Shape of the $\vec{t}\cdot\vec{T}$ Potential in (^{3}He, t) Scattering*

S. I. Hayakawa

Department of Physics, McGill University, Montreal, Canada

(Received 26 October 1972; revised manuscript received 31 April 1973)

Ground-state analog transitions in the (^{3}He,t) scatterings are adequately described by introducing shell effects into both the real and imaginary part of the asymmetry potential.

I. INTRODUCTION

Symmetry dependence of the optical potential for mass-three projectiles has been subject to a number of investigations in recent years. Elastic scattering studies of ^{3}He and tritons indicate some evidence for a $\vec{t}\cdot\vec{T}$ interaction, but the results are rather ambiguous.[1-3] Another way of studying the asymmetry potential is to use quasielastic (^{3}He, t) scatterings leading to the excitation of the ground-state analog of a target nucleus. In this reaction the transition is assumed to proceed via the isospin flip through the $\vec{t}\cdot\vec{T}$ term (Lane potential)[4] in the optical potential. The optical potential is, thus, written in the form of $U(r) = U_0(r) + U_1(r)\,\vec{t}\cdot\vec{T}$, where $U_0(r)$ and $U_1(r)$ are both complex. $\vec{t}$ and $\vec{T}$ are the isospin operators for the projectile and the target nucleus, respectively. Earlier investigations of this reaction on Ti and Ni isotopes and various $N=28$ nuclei[5-7] gave the data which were well described by the combination of Woods-Saxon and its derivative, namely

$$U_1(r) = \frac{V_1}{A}(1+e^x)^{-1} - i\frac{W_1}{A}\frac{d}{dx'}(1+e^{x'})^{-1}, \qquad (1)$$

where the notations are standard.[3] This form of $U_1(r)\,\vec{t}\cdot\vec{T}$ is often termed the macroscopic potential. More recent studies,[8,9] however, showed that this reaction had a strong energy dependence. Furthermore, shapes of the triton angular distributions had distinct features for each nucleus studied at different incident energies. The macroscopic model could not reproduce such nucleus dependence in the experimental angular distributions consistently. Thus, the microscopic model,[10] which accomodates "shell effects" explicitly in the form factor of distorted-wave calculations, was closely examined. However, over-all agreement between the data and the theory was rather poor.[8] Recently, Becchetti, Makofske, and Greenlees[11] made a similar study on Ti and Ni isotopes. Their conclusions were that the data were well described by the microscopic model, even though the range or form of the effective force was not well determined; and the effective interaction was energy-dependent in a manner similar to that observed for optical-model potentials.

It has been reported[12,13] that two-step processes such as (^{3}He $\to \alpha \to t$) were vital in this reaction, particularly in the transitions which required large angular momentum transfers. It is not clear at present how important such two-step processes are in quasielastic scatterings leading to the excitation of ground-state analogs.[14,15] Furthermore, complete analyses, including such two-step processes, as well as single step, require considerable computations. Experimentally, the excitation of ground-state analogs is always the strongest in this reaction and it is worthwhile to explore a simpler description of this transition in the spirit of the optical potential. Recently, we pointed out[16] that the introduction of an imaginary form factor in microscopic distorted-wave calculations produced significant changes in predicted triton angular distributions. One of the implications of that study is that accomodation of shell effects in both the real and the imaginary part of the $\vec{t}\cdot\vec{T}$ potential could lead to a more consistent description of the quasielastic process. In this paper we wish to show that that is indeed the case.

II. EVALUATION OF THE $\vec{t}\cdot\vec{T}$ POTENTIAL

The real part of the $U_1(r)$ for mass-three projectiles can be written as

$$\mathrm{Re}U_1(r) = \frac{1}{4\pi}\int \rho_{xn}(r')V_t(r'r)r'^2\,dr', \qquad (2)$$

where $\rho_{xn}(r')$ is a normalized density distribution of the excess neutrons in the target nucleus which participate in the analog transition. $V_t(r', r)$ is the first term in the radial harmonics of the effective $\vec{t}\cdot\vec{t}'$ interaction between the projectile and a nucleon in the target nucleus. Assuming the simple shell model we can estimate $\rho_{xn}(r)$ for the nuclei of our interest. For ^{50}Cr only the $1f_{7/2}$ orbital contributes in the analog transitions, thus $\rho_{xn}(r) = u^2_{1f7/2}{}^2(r)$ where $u_j(r)$ is the radial part of a single-particle bound-state wave function. Similarly, $\rho_{xn}(r) = u_{1g\,9/2}{}^2(r)$ for ^{90}Zr. In the case of ^{62}Ni all the levels through $1f_{7/2}$ are filled with both neutrons and protons, and $\rho_{xn}(r)$ is assumed to be $\rho_{xn}(r) = 0.61 u_{2p\,3/2}{}^2(r) + 0.39\, u_{1f5/2}{}^2(r)$.[17] The bound-state wave functions $u_j(r)$ were generated from a Woods-Saxon potential with a radius of $1.25A^{1/3}$ fm, diffuseness parameter $a = 0.65$ fm and a spin-orbit strength 25 times the Thomas term. The depth of the nuclear potential was adjusted to give the neutron binding energies of 12.93, 10.60, and 11.997 MeV for ^{50}Cr, ^{62}Ni, and ^{90}Zr, respectively. In order to estimate the effective interaction between the projectile and a nucleon in the target nucleus we follow Madsen's microscopic descriptions.[10] We chose a Gaussian shape, rather than a Yukawa shape, for the effective interaction so that the (^{3}He, t) scatterings could be easily compared with the corresponding (p, n) reactions. There is also an indication[18] that the Gaussian might be a more appropriate form to use in this type of problem. A range parameter of 0.229 fm^{-2}, which was used in the present work, was obtained by assuming a Gaussian wave function for the ^{3}He projectile and a range parameter of 0.46 fm^{-2} for the isovector part of the nucleon-nucleon interactions.[19] The strength of this interaction, V_{ot}, is difficult to evaluate. However, one can get some feeling about this strength by requiring the volume integral of this interaction to be 241 MeV fm^3, which fitted the quasielastic data of ^{48}Ti(p, n)-^{48}V$_{\rm IGS}$ at $E_p = 18.5$ MeV in a microscopic distorted-wave calculation.[5] This estimate gives a value of 4.7 MeV for the strength.

The geometry of the potential thus obtained, is now strongly nucleus-dependent. The radial shapes of the nucleus-dependent potential, thus obtained, are shown as Re$U_1(r)$ in Fig. 1. A value of 3.2 MeV instead of 4.7 MeV was used for V_{ot} (see discussions below). They are compared with a usual Woods-Saxon shape for ^{62}Ni. The geometries of the Woods-Saxon shape used here are $V_1 = 54$ MeV, $r_0 = 1.14$ fm, and $a_0 = 0.712$ fm.[6] Because of the striking differences between the macroscopic and nucleus-dependent potentials, one would expect that a clear choice could be made between the two potentials by analyzing data. However, past experiences[5-9] show that both potentials underestimate cross sections by a factor of 3 to 5 or more, for the data of $E_{^3\rm He} \leq 37.5$ MeV. In macroscopic models it was therefore found necessary to introduce a surface-peaked imaginary part with a strength usually being set equal to the real strength, while the other geometries are the same as those of the imaginary part of the ^{3}He optical potential. There are two interesting features which emerged through the application of this macroscopic model. One is that the strength needed to reproduce experimental yields has a strong energy dependence, but it decreases monotonically with the incident energy. Another feature is that the shapes of the predicted angular distributions have no strong nucleus dependence, which is of course expected. The first feature implies that the mechanism of this reaction could be quite complex. And the second feature is in contrast with the experimental data which showed rather strong nucleus dependence in the shapes of triton angular distributions at different incident energies.[8,9] Even from a phenomenological point of view, the macroscopic model as it is presently used, is quite inadequate. In the present approach, the estimation of the real part produced strong nucleus dependence. It should be pointed out that this real potential is equivalent to the form factor of the microscopic model in distorted-wave calculations. Therefore, the problem of underestimating cross sections still remains. One of the ways to solve this problem is to renormalize the effective projectile-nucleon (in the target nucleus) interactions by fitting the experimental data. If one does this, the strength of the effective interaction becomes strongly energy-dependent. The ratios of the strengths for the ^{62}Ni target are typically 1:0.7:0.3 at $E_{^3\rm He} = 21.4$, 37.5, and 70 MeV, respectively.[8,9] In addition the shapes and magnitudes of the triton angular distributions are strongly dependent upon the optical parameters used for the ^{3}He and t channels as well as the geometries of the effective interactions. These

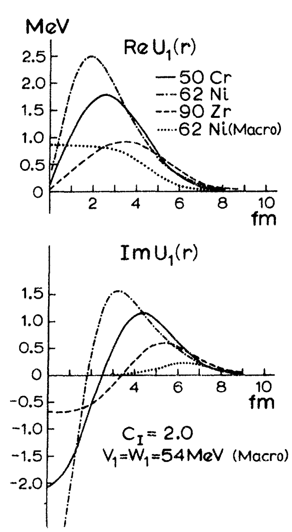

FIG. 1. Shapes of nucleus-dependent potential for ^{50}Cr, ^{62}Ni, and ^{90}Zr. $V_{ot} = 3.2$ MeV and $C_I = 2.0$ was used for all the nuclei. A macroscopic potential for ^{62}Ni is shown with a dotted line for comparison.

problems were well studied for the Ti and Ni isotopes by Becchetti, Makofske, and Greenlees.[11]

In the microscopic model it was conceptually a little difficult to introduce an imaginary part in the form factor. However, in the spirit of the optical potential, the asymmetry potential is expected to be complex almost by definition. Since this asymmetry results from the optical potential differences between the $T_>$ and the $T_<$ channels, and both channels are open in our case. We are indebted to D. Robson for this consideration. So, we are left with a problem of estimating the corresponding imaginary part. A realistic estimation of this imaginary part starting from the basic beginning of the optical model[20] is extremely difficult in our case. However, we can take advantage of the past experiences in phenomenological treatment of the optical potential. For low-energy proton elastic scattering (9 MeV $\leq E_p \leq$ 22 MeV) Perey[21] showed that the data could be well described by restricting the optical parameters to six, where he assumed a real volume and a surface imaginary with $R_R = R_I = R_{so}$ and $a_R = a_{so}$. A more recent study of nuclear-matter radii from a reformulated optical model by Greenlees, Pyle, and Tang[22] also confirms Perey's finding for $E_p \leq$ 20 MeV. The point is that the surface imaginary dominates the imaginary part of the optical potential for low-energy nucleon scatterings. What is particularly significant is that in Perey's study when he restricted the radius and diffuseness parameters to be identical for both the real and the imaginary part the results were almost indentical to those obtained from his standard six parameters. Slight deviations occurred at backward angles ($\theta_{c.m.} \geq 120°$) where cross sections were small. This implies that for the first approximation a derivative of the real part can be used as the shape of the imaginary part. Since we estimated the real part by folding nucleon-nucleon interactions their findings are particularly relevant to the present work. If we extend this method to composite projectiles, we would also expect similar features in the optical potentials of ^{3}He and tritons. Namely, elastic scattering should be well described by a potential of a real volume and a surface-imaginary shape with the same geometrical parameters. However, it is not clear at present how good such an extrapolation is for the mass-three projectiles. There is a considerable amount of elastic data for the mass-three projectiles; but they are relatively incomplete in comparison with those for protons and neutrons. For example, there are not many accurate measurements of the total reaction cross sections in conjuction with the elastic data; and, polarization data at off resonances which are relevant to the spin-orbit force are practically nonexistent, even though one study[23] indicated that the spin-orbit force for ^{3}He was weak (one third of the protons at best). There are well known ambiguities found in the optical-model analyses of the mass-three projectiles. The strength of the real well varies discretely from about 130 to 170 MeV.[24] The shape of the imaginary part is not well determined. Nevertheless, the data are well fitted in general by assuming Woods-Saxon shapes for both the real and the imaginary part of the potential, where the radius and diffuseness of the imaginary part are usually 10 ~ 30% larger than those of the real part. However, in a recent study of energy dependence of the ^{3}He optical potential Marchese, Clarke, and Griffiths[25] found interesting features which are relevant to the present study. They classified the ambiguities of the real part in terms of the volume integral per particle pair, which is the same approach used by Greenlees, Pyle, and Tang[22] for protons. So that they could compare their results to the corresponding proton results. A real volume and surface imaginary shape was chosen in their study. In brief, their results showed that the volume integral per nucleon pair was independent of the target nuclei, and the energy dependence of one of the optical potential sets which had an average volume integral of 440 MeV fm^3 per pair was very similar to that expected from the proton results[26] for $E_{^3\mathrm{He}} \geq 35$ MeV. For $E_{^3\mathrm{He}} < 35$ MeV elastic cross sections were only sensitive to the extreme tail of the potential, so that they could not determine the full interaction potential. The geometries used for the potential were $r_R = 1.092$, $a_R = 0.753$ and $r_D = 1.17$, $a_D = 0.877$ for the real and the imaginary part, respectively. It is not known whether or not similar systematics emerge if the radius and diffuseness parameters are restricted to be the same for both the real and the imaginary part. However, their results certainly give some credence to the folding method in estimating the real part as well as to the contention that a derivative of the real part could be used as the shape of the imaginary part for the first approximation.

Thus, we introduce a phonomenological imaginary part, which is related to the real part of the potential in the simple form

$$\mathrm{Im}U_1(r) = -C_I \frac{\partial \mathrm{Re}U_1(r)}{\partial r} , \tag{3}$$

where C_I is a constant, which can be empirically determined by fitting the experimental data. The radial shapes of this imaginary part obtained from Eq. (3) are shown in the lower part of Fig. 1. A value of $C_I = 2.0$ was used for the figure. The asymmetry potential is now strongly nucleus-dependent in both the real and the imaginary part.

Thus, the N-Z dependence of the Lane potential is not expected to be maintained.[27] It can be easily seen in the figure that most strength of the nucleus-dependent potential is localized inside the nuclear radius. On the other hand, the imaginary part of macroscopic potential which dominates the transitions is localized in the outer surface region due to the geometries used here. But these geometries ($r_0 = 1.143$, $a_0 = 0.712$, $r_0' = 1.60$, and $a_0' = 0.829$) have been shown to give a good description of the Ni data at 37.7 MeV.[6] It is important to note, however, that tails of both imaginary parts for ^{62}Ni have very similar slopes and these slopes are very critical for the transitions at lower incident energies ($E_{^3\mathrm{He}} < 40$ MeV). Triton angular distributions from the nucleus-dependent potential were obtained by using the distorted-wave computer code DWUCK .[28]

Some of the features of this potential found in the present study are: (i) the imaginary part dominates the transition ampliplitudes for lower incident energies ($E_{^3\mathrm{He}} < 40$ MeV); (ii) for $E_{^3\mathrm{He}} =$ 70 MeV the effect of the imaginary part is considerably reduced; (iii) the upper limit of the strength of the effective projectile-nucleon (in the nucleus) interaction, V_{ot} is estimated to be 3.8 MeV for the range parameter of 0.229 fm^{-2}. This value of 3.8 MeV was obtained by fitting the first maximum of the ^{62}Ni data at 70 MeV with $C_I = 0$. The predicted yield at this maximum was rather insensitive to the choice of the optical parameters used for the entrance and exit channels. On the other hand, the shapes of the angular distributions at higher angles strongly depended on optical parameters. The final value of 3.2 MeV for V_{ot}, which was used for Fig. 1, was obtained by optimizing the fits to the 70-MeV data with $C_I = 0.4$ and the Bechetti-Greenlees optical parameters[2] which were extended to this energy. For the data at lower energies[8] C_I was chosen to reproduce the experimental absolute yields while V_{ot} was fixed to be 3.2 MeV for all the calculations. The results are shown in solid lines in Fig. 2. Predicted triton angular distributions in forward angles ($\theta_{\mathrm{c.m.}} \leq 40°$) were found to be rather insensitive to the choice of optical parameters at these low energies except for the ^{90}Zr. In the case of ^{90}Zr predicted triton yields could vary by a factor of 2 in some extreme cases depending upon optical parameter sets used, even though shapes of the angular distributions were very similar. The fits for these low-energy data were obtained by using optical parameters of Urone *et al.*.[29] Even though the Bechetti-Greenlees parameters[2] yielded similar results, their fits were slightly inferior to those shown in the figure. The corresponding fits from the macroscopic model are shown in dashed lines in the figure. The geometries of the form factors were the same as those of the ^{3}He optical potentials used for the entrance channels. The strengths of the real and the imaginary part were set to be equal. They were adjusted such that the triton angular distributions in low-angle regions gave the same magnitudes as those of the nucleus-dependent potential. The strengths, W_1 are also shown in the figure.

III. DISCUSSIONS AND SUMMARY

In the previous section we have seen that a reasonable description of the data, which has strong nucleus dependence, can be achieved by introducing shell effects into both the real and the imaginary part of the asymmetry potential. However, a strong energy dependence found in the

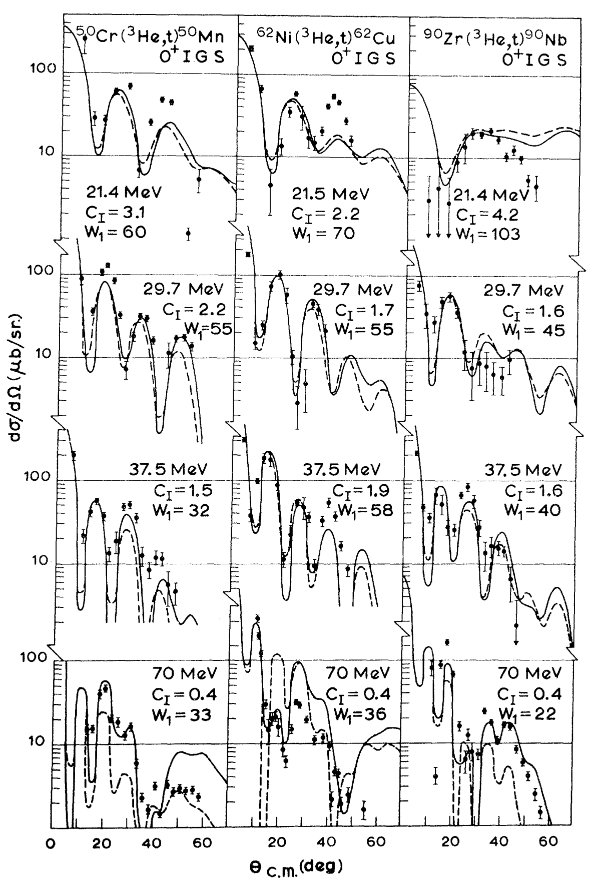

FIG. 2. Solid lines are distorted-wave Born-approximation (DWBA) fits with nucleus-dependent potentials. The experimental data used here are from Refs. 8 and 9. DWBA calculations with the macroscopic potentials are shown with dashed lines for comparison. $V_1 = W_1$ was assumed and values of W_1 shown in the figure are in MeV.

present work is very similar to the one found by using the macroscopic descriptions.[8,9] Namely, the magnitude of C_I, which was needed to reproduce the experimental absolute yields decreases as the incident energy increases. What is significant is the result at 70 MeV, where the imaginary part with $C_I = 0.4$ contributes only about 30% to the yields. This implies that the analog transitions proceed primarily via a single step. At this point one may question the validity of the effective projectile-nucleon (in target nucleus) interactions deduced from the 70-MeV data.[9] In this regard we should point out that our estimate of the effective projectile-nucleon interaction is in reasonable agreement with an estimate of the effective nucleon-nucleon interactions in the corresponding (p, n) reaction. As mentioned in the previous section a volume integral of 241 MeV fm^3 was needed to fit ^{48}Ti(p, n) ^{48}V$_{IGS}$ data at $E_p = 18.5$ MeV.[5] Further, as Satchler[30] recently pointed out, the effective nucleon-nucleon interactions could be considered to be complex, and the (p, n) reaction seems to be getting more complicated as days go by.[31] Therefore, this value of 241 MeV fm^3 should be regarded as an upper limit of the effective (real) interaction. Our values of V_{ot} = 3.2 MeV and a range parameter of 0.229 fm^{-2} give a volume integral of 163 MeV fm^3.

For the cases of lower incident energies ($E_{^3He} \leq$ 37.5 MeV), the imaginary part dominates the transition amplitudes. For $E_{^3He} = 37.5$ MeV the real part contributes less than 20% of the total yield; however, detailed shapes of angular distribution in higher angles ($\theta_{c.m.} > 20°$) depend on the relative strengths and the slopes of the two tails (cf. Fig. 1). Particularly, the ratios of the first two maxima in the angular distributions were dependent on the choice of C_I. However, when C_I was chosen to reproduce the experimental absolute yields, shapes of predicted angular distributions tended to be near optimum. During this study we obtained better fits than those shown in Fig. 2 by choosing different combinations of optical parameters at different energies, or shifting the locations of the imaginary parts, or completely disregarding the normalizations. But no significant systematics were obtained from such efforts. What is interesting is that values of C_I, as shown in Fig. 2, do not appreciably vary from nucleus to nucleus for a given incident energy. And an over-all trend is that C_I decreases as the incident energy increases. Recent studies[12,13,15,31,32] indicate that two-step processes such as (^{3}He$\rightarrow \alpha \rightarrow t$) and (^{3}He$\rightarrow d \rightarrow t$) could be vital in the (^{3}He, t) reaction at lower incident energies. Such a contention seems to be consistent with the present result that the imaginary part of the potential strongly contributes to the analog transitions. There is, in fact, an indication[33] that such two-step processes could be accomodated in distorted-wave treatment by an effective imaginary form factor.

Shapes of the triton angular distributions obtained from the present method are not so much different from those of the macroscopic model (cf. Fig. 2). However, as discussed in previous studies,[8,9] the macroscopic model gives predictable shapes of the angular distributions, regardless of the target nucleus for $E_{^3He} < 40$ MeV. The only remedy to accomodate the nucleus-dependent effects seen in the data is to change the ratio of the real and the imaginary strengths, V_1/W_1. For example, if both the strengths are set to be equal, the macroscopic model calculations cannot produce the first two maxima of the angular distributions with the same magnitude, which is the experimental case for the ^{50}Cr and ^{90}Zr at 37.5 MeV. This peak ratio is quite independent of the optical parameters used for the ^{3}He and t channels. The ratios, V_1/W_1 required to fit the data range from one for the ^{62}Ni to nine for the ^{90}Zr at 37.5 MeV.[8,9] Since the imaginary part still dominates the yields for $E_{^3He} < 40$ MeV, this leads to an unphysical situation where the real strength of the $\vec{t}\cdot\vec{T}$ potential varies from about 50 to 300 MeV depending upon the target nucleus. In the present method, shapes are somewhat more sensitive to the optical parameters used. For an example, the first two maxima of the ^{50}Cr data at 37.5 MeV can be perfectly fitted by using the ^{3}He parameters by Gibson *et al.*[34] for the ^{3}He channel and the same parameters for the triton channel with a slight asymmetry adjustment. Therefore, caution must be taken in evaluating the details of the shapes obtained from the present method. Nevertheless, the over-all trend is such that the shapes from the nucleus-dependent potential are consistently better than those from the usual macroscopic potential. The fact that the values of C_I do not significantly change from nucleus to nucleus for a given incident energy may seem rather surprising in view of the complexities of the (^{3}He, t) reaction mechanisms discussed elsewhere.[35] However, this fact seems to support *a posteriori* our contention that the quasielastic process can be described in a simple manner by introducing the shell effects into both the real and the imaginary part of the $\vec{t}\cdot\vec{T}$ potential.

It is unfortunate that the real part of the asymmetry potential is masked by the imaginary part, due to the reaction mechanisms at lower incident energies. However, as is shown in the previous section the real part of the $\vec{t}\cdot\vec{T}$ potential is strongly nucleus-dependent and cannot possibly be accomodated by a Woods-Saxon shape or its derivative from. The significance of shell effects on isospin splitting has been recently pointed out by Philpott.[36] It is plausible that such shell effects are pronounced in the energy dependence of the (^{3}He, t) scatterings, since the tail, rather than the total strength of the interaction is very critical for this reaction. Thus, the present result strongly suggests that the shell effects in the asymmetry potential deserve more elaborate studies.

ACKNOWLEDGMENT

The author wishes to thank Dr. R. A. Hinrichs and Dr. D. L. Show for making their data available prior to publication. Dr. D. Robson is acknowledged for interesting conversations. Miss L. Buja-Bijunas is thanked for her valuable assistance in the study.

*Work supported in part by the Atomic Energy Control Board of Canada.

[1]R. M. Drisco, P. G. Roos, and R. H. Bassel, in *Proceedings of the International Conference on Nuclear Structure, Tokyo, Japan, 7–13 September 1967;* G. R. Satchler, in *Isospin in Nuclear Physics,* edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).

[2]F. D. Becchetti, Jr., and G. W. Greenlees, in *Polarization Phenomena in Nuclear Reactions,* edited by H. H. Barschall and W. Haeberli, (Univ. of Wisconsin Press, Madison, 1970).

[3]P. P. Urone, L. W. Put, H. H. Chang, and B. W. Ridley, Nucl. Phys. A163, 225 (1971).

[4]A. M. Lane, Phys. Rev. Lett. 8, 171 (1962).

[5]J. J. Wesolowski, E. H. Schwartz, P. G. Roos, and C. A. Ludemann, Phys. Rev. 169, 878 (1968).

[6]P. D. Kunz, E. Rost, G. D. Jones, R. R. Johnson, and S. I. Hayakawa, Phys. Rev. 185, 1528 (1969).

[7]G. Bruge, A. Bussiere, H. Faraggi, P. Kossanyi-Demay, J. M. Loiseaux, P. Roussel, and L. Valentin, Nucl. Phys. A129, 417 (1969).

[8]W. L. Fadner, J. J. Kraushaar, and S. I. Hayakawa, Phys. Rev. C 5, 859 (1972).

[9]R. A. Hinrichs and D. L. Show, Phys. Rev. C 6, 1257 (1972).

[10]V. A. Madsen, Nucl. Phys. 80, 177 (1966).

[11]F. D. Becchetti, W. Makofske, and G. W. Greenlees, Nucl. Phys. A190, 437 (1972).

[12]M. Toyama, Phys. Lett. 38B, 147 (1972).

[13]R. Schaeffer and G. F. Bertsch, Phys. Lett. 38B, 159 (1972).

[14]P. D. Kunz and E. Rost, Bull. Am. Phys. Soc. 17, 509 (1972).

[15]W. R. Coker, T. Udagawa, and H. H. Wolter, Phys. Rev. C 7, 1154 (1973).

[16]W. L. Fadner, J. J. Kraushaar, and S. I. Hayakawa, Phys. Lett. 40B, 49 (1972).

[17]S. Cohen, R. D. Lawson, M. H. MacFarlane, S. P. Pandya, and M. Soga, Phys. Rev. 160, 903 (1967).

[18]G. W. Greenlees, W. Makofske, and G. J. Phle, Phys. Rev. C 7, 1145 (1970).

[19]Y. C. Tang, E. Schmid, and K. Wildermuth, Phys. Rev. 131, 2631 (1963).

[20]H. Feshbach, C. F. Porter, and V. F. Weisskopf, Phys. Rev. 96, 448 (1954).

[21]F. G. Perey, Phys. Rev. 131, 745 (1963).

[22]G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. 171, 1115 (1968).

[23]R. L. Hutson, S. Hayakawa, M. Chabre, J. J. Kraushaar, B. W. Ridley, and E. T. Boschitz, Phys. Lett. 27B, 153 (1968).

[24]See, for examples, Refs. 3 and 25 and the references cited therein.

[25]C. J. Marchese, N. M. Clarke, and R. J. Griffiths, Phys. Rev. Lett. 29, 660 (1972).

[26]F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[27]A significant deviation from the N-Z rule was observed in ^{64}Ni. See Ref. 6.

[28]The author wishes to thank Dr. P. D. Kunz for his advice concerning the code DWUCK.

[29]See Table I in Ref. 3.

[30]G. R. Satchler, Phys. Lett. 35B, 279 (1971).

[31]See recent studies by various groups, namely R. F. Bently and C. D. Zafiratos, Bull. Am. Phys. Soc. 17, 903 (1972); L. D. Rickertsen and P. D. Kunz, *ibid.* 17, 903 (1972); W. R. Coker, T. Udagawa, and H. H. Wolter, *ibid.* 17, 903 (1972).

[32]N. DeTakacsy, Phys. Lett. 42B, 1 (1972).

[33]L. Rickertsen and P. D. Kunz, Nuclear Physics Laboratory, University of Colorado Technical Progress Report, 1972 (unpublished).

[34]E. F. Gibson, B. W. Ridley, J. J. Kraushaar, M. E. Rickey, and R. H. Bassel, Phys. Rev. 155, 1194 (1967).

[35]See Ref. 15 and the references cited therein.

[36]R. J. Philpott, Nucl. Phys. A179, 113 (1972).