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Photoproduction of Positive Pions from ^{27}Al and ^{51}V

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The reactions $^{27}\mathrm{Al}(\gamma,\pi^+)^{27}\mathrm{Mg}$ and $^{51}\mathrm{V}(\gamma,\pi^+)^{51}\mathrm{Ti}$ are studied using the free-nucleon photopion production amplitude and the shell model for the nucleus. Numerical calculations are done assuming surface production of pions and the results are compared with the available experimental data.

I. INTRODUCTION

Theoretical study of reactions of the type $A(\gamma,\pi)B$ has been made earlier by Laing and Moorhouse,[1] Devanathan *et al.*,[2-6] and Saunders,[7] and it has been pointed out that the shell-model calculations using the free-nucleon photopion production amplitude usually overestimate the cross sections. This may be attributed to the uncertainty in the nuclear wave functions and to the partial absorption of the emitted pion. In the case of the reaction $^{16}\mathrm{O}(\gamma,\pi^+)^{16}\mathrm{N}$ for which the initial and the final nuclear wave functions are fairly well understood, it is found[3,6] that the use of improved wave functions, which include the effects of configuration mixing, yields a lower value to the cross sections when compared with the results of the nuclear shell model. The inclusion of the final-state interaction of the emitted pion with the residual nucleus can also be investigated by using an optical potential for the nucleus and expanding the pion wave function into partial waves as outlined in Ref. 5, but this involves lengthy numerical calculations. However, it is found that a reasonable agreement between shell-model calculations and experiments can still be obtained by invoking the surface-production mechanism for pions. It may be observed that it is purely a phenomenological model, but it turns out to be a useful one since it has been applied earlier with success[1,4,6] to the reactions $^{16}\mathrm{O}(\gamma,\pi^+)^{16}\mathrm{N}$, $^{11}\mathrm{B}(\gamma,\pi^+)^{11}\mathrm{Be}$, and $^{11}\mathrm{B}(\gamma,\pi^-)^{11}\mathrm{C}$. In the present contribution, the study is extended to two more photopion reactions, $^{27}\mathrm{Al}(\gamma,\pi^+)^{27}\mathrm{Mg}$ and $^{51}\mathrm{V}(\gamma,\pi^+)$-$^{51}\mathrm{Ti}$, for which experimental results are available.[8-10]

The cross sections for the two reactions under study have been determined experimentally from a measurement of β activity of the final nuclei. So, in each case, the final nuclear states that contribute to the reaction are the low-lying bound states which are stable against nucleon emission but undergo β decay after transition to the ground state by γ emission. Complete information regarding the level scheme and the spin-parity assignments of these low-lying states of the final nuclei is not available from the spectroscopic data. The best one can do in these circumstances is to sum over the final nuclear spin states resulting from single-particle transitions.

II. MATRIX ELEMENT

The nuclear matrix element for the processes under study can be written as

$$Q=\langle \psi_f(J_f M_f)|\sum_j t_j|\psi_i(J_i M_i)\rangle\,, \tag{1}$$

where t_j represents the single-nucleon photopion production amplitude and the summation extends over all the nucleons in the nucleus. Since we are interested in the transition to the low-lying bound states of the final nucleus, we need consider only the single-nucleon transitions from the outermost shell of the initial nucleus. This amounts to neglecting closed shells and taking the summation of the index j only over the outer-shell nucleons. Since the initial and the final nuclear wave functions are antisymmetric, the matrix element (Eq. 1) can be written as

$$Q=N\langle \psi_f(J_f M_f)|t_N|\psi_i(J_i M_i)\rangle\,, \tag{2}$$

where N is the number of nucleons in the outermost shell. In what follows we suppress the isospin quantum number so that N will denote the number of protons in the present case of positive-pion production. Following Ref. 2, the transition operator

$$t=\sum_{n=0,1}\vec{\sigma}_n\cdot\vec{K}_n e^{i\vec{k}\cdot\vec{r}} \tag{3}$$

can be expanded into spherical tensors of rank λ:

$$t=\sum_{nl\lambda m_\lambda} A_{nl\lambda m_\lambda}\, j_l(kr)[\,Y_l(\hat{r})\times\vec{\sigma}_n\,]_\lambda^{m_\lambda} \tag{4}$$

with

$$A_{nl\lambda m_\lambda}=4\pi(i)^l(-1)^{l+n-\lambda}(-1)^{m_\lambda}[\,Y_l(\hat{k})\times\vec{\mathrm{K}}_n\,]_\lambda^{m_\lambda}\,. \tag{5}$$

The momentum transfer to the nucleon is denoted by $\vec{\mathrm{k}}$. The quantities K_0 and K_1 are the spin-independent and spin-dependent amplitudes of the photoproduction of positive pions from protons. These are taken from the theory of Chew *et al.*[11] The operators σ_1 and σ_0 denote, respectively, the Pauli operator and the unit operator in the spin space of the nucleon.

Using the coefficient of fractional parentage, the wave function of the initial nucleus is expanded into product wave functions of $N-1$ nucleons and the Nth nucleon. In the standard notation,

$$\psi_i(J_iM_i)=\sum_{\alpha'J'}\langle J_a^{N-1}(\alpha'J')J_a;\,\alpha_iJ_i\,]\,J_a^N\alpha_iJ_i\rangle\,\psi\big((J_a)_{1,2,\ldots,N-1}^{N-1}(\alpha'J')(J_a)_N;\,\alpha_iJ_iM_i\big)\,. \tag{6}$$

Since the final nuclear state arises from the single-nucleon transition $J_a\rightarrow J_b$, its antisymmetrized wave function can be constructed by using the operator P_{jN} which exchanges the nucleons with indices j and N:

$$\psi_f(J_fM_f)=\frac{1}{\sqrt{N}}\sum_{\alpha''J''}\left(1-\sum_{j=1}^{N-1}P_{jN}\right)\psi\big((J_a)_{1,2,\ldots,j,\ldots,N-1}^{N-1}(\alpha''J'')(J_b)_N;\,\alpha_fJ_fM_f\big)\,. \tag{7}$$

Substituting Eqs. (4), (6), and (7) into Eq. (2), we obtain after some simplification

$$Q=\sqrt{N}\sum_{nl\lambda m_\lambda}\sum_{\alpha'J'}\langle J_a^{N-1}(\alpha'J')J_a;\,\alpha_iJ_i\,]\,J_a^N\alpha_iJ_i\rangle\,C(J_i\lambda J_f,\,M_i,\,M_f-M_i)(2J_i+1)^{1/2}(2J_b+1)^{1/2}$$
$$\times W(J'J_aJ_f\lambda,J_iJ_b)A_{nl\lambda m_\lambda}\langle j_l(kr)\rangle\langle l_b\tfrac{1}{2}J_b\,\|\,(Y_l(\hat{r})\times\sigma_n)_\lambda\,\|\,l_a\tfrac{1}{2}J_a\rangle\,, \tag{8}$$

where $\langle j_l(kr)\rangle$ is the radial integral defined by

$$\langle j_l(kr)\rangle=\int_{r_0}^{\infty}u_{n_bl_b}^{*}(r)j_l(kr)u_{n_al_a}(r)r^2dr\,. \tag{9}$$

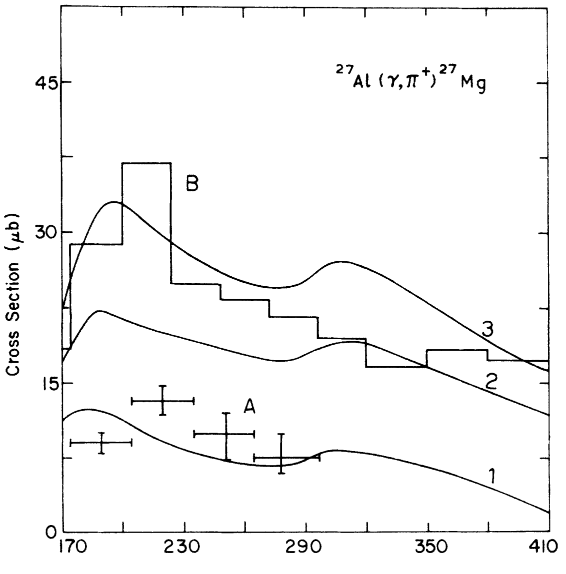

FIG. 1. Cross section for the reaction ^{27}Al(γ,π^+)^{27}Mg. Experimental data A are from Walters and Hummel (Ref. 8) and the histogram B is from Blomqvist *et al.* (Ref. 10).

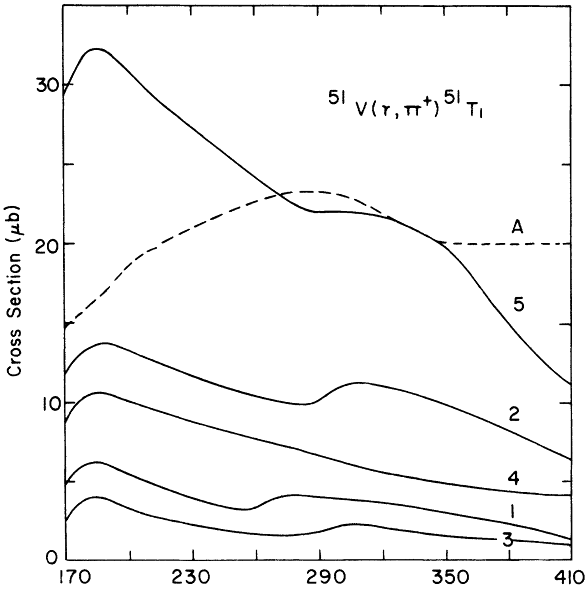

FIG. 2. Cross section for the reaction ^{51}V(γ,π^+)^{51}Ti. Curve A denotes the experimental results of Nydhal and Forkman (Ref. 9).

In Eq. (8) the notation of Rose[12] for angular momentum coefficients and his definition of reduced matrix elements are used. The limits of the radial integral are r_0 and ∞ in the case of the surface-production model. The lower limit r_0 is chosen to correspond to the rms radius of the nucleus.[13] For the radial functions $u_{nl}(r)$ we have used oscillator functions with the length parameter b [14] determined from electron scattering data.

For further details regarding the evaluation of the cross section, the reader is referred to Refs. 2 and 3.

III. NUMERICAL RESULTS

The cross sections for the reaction $^{27}\mathrm{Al}(\gamma, \pi^+)$-$^{27}\mathrm{Mg}$ are plotted in Fig. 1 as a function of the incident photon energy, assuming surface production[15] of pions. The values of the parameters b and r_0 are taken as 1.816 and 2.828 fm, respectively. Curves 1 and 2 represent the cross sections corresponding to the single-nucleon transitions $1d_{5/2} \rightarrow 2s_{1/2}$ and $1d_{5/2} \rightarrow 1d_{3/2}$, respectively. The sum of these two cross sections is represented by curve 3. The experimental results are those of Walters and Hummel[8] and Blomqvist, Nydhal, and Forkman,[10] and they are in disagreement. However, there is a good agreement between our theoretical curve 1 and the experimental results of Walters and Hummel. This may be interpreted to indicate that the low-lying bound states of $^{27}\mathrm{Mg}$, stable against nucleon emission, arise from the single-nucleon transition $1d_{5/2} \rightarrow 2s_{1/2}$. Also, there is a similar agreement between the theoretical curve 3 and the experimental results of Blomqvist *et al.* The latter agreement suggests that the final nuclear states of $^{27}\mathrm{Mg}$ are obtained from both the single-nucleon transitions $1d_{5/2} \rightarrow 2s_{1/2}$ and $1d_{3/2}$. To draw definite conclusions, we must have unambiguous experimental results; an independent experimental check is required to remove the present uncertainty.

Figure 2 represents the theoretical cross sections for the reaction $^{51}\mathrm{V}(\gamma, \pi^+)^{51}\mathrm{Ti}$ assuming surface production of pions. The parameters b and r_0 are assumed to be equal to 2.314 and 3.676 fm, respectively. Since detailed information about the level structure of $^{51}\mathrm{Ti}$ is not available we have computed separately the cross sections for this reaction corresponding to single-particle transitions $1f_{7/2} \rightarrow 2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$. Curves 1, 2, 3, and 4 represent separately the cross sections arising from the individual transitions, and curve 5 is obtained by taking the sum of all these contributions. The theoretical results are compared with the experimental data of Nydhal and Forkman[9] and there is a reasonable agreement in the energy range 260-350 MeV of the incident photon.

It may be observed from Figs. 1 and 2 that the curves show two peaks, one around 200 MeV and the other around the pion-nucleon resonance. This feature is essentially due to the fact that the cross section can approximately be factorized into two parts; the one arising from the nuclear form factor, which is a monotonically decreasing function with the increasing momentum transfer, and the other due to the free-nucleon cross section which exhibits a (3, 3) resonance near 320 MeV. The resultant effect is to smooth out considerably the variation of cross section with incident photon energy and gives rise to two small peaks as observed in Figs. 1 and 2.

IV. CONCLUSIONS

In this article we have studied the reactions $^{27}\mathrm{Al}(\gamma, \pi^+)^{27}\mathrm{Mg}$ and $^{51}\mathrm{V}(\gamma, \pi^+)^{51}\mathrm{Ti}$ for which experimental data are available. The final bound states $^{27}\mathrm{Mg}$ and $^{51}\mathrm{Ti}$ obtained in these reactions are the low-lying states which are stable against nucleon emission and for which information regarding the level sequence and the spin-parity assignments is not available. So we have summed over all the final spin states arising from single-nucleon transition from the outermost shell to the neighboring subshells. The surface-production mechanism is used for the photopions and it is found to be a simple but useful model to account for reabsorption of emitted pions and to obtain agreement with experimental results.

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[14] The oscillator length parameter b is calculated using Eq. (2.69) given in L.R.B. Elton, *Nuclear Sizes* (Oxford U.P., Oxford, England, 1961), p. 22.

[15] The numerical integration is done in steps of 0.1 pion Compton wavelength. For this purpose the lower limit of integration r_0 is taken to correspond to the rms radius, correct to the first decimal in units of pion Compton wavelength, viz., 2.0 units in pion Compton wavelength for ^{27}Al corresponding to $\langle r_0 \rangle = 2.828$ fm and 2.6 units in pion Compton wavelength for ^{51}V corresponding to $\langle r_0 \rangle = 3.676$ fm. The values so chosen are very close to the values obtained using the relations

$$\langle r_0 \rangle = (3/5)^{1/2} R \quad \text{and} \quad R = a A^{1/3},$$

with $a = 1.197$ fm for ^{27}Al and $a = 1.25$ fm for ^{51}V as given in Ref. 13. The corresponding values for $\langle r_0 \rangle$ are 2.782 fm for ^{27}Al and 3.594 fm for ^{51}V. The recent work [H. Uberall, *Electron Scattering from Complex Nuclei*, (Academic, New York, 1971), Vol. 1, p. 315] has shown that $\langle r_0 \rangle$ for ^{27}Al lies between 2.81 and 3.11 fm, and this is also in support of our choice.