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Exchange Effects in Radiative n-p Capture

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The contribution of the one-pion-exchange interaction current to the transition amplitude for radiative n-p capture at thermal energies is evaluated using the Chew-Low static model for the amplitude for photoproduction of pions off nucleons. A correct treatment of the pion-rescattering term in that amplitude gives essential agreement with the static-quark-model calculation of Riska and Brown which considers only nucleon excitation to the $\Delta(1236)$ state.

The object of this note is to analyze and add to recent calculations of the role of meson-exchange effects[1] (deuteron interaction current)[2] in radiative n-p capture at thermal energies. It is well known[3] that the experimental cross section for this process ($\sigma_{\exp} = 334.2 \pm 0.5$ mb) is larger by about 10% than the theoretical value predicted for it using the usual single-particle magnetic moment operators. In Refs. 1 and 2 it is shown that if the one-pion-exchange-current effects are included and realistic wave functions with a finite D-wave component for the deuteron are used one can account for essentially all the discrepancy.

In Ref. 1, this is done by evaluating the pion current and catastrophic terms which have both been known for a long time[4] and evaluating the contribution of $\Delta(1236)$ excitation of a nucleon using the static quark model. In Ref. 2, on the other hand, the pion-exchange contribution was calculated using the Chew-Low static model[5] for photoproduction of a pion off a nucleon, but we believe the model to have been used incorrectly. The complete amplitude for the photoproduction of a pion of momentum $\vec{q}$ and isospin β off a nucleon by a photon of momentum $\vec{k}$ and polarization $\vec{\epsilon}$ is[6]

$$\vec{\epsilon}\cdot\vec{M}(\vec{k},\vec{q})_\beta = e\left(\frac{f_r}{m_\pi}\right)\left[\vec{\sigma}\cdot\vec{\epsilon} - \frac{2\vec{\sigma}\cdot(\vec{q}-\vec{k})\vec{q}\cdot\vec{\epsilon}}{(\vec{q}-\vec{k})^2 + m_\pi^2}\right]\epsilon_{3\beta\gamma}\tau_\gamma + \frac{e}{2M}\left(\frac{f_r}{m_\pi}\right)^{-1}\mu_v\left[4\pi\sum_\alpha P_\alpha(\vec{q},\vec{p})h_\alpha(\omega)\right]. \tag{1}$$

Here $f_r^2/4\pi = 0.082$ represents the pion-nucleon coupling constant, $\mu_v = 2.353$ is the isovector magnetic moment of the nucleon, $P_\alpha(\vec{q},\vec{p})$ are the usual angular momentum isospin projection operators for the meson-nucleon system, and $h_\alpha(\omega) = q^{-3}e^{i\delta_\alpha}\sin\delta_\alpha$ for $\omega = (q^2+\mu^2)^{1/2} > \mu$ is related to the pion-nucleon scattering amplitude. The index α varies over the four allowed angular momentum-isospin labels in the static model and $\vec{p}$ stands for a π^0 meson state of momentum $\vec{k}\times\vec{\epsilon}$. The contribution of the first two terms is equivalent to that of the pion current and catastrophic terms in Ref. 1 and is not in dispute. Its magnitude is relatively insensitive to the form of the two-nucleon wave functions used in the calculation.[2]

The third term in Eq. (1), called the rescattering term, was found to contribute only to S-D transitions. If, as in Ref. 2, we consider only the resonant contribution, we must evaluate $h_3(\omega)$ at $\omega = 0$.[7] It is easily seen that the pole terms corresponding to the Born diagrams are already included in the zero-order result and should be omitted here to avoid double counting. If now we use an effective-range approximation[5] for the $h_\alpha(\omega)$

$$h_\alpha(z) = \frac{\lambda_\alpha}{z}\frac{1}{1 - z r_\alpha} \tag{2}$$

one gets

$$\tilde{h}_\alpha(0) = \left(h_\alpha(z) - \frac{\lambda_\alpha}{z}\right)_{z=0} = \lambda_\alpha r_\alpha. \tag{3}$$

For the 33 state this gives[8]

$$\tilde{h}_3(0) = \tfrac{4}{3}(f_r^2/4\pi m_\pi^2)(1/\omega^*), \tag{4}$$

where $\omega^* = M^* - M$ is the pion energy at resonance which is equal to the difference between the masses of the $\Delta(1236)$ and the nucleon. In any case one cannot stop here since the result as it stands would lead to a contradiction with Hermiticity of the magnetic moment operator. To avoid this, in Ref. 2 there is added a term corresponding to the interchange of $\vec{q}$ and $\vec{p}$ and the corresponding isospin labels. However, the failure of Hermiticity is avoided at $\omega = 0$ when the contribution of all other P-wave amplitudes is included. While these are known to be small for the photoproduction amplitude between the physical threshold and the 33 resonance, this smallness is due to the cancellation between the large negative Born terms and large positive higher-order contribution, particularly, the contribution of the 33 state in the crossed channel. Since we require the amplitudes $\tilde{h}(\omega)$ with the Born term omitted, these amplitudes will make substantial contributions. If we use the Low equations[5] and neglect the scattering cross section for all but the 33 state in the dispersion integrals we get the following results:

$$\tilde{h}_{11}(0)=\tfrac{8}{5}\tilde{h}_3(0)\,,$$

$$\tilde{h}_{31}(0)=\tilde{h}_{13}(0)=\tfrac{2}{5}\tilde{h}_3(0)\,.$$

Including these and writing the result for the rescattering contribution to the exchange moment in the phenomenology of Chemtob and Rho[9] we find[10]

$$\vec{\mathrm{M}}=\tfrac{1}{2}\frac{e}{2M}\{(\vec{\tau}^{(1)}-\vec{\tau}^{(2)})_3[(\vec{\sigma}^{(1)}-\vec{\sigma}^{(2)})h_{\mathrm{I}}+T^{-}_{12}h_{\mathrm{II}}] + (\vec{\tau}^{(1)}\times\vec{\tau}^{(2)})_3[(\vec{\sigma}^{(1)}\times\vec{\sigma}^{(2)})g_{\mathrm{I}}+T^{\times}_{12}g_{\mathrm{II}}]\}\,, \tag{5}$$

$$h_{\mathrm{I}}=-g_{\mathrm{I}}=-\tfrac{1}{3}\xi Y_0(m_\pi r),\quad h_{\mathrm{II}}=2g_{\mathrm{II}}=-\xi Y_2(m_\pi r)\,, \tag{6}$$

where

$$T^{\odot}_{12}=(\vec{\sigma}^{(1)}\odot\vec{\sigma}^{(2)})\cdot\hat{r}\hat{r}-\tfrac{1}{3}(\vec{\sigma}^{(1)}\odot\vec{\sigma}^{(2)}),\quad \odot=-\,,\times,$$

$$Y_0(x)=e^{-x}/x,\quad Y_2(x)=(1+\frac{3}{x}+\frac{3}{x^2})Y_0(x)\,,$$

and

$$\xi=\frac{16}{5}\mu_v\frac{f_r^{\,2}}{4\pi}\frac{m_\pi}{M^*-M}\;. \tag{7}$$

This is to be compared to the value

$$\xi=\frac{64}{25}\mu_v\frac{f_r^{\,2}}{4\pi}\frac{m_\pi}{M^*-M}$$

of Riska and Brown.[11]

In summary we have shown that a correct treatment of the pion-rescattering term in the Chew-Low model yields exactly the same result as obtained in the quark model by Riska and Brown except for a numerical factor of $\frac{5}{4}$. This, then, substantiates the conclusion of these authors that a proper treatment of one-pion-exchange contributions to the magnetic moment operator resolves the long-standing discrepancy between theory and experiment for radiative n-p capture at thermal energies.

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[6]In Ref. 2 the photoproduction amplitude was obtained from the book *Elementary Quantum Field Theory* by E. M. Henley and W. Thirring (McGraw-Hill, New York, 1962). Both contain an expression for the rescattering term which is too small by a factor of 4π. The isoscalar term has been omitted here as in Ref. 2. Note also that here as in Ref. 5 the subscript α stands either for a two-digit subscript $(2T, 2J)$ or the more compact one-digit subscript $(J+T)$.

[7]The justification for this is given in Ref. 2.

[8]The expression for $h_3(\omega)$ in Ref. 2 is independent of the pion-nucleon coupling constant $(f_r^{\,2}/4\pi)$, a fact noted but passed over by the authors; this is a consequence of the improper continuation of $h_3(\omega)$.

[9]M. Chemtob and M. Rho, Nucl. Phys. A163, 1 (1971). Similar results to ours for the exchange moment operator are obtained in this reference.

[10]Equation 5 contains only the isovector part of the rescattering contribution and also omits a term which makes contributions only to isotriplet-isotriplet—triplet-triplet transitions and hence does not contribute to the process at hand. This term is identical to the first term of (5) except for the reversal of sign of $\vec{\tau}^{(2)}$ and $\vec{\sigma}^{(2)}$ both in the parentheses and in T^{-}_{12}.

[11]As a historical remark we note that the rescattering term appears to have been first derived by Kuroboshi and Hara [Prog. Theor. Phys. 20, 163 (1958)] who find all the same terms as in Eq. (5) augmented by that mentioned in Ref. 10. The coefficients of the terms are in the same ratio as here except for a reversal in sign of what is here called g_I. These authors use dispersion integrals as do Chemtob and Rho to evaluate ξ and except for the fact that they neglect all but the 3-3 pion-nucleon scattering cross section in the dispersion integrals, they agree with respect to the value of the dispersion integral. However their coefficients over-all appear to be a factor of 2 smaller than those found by Chemtob and Rho. The numerical value of ξ found by Chemtob and Rho from their dispersion integrals appears to be about 30% lower than our value.