone, and Prosser¹⁹ and the absolute value of $\Gamma_\gamma\Gamma_\alpha/\Gamma$ for the 1.790-MeV resonance measured

TABLE IV. Partial widths (in Weisskopf units) for the decay of the 7.10-MeV level.

Transition (MeV)	Multipolarity	Exp.	Γ_γ Theory ^a
7.10 \rightarrow 1.98	$E2$	7.28 ± 1.86	0.40
7.10 \rightarrow 3.56	$M1$	0.026 ± 0.006	0.041
7.10 \rightarrow 3.56	$E2$	$0.007_{-0.002}^{+0.007}$	0.026

^a Reference 7.

by Gove and Litherland.²⁰ Together with our measurement of $\Gamma_\gamma/\Gamma_\alpha = 0.9 \pm 0.1$, we arrive at $\Gamma_\gamma = 7.9 \times 10^{-2}$ eV, for which we estimate a 25% error. Using the γ -ray branching ratios measured here, and $\delta(7.10 \rightarrow 3.56) = 0.02 \pm 0.02$ (a weighted average of the results of Gove and Litherland, Lee, Krone, and Prosser, and this work), we arrive at the partial widths quoted in Table IV. Ellis and Engeland have given wave functions for these states as well as electromagnetic transition rates. These are also given in Table IV. In this model, both the 1.98- and 3.55-MeV levels are predominantly 2p-0h structure, while the 7.10-MeV state is a mixture of 2p-0h and 4p-2h states. We see that this picture gives the relative strengths of the $E2$ transitions correctly, but underestimates the relatively strong 7.10 \rightarrow 1.98 transition by a factor ≈ 17 . The $M1$ strength is correctly given. The value $\Gamma_\alpha = 8.9 \times 10^{-2}$ eV is approximately a factor of 3 greater than that estimated from the average α -particle transition widths given by Wilkinson.²⁹ These results confirm nicely the suggestion some time ago of Gove and Litherland, that the 7.10-MeV level had comparable α -particle and γ -ray widths. For completeness, we note that Lee, Krone, and Prosser report a transition 7.10 \rightarrow 3.92, with a 4% branching ratio. This leads to an experimental strength $|M|^2 = 4.4 \pm 1.6$, a factor of 9 stronger than predicted by Engeland and Ellis.

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