PHYSICAL REVIEW C VOLUME 8, NUMBER 5 NOVEMBER 1973

Meson-Exchange Effects in Neutron-Proton Bremsstrahlung*

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(Received 9 July 1973)

A prescription for including exchange effects in neutron-proton bremsstrahlung ($np\gamma$) by requiring gauge invariance of the complete $np\gamma$ amplitude is given entirely within a potential framework. To lowest order in the photon momentum (K^0) this prescription is unambiguous and it includes, as well as exchange effects, any other nonlocal effects in the nuclear potential. The lowest-order contribution is written, with the use of the Schrödinger equation, so as to eliminate the necessity of integrating over the nuclear potential. The higher-order exchange contributions are treated through order K^2 using the one-pion exchange (OPE) only. (Because the effective expansion is K/μ, where μ is the mass of the exchanged meson, the pion is expected to dominate.) These higher-order terms are the same as the Feynman-diagram prescription would give for $np\gamma$ due to OPE. Calculations with the Hamada-Johnston (HJ) and Bryan-Scott III (BS) potentials are compared to the experimental results at 208 MeV by Brady *et al.* and at 130 MeV by Edgington *et al.* This inclusion of the exchange bremsstrahlung (order K^0) increases the cross section by roughly a factor of 2, providing generally good agreement with both experiments. An estimate of the order-K^2 terms indicates that they contribute $< 2\%$; possible implications of this result on order-K contributions are discussed. Contributions arising from nonlocal terms other than exchange such as momentum-dependent and spin-orbit effects amount to about 1%. There is little difference between the $np\gamma$ predictions for the HJ and BS potentials. Our $np\gamma$ coplanar results for the HJ potential are compared to those obtained in a calculation, which uses the low-energy theorem for internal radiation, by Celenza *et al.*

[NUCLEAR REACTIONS Neutron-proton bremsstrahlung ($np\gamma$), $E=130$, 200 MeV; calculated coplanar $\sigma(\theta_n,\theta_p,\theta_\gamma)$, $\sigma(\theta_n,\theta_p)$ including meson-exchange contributions, comparison to experiment. Developed higher-order corrections arising from OPE.]

I. INTRODUCTION

Neutron-proton bremsstrahlung ($np\gamma$) is of interest as a possible means of using off-shell behavior to distinguish among various nucleon-nucleon potentials and also as a means for testing, in the simple two-nucleon system, theoretical models for exchange effects in photonuclear reactions. For proton-proton bremsstrahlung ($pp\gamma$), which is easier to measure experimentally, the exchange effects are mostly absent and fairly good agreement with experiment has been obtained[1] with either of two potentials, the hard-core Hamada-Johnston[2] (HJ) and the momentum-dependent, meson-exchange potential of Bryan and Scott III[3] (BS).

In earlier $np\gamma$ calculations,[4] without exchange effects, there was little difference between the HJ and BS potentials and, for each, the theoretical predictions were considerably smaller than experiment.[5] In the present work[6] we include the exchange effects appropriate to the nuclear potential (HJ or BS) used to fit the elastic np phase shifts. We find that inclusion of these exchange effects greatly increases the predicted $np\gamma$ cross section and gives generally good agreement with both the experiment of Brady and Young[5] and the recent experimental results of Edgington *et al.*,[7] although some disagreement still persists at some angles.

We still find little difference between the $np\gamma$ predictions for the HJ and BS potentials for a wide range of np opening angles. In fact, there seems to be even less dependence on the nuclear potential in $np\gamma$ than was found in $pp\gamma$. This is discouraging from the point of view of using bremsstrahlung to distinguish between potentials but, on the other hand, encouraging in that it indicates that potentials might constitute an appropriate way to calculate some off-energy-shell effects.

We formulate (in Sec. II) a prescription for including exchange effects in $np\gamma$ by requiring gauge invariance of the complete $np\gamma$ amplitude. We do this by making the usual minimal electromagnetic replacement ($\vec{\nabla}\to\vec{\nabla}-ie\vec{A}$) in the momentum representation of the nuclear potential as well as in the kinetic-energy part of the Hamiltonian. This procedure is unambiguous to lowest order (K^0) in the photon momentum K and includes, as well as exchange effects, any other effects (to order K^0) due to nonlocality (including explicit momentum dependence, or spin-orbit terms)[8] of the nuclear potential. To this lowest order, all the exchange and nonlocal contributions are given quite simply in terms of the original nuclear potential. Although we use the momentum representation in our derivation, we show that our lowest-order result should apply even to phenomenological potentials for which a momentum representation may not exist. To this order in K, our exchange contribution is the same as has been derived by previous authors.[9]

Beyond lowest order in K, imposition of gauge invariance alone does not unambiguously define the exchange contribution.[10] We discuss this in Appendix B and develop a reasonable prescription for generating the exchange contribution for higher order in K from the explicit momentum representation of the potential. The prescription, given entirely within a potential framework, leads to the same exchange contribution (to all orders in K) for one pion exchange (OPE) as does the application of field theory[11] to describe the pion. Going to higher order in K requires an explicit momentum representation of the potential, which either does not exist or is impractical to calculate for most phenomenological potentials (such as HJ). But we indicate that OPE (which is a common feature of almost all reasonable potentials), with an explicit momentum representation, should dominate these higher orders (although all exchanges are important to lowest order) because of the pion's lighter mass (or, equivalently, the longer range of OPE).

The various contributions to $np\gamma$ can be categorized as follows: (1) The electric pole (EP) terms arising from the minimal electromagnetic substitution in the kinetic-energy part of the Hamiltonian. (2) The magnetic pole (MP) terms arising from the physical (Dirac + anomalous) magnetic moments of the nucleon. These pole contributions (EP) and (MP) taken together are usually called the external radiation (EXT). Then, there are a number of so called internal radiation (INT) terms: (3) The so called (RES) rescattering term arising from the above electric and magnetic terms acting between successive nuclear scatterings. (4) Exchange (EXCH) contributions arising from an explicit exchange part of the nuclear potential. (5) Other contributions (NL) arising from nonlocality (or explicit momentum dependence) of the nuclear potential.

In previous $np\gamma$ work EP, MP, and the internal radiation RES, all to all orders in K, were included. In the present work[6] we include calculations for the rest of the internal radiation EXCH + NL to order K^0. In Appendix B we expand the OPE exchange contribution to order K^1 and K^2. We have explicitly calculated those order-K^2 terms that can be incorporated simply into our present calculation and find they have little effect. The possible relative importance of the order-K^1 and order-K^2 terms is discussed in Appendix B.

To order K^0 we show that the Schrödinger equation can be used to eliminate the nuclear potential from the (EXCH + NL) contribution so that both effects can be included quite simply. We have also made some calculations of EXCH only, using the explicit exchange part of the potential and find that leaving NL out of the momentum-dependent BS potential has very little effect.

In Sec. II, we give the theoretical background for our calculations. Also displayed in Sec. II is the difference between our treatment of the internal radiation scattering and that of Celenza, Gibson, Liou, and Sobel (CGLS).[12] This comparison is included because CGLS use the HJ potential and differs from the present calculation only in the treatment of the internal radiation. Earlier calculations[13] of $np\gamma$ using different methods or approximations have been discussed elsewhere.[4]

In Sec. III, our calculational procedure is described. Section IV contains our results for the HJ and BS potentials as compared to experiment[5,7] and the CGLS results.

In Sec. V, we summarize our conclusions. In Appendix A, the connection between nonlocality and momentum dependence of potentials is illustrated. Appendix B extends the theoretical treatment of Sec. II to higher order in K.

II. THEORETICAL BACKGROUND

We start with a completely general nonlocal potential[8] $V(\vec{r}_i)\equiv V(\vec{r}_1^{\,\prime},\,\vec{r}_2^{\,\prime},\,\vec{r}_1,\,\vec{r}_2)$ which we will relate to an equivalent form in terms of the differential operators $\vec{\nabla}_i$. Any matrix element of the potential is given by

$$M_{\beta\alpha}=\int d\vec{r}_i\,\psi_\beta^*(\vec{r}_1^{\,\prime},\vec{r}_2^{\,\prime})V(\vec{r}_i)\psi_\alpha(\vec{r}_1,\,\vec{r}_2). \qquad (2.1)$$

We expand the wave functions in terms of plane waves to give

$$M_{\beta\alpha}=(2\pi)^{-6}\int d\vec{r}_i\,d\vec{p}_i\,e^{-i(\vec{p}_1^{\,\prime}\cdot\vec{r}_1^{\,\prime}+\vec{p}_2^{\,\prime}\cdot\vec{r}_2^{\,\prime})}\phi_\beta^*(\vec{p}_1^{\,\prime},\,\vec{p}_2^{\,\prime})\times V(\vec{r}_i)e^{i(\vec{p}_1\cdot\vec{r}_1+\vec{p}_2\cdot\vec{r}_2)}\phi_\alpha(\vec{p}_1,\,\vec{p}_2) \qquad (2.2)$$

$$=\int d\vec{p}_i\,\phi_\beta^*(\vec{p}_1^{\,\prime},\,\vec{p}_2^{\,\prime})U(\vec{p}_i)\phi_\alpha(\vec{p}_1,\,\vec{p}_2), \qquad (2.3)$$

where

$$U(\vec{p}_i) = (2\pi)^{-6}\int d\vec{r}_i\, e^{-i(\vec{p}_1'\cdot\vec{r}_1' + \vec{p}_2'\cdot\vec{r}_2')} \times V(\vec{r}_i) e^{i(\vec{p}_1\cdot\vec{r}_1 + \vec{p}_2\cdot\vec{r}_2)} \tag{2.4}$$

is the potential in the momentum representation.

Now we reintroduce the original spatial wave functions so that

$$M_{\beta\alpha} = (2\pi)^{-6}\int d\vec{p}_i d\vec{r}_i\, \psi_\beta^*(\vec{r}_1', \vec{r}_2') e^{i(\vec{p}_1'\cdot\vec{r}_1' + \vec{p}_2'\cdot\vec{r}_2')} \times U(\vec{p}_i) e^{-i(\vec{p}_1\cdot\vec{r}_1 + \vec{p}_2\cdot\vec{r}_2)} \psi_\alpha(\vec{r}_1, \vec{r}_2)\,, \tag{2.5}$$

which can be written as

$$M_{\beta\alpha} = (2\pi)^{-6}\int d\vec{p}_i d\vec{r}_i \psi_\beta^*(\vec{r}_1', \vec{r}_2')\psi_\alpha(\vec{r}_1, \vec{r}_2)\, U(\vec{\nabla}_1'/i, \vec{\nabla}_2'/i, -\vec{\nabla}_1/i, -\vec{\nabla}_2/i) e^{i(\vec{p}_1'\cdot\vec{r}_1' + \vec{p}_2'\cdot\vec{r}_2' - \vec{p}_1\cdot\vec{r}_1 - \vec{p}_2\cdot\vec{r}_2)}\,. \tag{2.6}$$

We integrate by parts repeatedly so that

$$M_{\beta\alpha} = (2\pi)^{-6}\int d\vec{p}_i\, d\vec{r}_i\, e^{i(\vec{p}_1'\cdot\vec{r}_1' + \vec{p}_2'\cdot\vec{r}_2' - \vec{p}_1\cdot\vec{r}_1 - \vec{p}_2\cdot\vec{r}_2)} U(-\vec{\nabla}_1'/i, -\vec{\nabla}_2'/i, \vec{\nabla}_1/i, \vec{\nabla}_2/i)\psi_\beta^*(\vec{r}_1', \vec{r}_2')\psi_\alpha(\vec{r}_1, \vec{r}_2)\,. \tag{2.7}$$

Any surface terms from the $\vec{r}_i$ integration in Eq. (2.7) will go out in the integration over $\vec{p}_i$. We can now perform the $\vec{p}_i$ integrations leading to δ functions, so that the $\vec{r}_i$ integrations can be done, resulting in

$$M_{\beta\alpha} = \lim_{r_i\to 0} U(-\vec{\nabla}_1'/i, -\vec{\nabla}_2'/i, \vec{\nabla}_1/i, \vec{\nabla}_2/i)\psi_\beta^*(\vec{r}_1', \vec{r}_2')\psi_\alpha(\vec{r}_1, \vec{r}_2)\,. \tag{2.8}$$

Equation (2.8) shows that the nuclear potential can be represented by its corresponding momentum representation with the momenta replaced by the appropriate differential operators. Now we can introduce the vector potential into Eq. (2.7) or (2.8) by the usual gauge-invariant replacement

$$\vec{\nabla}_1 \to \vec{\nabla}_1 - iq_1\vec{A}(\vec{r}_1), \tag{2.9}$$

$$\vec{\nabla}_1' \to \vec{\nabla}_1' + iq_1'\vec{A}(\vec{r}_1'), \tag{2.9$'$}$$

and similarly for particle 2. Here q_i is the charge of particle i.

We now rework our steps to go back to define an induced electromagnetic potential $V_{\rm em}^{(2)}(\vec{r}_i)$. Making the replacement (2.9) in Eq. (2.7) and working backwards, we see that the nuclear potential $V_n(\vec{r}_i)$ gets replaced by

$$V_{{\rm em}+n}(\vec{r}_i) = (2\pi)^{-6}\int d\vec{p}_i U_n[\vec{\nabla}_1'/i - q_1'\vec{A}(\vec{r}_1'), \vec{\nabla}_2'/i - q_2'\vec{A}(\vec{r}_2'), -\vec{\nabla}_1/i - q_1\vec{A}(\vec{r}_1), -\vec{\nabla}_2/i - q_2\vec{A}(\vec{r}_2)] \times e^{i(\vec{p}_1'\cdot\vec{r}_1' + \vec{p}_2'\cdot\vec{r}_2' - \vec{p}_1\cdot\vec{r}_1 - \vec{p}_2\cdot\vec{r}_2)}\,. \tag{2.10}$$

The induced electromagnetic potential is then given by

$$V_{\rm em}^{(2)}(\vec{r}_i) = V_{{\rm em}+n}(\vec{r}_i) - V_n(\vec{r}_i)\,. \tag{2.11}$$

The radiation vector potential is given by

$$\vec{A}(\vec{r}) = (2\pi/K)^{1/2}\hat{\epsilon}_{K,\lambda} e^{-i\vec{K}\cdot\vec{r}} a_{K,\lambda}^{+} + \text{H.c.} \tag{2.12}$$

for the emission or absorption of a photon of momentum $\vec{K}$ and polarization λ. Because $\vec{\nabla}$ and $\vec{A}(\vec{r})$ do not commute, we cannot simply make the replacement $\vec{\nabla}\to i\vec{p}$ in Eq. (2.10). However to zeroth order in K in the exponential $e^{-i\vec{K}\cdot\vec{r}}$, $\vec{\nabla}$ and $\vec{A}(\vec{r})$ do commute. We continue this derivation here to zeroth order in K and, in Appendix B, we investigate the contribution of higher order terms in K for a specific case (pion exchange). To this order we can write

$$V_{\rm em}^{(2)}(\vec{r}_i) = (2\pi)^{-6}\int d\vec{p}_i[U_n(\vec{p}_1' - q_1'\vec{A}, \vec{p}_2' - q_2'\vec{A}, \vec{p}_1 - q_1\vec{A}, \vec{p}_2 - q_2\vec{A}) - U_n(\vec{p}_i)] e^{i(\vec{p}_1'\cdot\vec{r}_1' + \vec{p}_2'\cdot\vec{r}_2' - \vec{p}_1\cdot\vec{r}_1 - \vec{p}_2\cdot\vec{r}_2)}\,. \tag{2.13}$$

For application to bremsstrahlung without radiative corrections only first order in $\vec{A}$ (to create one photon) is desired and, to this order, we have

$$V_{\rm em}^{(2)}(\vec{r}_i)=(2\pi)^{-6}\int d\vec{p}_i e^{i(\vec{p}_1'\cdot\vec{r}_1'+\vec{p}_2'\cdot\vec{r}_2'-\vec{p}_1\cdot\vec{r}_1-\vec{p}_2\cdot\vec{r}_2)}\vec{A}\cdot[-q_1'\vec{\nabla}_{p_1'}-q_2'\vec{\nabla}_{p_2'}-q_1\vec{\nabla}_{p_1}-q_2\vec{\nabla}_{p_2}]U_n(\vec{p}_i)\ . \tag{2.14}$$

We next integrate by parts giving [there are surface terms to Eq. (2.15) but these vanish when $V_{\rm em}(\vec{r}_i)$ is integrated over, as in Eq. (2.1), to define a matrix element]

$$V_{\rm em}^{(2)}(\vec{r}_i)=i\vec{A}\cdot(q_1'\vec{r}_1'+q_2'\vec{r}_2'-q_1\vec{r}_1-q_2\vec{r}_2)(2\pi)^{-6}\int d\vec{p}_i\, e^{i(\vec{p}_1'\cdot\vec{r}_1'+\vec{p}_2'\cdot\vec{r}_2')}U_n(\vec{p}_i)e^{-i(\vec{p}_1\cdot\vec{r}_1+\vec{p}_2\cdot\vec{r}_2)} \tag{2.15}$$

$$=i\vec{A}\cdot(q_1'\vec{r}_1'+q_2'\vec{r}_2'-q_1\vec{r}_1-q_2\vec{r}_2)V_n(\vec{r}_i)\ . \tag{2.16}$$

Thus the electromagnetic potential required by gauge invariance is given, to this order, by a simple function of the vector potential multiplying the original nuclear potential. Equation (2.16) is general in the sense that it does not matter whether the potential $V_n(\vec{r}_i)$ came from a meson theory, but only that it have a well-defined momentum representation. For potentials that are generated by meson exchange Eq. (2.16) leads to the same result (to zeroth order in K) as using the Feynman diagram for photon emission by the exchanged meson. There are ambiguities to higher order in K, but in Appendix B we show (for pion exchange) that Eq. (2.10) can be used to give the same result as the Feynman diagram to each order in K. A hard-core potential (such as the Hamada-Johnston potential) can be used in Eq. (2.16) if the core is considered to be an abstraction of a large, but not infinite, repulsion. Then, although the infinite-core potential would have no momentum representation, use of the hard-core potential in Eq. (2.16) to calculate electromagnetic effects leads to no problems.

It is convenient to introduce the usual center-of-mass coordinates for equal mass scattering

$$\vec{R}=\tfrac{1}{2}(\vec{r}_1+\vec{r}_2),\quad \vec{r}=(\vec{r}_1-\vec{r}_2) \tag{2.17}$$

and similarly for the primed coordinates. In terms of center-of-mass coordinates the nuclear potential is given by

$$V_n(\vec{r}_i)=\delta(\vec{R}-\vec{R}')V_n(\vec{r}',\vec{r})\ . \tag{2.18}$$

The corresponding photon-emission potential is given by

$$V_{\rm em}^{(2)}(\vec{r}_i)=e^{-i\vec{K}\cdot\vec{R}}\delta(\vec{R}-\vec{R}')V_{\rm em}^{(2)}(\vec{r}',\vec{r})\ , \tag{2.19}$$

with

$$V_{\rm em}^{(2)}(\vec{r}',\vec{r})=i(2\pi/K)^{1/2}\,\tfrac{1}{2}\hat{\epsilon}_{K,\lambda}\cdot[(q_1'-q_2')\vec{r}'-(q_1-q_2)\vec{r}]V_n(\vec{r}',\vec{r})\ . \tag{2.20}$$

The factor $e^{-i\vec{K}\cdot\vec{R}}$ in Eq. (2.19) leads to the modification of momentum conservation ($\vec{P}'+\vec{K}=\vec{P}$) appropriate to photon emission. In the expansion in powers of K, the $e^{-i\vec{K}\cdot\vec{R}}$ cannot be expanded because the $\vec{R}$ integration is taken over all space (leading to momentum conservation). However, the magnitudes of $\vec{r}$ and $\vec{r}'$ are limited by the nuclear range allowing an expansion of $e^{-i\vec{K}\cdot\vec{r}}$ and $e^{-i\vec{K}\cdot\vec{r}'}$ in terms of (K/μ) where $1/\mu$ represents the range of the nuclear potential.

For proton-proton bremsstrahlung, Eq. (2.20) leads to no modification to this order (K^0), since all the charges are equal. For neutron-proton bremsstrahlung, we use the isotopic spin formalism with the charge operators given by

$$q_{1,2}=(\tfrac{1}{2}e)(1-\tau_3^{1,2})\ . \tag{2.21}$$

We can rewrite Eq. (2.20) as

$$V_{\rm em}^{(2)}(\vec{r}',\vec{r})=\langle\vec{r}',q_1',q_2'|V_{\rm em}^{(2)}|\vec{r},q_1,q_2\rangle\ , \tag{2.22}$$

where $|\vec{r},q_1,q_2\rangle$ is a definite charge state and $V_{\rm em}^{(2)}$ is an operator in both coordinate and isospin space given by[14]

$$V_{\rm em}^{(2)}=-\tfrac{1}{2}i\,e(2\pi/K)^{1/2}\hat{\epsilon}_{K,\lambda}\cdot[\tfrac{1}{2}(\tau_3^1-\tau_3^2)\vec{r},V_n]\ . \tag{2.23}$$

The only effect of the operator $\frac{1}{2}(\tau_3^1-\tau_3^2)$ is to change the total isotopic spin of an n-p state while V_n conserves isospin. Therefore $V_{\rm em}^{(2)}$ only connects n-p states of different isospin.

The exchange and nonlocal contribution to np bremsstrahlung is given by $\langle I', \vec{p}'^{(-)}| V_{em}^{(2)} | I, \vec{p}^{(+)}\rangle$, where $|I, \vec{p}^{(\pm)}\rangle$ is an eigenstate of the strong two-nucleon Hamiltonian corresponding to relative momentum $\vec{p}$, isotopic spin I, and (outgoing/incoming) wave boundary conditions. We can eliminate the nuclear potential from this matrix element by using the Schrödinger equation

$$V_n |I, \vec{p}^{(+)}\rangle = [(p^2 + \vec{\nabla}^2)/m] \, |I, \vec{p}^{(+)}\rangle \tag{2.24}$$

and its Hermitian conjugate

$$\langle I', \vec{p}'^{(-)}| V_n = \langle I', \vec{p}'^{(-)}| [p'^2 + \overleftarrow{\nabla}^2]/m, \tag{2.24'}$$

where the notation $\overleftarrow{\nabla}^2$ means that the differential operator acts to the left,[15] and m is the nucleon mass (assuming $m_n = m_p$). Applying this to the bremsstrahlung matrix element for $V_{em}^{(2)}$ gives

$$\langle I', \vec{p}'^{(-)}| V_{em}^{(2)} | I, \vec{p}^{(+)}\rangle = (ie/2m)(2\pi/K)^{1/2} \hat{\epsilon}_{K,\lambda} \cdot \delta_{I', 1-I} \langle I', \vec{p}'^{(-)}| (\overleftarrow{\nabla}^2 + p'^2)\vec{r} - \vec{r}(\vec{\nabla}^2 + p^2) | I, \vec{p}^{(+)}\rangle, \tag{2.25}$$

so that the initial and final states must have different isospin. This result includes all nonlocal (or momentum-dependence and spin-orbit) effects as well as exchange contributions, all to lowest order in K. It is to be added to the corresponding matrix element of $V_{em}^{(1)}$ given by Eq. (3.2), which includes the "pole" and "rescattering" terms arising from the kinetic-energy part of the Hamiltonian. In Sec. III we indicate how to separate the exchange contribution in Eq. (2.25) from the nonlocal contribution. Explicit calculations then indicate that the exchange emission gives the dominant contribution to Eq. (2.25).

It is of interest to observe that, to zeroth order in K, the kinetic-energy photon-emission term is given by [the prime on V_{em} and $V_{em}^{(1)}$ in Eqs. (2.26)–(2.31) indicates that $e^{i\mathbf{K}\cdot\mathbf{r}}$ is taken to lowest order in K, whereas $V_{em}^{(2)}$ already includes this approximation]

$$V'^{(1)}_{em} = -(ie/m)(2\pi/K)^{1/2} \tfrac{1}{2}(\tau_3^1 - \tau_3^2)\, \hat{\epsilon}_{K,\lambda} \cdot \vec{\nabla} = \tfrac{1}{2} ie (2\pi/K)^{1/2} \tfrac{1}{2}(\tau_3^1 - \tau_3^2) \hat{\epsilon}_{K,\lambda} \cdot [\vec{r}, \nabla^2/m]\,. \tag{2.26}$$

Adding Eqs. (2.23) and (2.26), we have

$$V'_{em} = V'^{(1)}_{em} + V_{em}^{(2)} = -\tfrac{1}{2} ie (2\pi/K)^{1/2} \hat{\epsilon}_{K,\lambda} [\tfrac{1}{2}(\tau_3^1 - \tau_3^2)\vec{r}, H_n] \tag{2.27}$$

for the total (kinetic energy + exchange + nonlocal) photon-emission potential in the extreme dipole limit (order K^0), in terms of the full nuclear Hamiltonian H_n.

Now we can use the Schrödinger equation to write the bremsstrahlung matrix element as

$$\langle \vec{p}'^{(-)}| V'_{em} | \vec{p}^{(+)}\rangle = -\tfrac{1}{2} ie (2\pi/K)^{1/2} (E - E') \langle \vec{p}'^{(-)}| \hat{\epsilon}_{K,\lambda} \cdot \vec{r} | \vec{p}^{(+)}\rangle\,, \tag{2.28}$$

where the initial and final state must have different isospin. For a process in which at least one of the states was bounded (e.g., photodisintegration of the deuteron or the decay of a nuclear or atomic system), Eq. (2.28) could be used directly and would include all electric exchange and nonlocal currents in the extreme dipole limit. For unbounded states (as in $np\gamma$) however, Eq. (2.28) cannot be used directly because all partial waves would contribute to this matrix element. Plane waves must be subtracted out of Eq. (2.28), and this leads to the usual development in terms of four "pole" terms and a "rescattering" term for the kinetic-energy part and potential contributions as given by Eq. (2.25).

It is instructive to show this plane wave subtraction explicitly. We have

$$\begin{aligned}\langle \vec{p}'^{(-)}| V'_{em} | \vec{p}^{(+)}\rangle &= (E - E') N (\psi_{p'}^{(-)}, Q \psi_p^{(+)}) \\ &= (E - E') N \{ (\phi_{p'}, Q[\psi_p^{(+)} - \phi_p]) + ([\psi_{p'}^{(-)} - \phi_{p'}], Q\phi_p) + (\phi_{p'}, Q\phi_p) + ([\psi_{p'}^{(-)} - \phi_{p'}], Q[\psi_p^{(+)} - \phi_p]) \},\end{aligned} \tag{2.29}$$

where $N = -\tfrac{1}{2} ie\,(2\pi/K)^{1/2}$, and Q is the operator $\tfrac{1}{2}(\tau_3^1 - \tau_3^2)\hat{\epsilon}\cdot\vec{r}$. The functions $\psi_p^{(\pm)}$ and ϕ_p satisfy the Schrödinger equations $H_n \psi_p^{(\pm)} = E \psi_p^{(\pm)}$ and $-(\nabla^2/m)\phi_p = E\phi_p$ ($\hbar = c = 1$), with $H_n = -\nabla^2/m + V_n$. We now use the Schrödinger equation and Eqs. (2.23) and (2.26) in the first three terms of (2.29) to give

$$\begin{aligned}\langle \vec{p}'^{(-)}| V'_{em} | \vec{p}^{(+)}\rangle &= (\phi_{p'}, V'^{(1)}_{em} [\psi_p^{(+)} - \phi_p]) + ([\psi_{p'}^{(-)} - \phi_{p'}], V'^{(1)}_{em} \phi_p) + (\phi_{p'}, V_{em}^{(2)} \psi_p^{(+)}) - (\psi_{p'}^{(-)}, V_{em}^{(2)} \phi_p) \\ &\quad + (\phi_{p'}, V'^{(1)}_{em} \phi_p) + (E - E') N ([\psi_{p'}^{(-)} - \phi_{p'}], Q[\psi_p^{(+)} - \phi_p])\,.\end{aligned} \tag{2.30}$$

At this stage we can relate Eq. (2.30) to the approximation used by Celenza, Gibson, Liou and Sobel[12] (CGLS) who calculate $np\gamma$ to zeroth order in K (except for the pole terms, which they treat to all orders). They drop the last term of Eq. (2.30) as being of order K and use the first four terms (and magnetic pole terms) to calculate $np\gamma$. However, independent of counting orders in K, we do not expect the last term in Eq. (2.30) to be small because the integral is taken over all space with no cutoff, while an additional power of r comes from the operator Q. The importance of this term probably accounts for the large difference between the CGLS results and ours. (We discuss other differences between the two calculations in Sec. IV.)

Continuing from Eq. (2.30) and applying the Schrödinger equation to the last term we find

$$\langle \vec{p}'^{(-)}|V'_{em}|\vec{p}^{(+)}\rangle = \{(\phi_{p'}, V'^{(1)}_{em}[\psi_p^{(+)}-\phi_p]) + ([\psi_{p'}^{(-)}-\phi_{p'}], V'^{(1)}_{em}\phi_p)\}_{EP} + ([\psi_{p'}^{(-)}-\phi_{p'}], V'^{(1)}_{em}[\psi_p^{(+)}-\phi_p])_{RES} + (\psi_{p'}^{(-)}, V^{(2)}_{em}\psi_p^{(+)})_{EXCH+NL} + (\phi_{p'}, V'^{(1)}_{em}\phi_p), \tag{2.31}$$

which, to lowest order in the expansion of $e^{i\vec{K}\cdot\vec{r}}$, comprises all the electric terms for $np\gamma$. For our practical calculation we take the EP and RES terms, as previously derived,[1,4] to all orders in K as well as including the magnetic part of $V^{(1)}_{em}$ to all orders in K. The potential $V^{(2)}_{em}$ has the short range of the corresponding nuclear potential, so it seems reasonable to keep the lowest order in K for the term $(\psi_{p'}^{(-)}, V^{(2)}_{em}\psi_p^{(+)})$. The term $(\phi_{p'}, V^{(1)}_{em}\phi_p)$, of course, does not contribute when over-all energy conservation is required for $np\gamma$.

We emphasize that, except for very small K, keeping the lowest orders of K is only a good approximation when the radial integrals are cutoff by a short range potential as with $V^{(2)}_{em}$. We believe that there is evidence for this in the difference between our results and those of CGLS,[12] who do not include the last term of (2.30) on the basis that it is of order K. Since their method corresponds to using the low-energy theorem,[16] we conclude that the low-energy theorem for orders K^{-1} and K^0 is not a good approximation for $np\gamma$ at reasonable photon energies.

III. CALCULATIONAL PROCEDURE

In this section we outline the method of calculation used for the results presented in this paper. The T matrix for $np\gamma$, treating the nuclear interaction exactly within the framework of a potential model and the electromagnetic interaction to first order, can be written (omitting spin and isospin indices) as

$$T = \langle \vec{p}'^{(-)}|V^{(1)}_{em} + V^{(2)}_{em}|\vec{p}^{(+)}\rangle, \tag{3.1}$$

The electromagnetic potential $V^{(1)}_{em}$ which arises from the kinetic-energy and magnetic-moment part of the nuclear Hamiltonian is given by[4]

$$V^{(1)}_{em} = (2\pi/K)^{1/2}\{[q_1 e^{-i\vec{K}\cdot\vec{r}/2} - q_2 e^{i\vec{K}\cdot\vec{r}/2}] i\hat{\epsilon}\cdot\vec{\nabla}/m + \tfrac{1}{2}e/m\,[\mu_1 e^{-i\vec{K}\cdot\vec{r}/2}\vec{\sigma}_1\cdot\vec{K}\times\hat{\epsilon} + \mu_2 e^{i\vec{K}\cdot\vec{r}/2}\vec{\sigma}_2\cdot\vec{K}\times\hat{\epsilon}]\}, \tag{3.2}$$

where $e=(1/137.04)^{1/2}$ is the charge of the proton, σ_1 and σ_2 are the Pauli spin operators, and m is the nucleon mass ($m_n = m_p$). The operator q_i is given by Eq. (2.21), and μ_i of Eq. (3.2) is given by

$$\mu_i = \tfrac{1}{2}(\mu_n + \mu_p)1 + \tfrac{1}{2}(\mu_n - \mu_p)\tau_3^i, \tag{3.3}$$

where $\mu_p = 2.793$ and $\mu_n = -1.913$ are the magnetic moments (in nuclear magnetons) of the proton and neutron, respectively.

The evaluation of the $np\gamma$ matrix element of Eq. (3.1) for $V^{(1)}_{em}$ involves the subtraction of plane waves from the initial and final states leading to the usual external radiation terms and that part of the internal radiation referred to as the rescattering term. For the rescattering term, $V^{(1)}_{em}$ is expanded in partial waves; the cutoff in the expansion of $V^{(1)}_{em}$ is limited only by the cutoff in the number of nuclear states which are included in the calculation. In the present work the nuclear states include all partial wave contributions for total angular momentum $J \leq 4$ (26 states). The method of calculation of these terms has been discussed elsewhere.[1,4]

For the determination of that part of the internal radiation scattering involving $V^{(2)}_{em}$ we use Eq. (2.25). If we perform an integration by parts, keeping in mind that the left (right) handed arrow operates on the final (initial) state wave function, we can express the radial contribution to Eq. (2.25) as

$$\mathcal{R}^{(2)}_{em} = R^3\left[\frac{d\psi_{p'}^{(-)*}}{dr}\psi_p^{(+)} - \frac{d\psi_p^{(+)}}{dr}\psi_{p'}^{(-)*}\right]_R + \int_0^R dr\, r^2\left[\psi_{p'}^{(-)*}\frac{d\psi_p^{(+)}}{dr} - \frac{d\psi_{p'}^{(-)*}}{dr}\psi_p^{(+)}\right] + \int_0^R dr\, r^3\psi_{p'}^{(-)*}\psi_p^{(+)}\left[p'^2 - p^2 - \frac{l'_f(l'_f+1)}{r^2} + \frac{l'_i(l'_i+1)}{r^2}\right], \tag{3.4}$$

where $\psi_p^{(\pm)}$ is the radial part of $|p^{(\pm)}\rangle$, and R is the matching radius used to determine the nuclear phase shifts. In the barycentric system the z axis is taken in the photon direction, and the polarization states of the photon are represented in the circular basis corresponding to left and right circular polarization. In the present approximation the effect of using the helicity representation for the photon is to project out of $V_{em}^{(2)}$ the spherical harmonics $Y_1^m(\hat{r})$ (with $m=\pm 1$ for the two photon polarization states). The angular integration then exhibits only electric-dipole transitions between the initial and final nuclear states.

The result using Eq. (3.4) includes exchange as well as other nonlocal effects to lowest order in the photon momentum. In an earlier calculation[6] only the exchange term was included and was referred to as exchange bremsstrahlung. To see how this separation is made, consider $V_{em}^{(2)}$ as given in Eq. (2.23), where V_n is an operator in isospin space and a nonlocal (or derivative) operator in coordinate space. In terms of direct (V_d) and exchange (V_x) potentials

$$V_n = V_d + V_x = v_d I + v_x T_x \ , \tag{3.5}$$

where I is the identity and

$$T_x = \tfrac{1}{2}(1 + \vec{\tau}^{\,1}\cdot\vec{\tau}^{\,2}) \tag{3.6}$$

is the exchange operator in isospin space. Since we are in the np subspace ($T_3 = 0$), we have

$$\tau_3^1 + \tau_3^2 = 0\,, \tag{3.7}$$

so that Eq. (2.23) can be written

$$V_{em}^{(2)} = -\tfrac{1}{2} ie(2\pi/K)^{1/2}\hat{\epsilon}\cdot[\tau_3^1\vec{r}, V_n]$$
$$= -\tfrac{1}{2} ie(2\pi/K)^{1/2}\hat{\epsilon}\{[\tau_3^1, V_n]\vec{r} + \tau_3^1[\vec{r}, V_n]\} \ . \tag{3.8}$$

The effect of the exchange potential is obtained by using only the first term in the curly bracket, since

$$[\tau_3^1, V_n]\vec{r} = -2V_x\,\tau_3^1\vec{r} \ . \tag{3.9}$$

The operator τ_3^1 insures that the initial and final nuclear states have different isospin. Since charge independence is assumed for the two-nucleon interaction, V_n can also be written

$$V_n = v_0 T_0 + v_1 T_1 \ , \tag{3.10}$$

where T_0 and T_1 are the isotopic-spin projection operators given by

$$T_1 = \tfrac{1}{4}(3 + \vec{\tau}^{\,1}\cdot\vec{\tau}^{\,2}) \tag{3.11}$$

and

$$T_0 = \tfrac{1}{4}(1 + \vec{\tau}^{\,1}\cdot\vec{\tau}^{\,2}) \ . \tag{3.12}$$

The exchange potential can be written in terms of the isospin potentials as

$$V_x = \tfrac{1}{2}(v_1 - v_0)T_x \ . \tag{3.13}$$

The Schrödinger equation cannot be used to eliminate V_x from the exchange bremsstrahlung matrix element; consequently, the evaluation is considerably more difficult than for the full contribution (2.25), where the potential has been eliminated. There are additional complications in the evaluation of the exchange-bremsstrahlung term in the case of a momentum-dependent nuclear potential (see Appendix C), although the complications do not arise using Eq. (2.25) for the full contribution.

IV. RESULTS

The cross section for $np\gamma$ has been calculated using the Hamada-Johnston[2] (HJ) and Bryan-Scott[3] III (BS) potentials at various coplanar geometries to compare with the experimental results at 130[7] and 208[5] MeV. The calculation includes all partial-wave contributions of the nuclear matrix elements with total angular momentum $J \leq 4$. The invariant form of the cross section appropriate for $np\gamma$ averaged over initial spins and summed over final spins and polarizations is used.[1,4]

The coplanar differential cross section $d\sigma/d\Omega_n d\Omega_p$, calculated at 200 MeV and various neutron (θ_n) and proton (θ_p) exit angles, is compared to the experimental results (208 MeV) of Brady and Young[5] in Table I. A comparison of the results for the HJ and BS potentials is included. The calculation includes $V_{em}^{(1)}$ to all orders of the photon momentum K that enter and $V_{em}^{(2)}$ to order K^0 as given by Eq. (2.25). The sum of these contributions in the present work is referred to as EXT + INT. The calculational results are within experimental error with the exception of the ($\theta_n = 38°$, $\theta_p = 38°$) cross section, and in two cases, (30, 30°) and (45, 30°), the agreement is good.

The results of Celenza, Gibson, Liou, and Sobel[12] for the HJ potential are also included in Table I. It is seen that our results for HJ are close to those of CGLS at (30, 30°), but the discrepancy increases

TABLE I. The coplanar $np\gamma$ cross section $d\sigma/d\Omega_n d\Omega_p$ in μb/sr^2 for incident laboratory energy $E = 200$ MeV and various neutron (θ_n) and proton (θ_p) exit angles. The results for the BS and HJ potentials calculated in the present work including EXT + INT are compared with the HJ results of Ref. 12 and the experimental results of Ref. 5.

θ_n (deg)	θ_p (deg)	BS (EXT + INT)	HJ (EXT + INT)	HJ (Ref. 12)	Experiment (Ref. 5)
30	30	34.1	34.6	30	35 ± 14
35	35	44.7	44.0	33	57 ± 13
38	38	71.4	69.8	49	116 ± 20
40	30	72.0	69.9		114 ± 44
45	30	128	121		132 ± 53

with inclusive nucleon exit angle. As discussed at the end of Sec. II, the calculation of CGLS uses the low energy theorem[16] to treat *all* the internal radiation (RES + EXCH + NL) to order K^0 in the $np\gamma$ matrix element, while we do the rescattering (RES) to all orders in K and the EXCH + NL to order K^0 in $V_{\text{em}}^{(2)}$. The main difference comes from the consequent absence of the last term of Eq. (2.30) from their calculation. Another difference arises from the fact that their K expansion for the $np\gamma$ internal radiation matrix element involves expanding the wave functions to order K^0 while we expand $V_{\text{em}}^{(2)}$ to order K^0 but treat the wave functions to all orders in K. They also do not include the magnetic RES terms, but our explicit calculations indicate that these are small. Their calculation includes some relativistic effects, but they find these corrections to be small.

A comparison to the experiment of Edgington *et al.*[7] at 130 MeV is made in Table II.[17] Some of our calculational results were presented in an earlier paper[6] and are included here for completeness. The agreement with the recent experimental results of Edgington *et al.*[7] is generally good, especially for small opening angles. It should be noted, however, that the uncertainties are comparable with the cross sections. Also, a substantial disagreement still exists at the largest opening angle (38, 32°). CGLS also give a value for 130 MeV (30, 30°) of $d\sigma/d\Omega_n d\Omega_p = 18\ \mu\text{b/sr}^2$. From Table II our predicted value for 130 MeV (29, 32°) is 31 $\mu\text{b/sr}^2$ [interpolation[17] in Table II to (30, 30°) would increase this value], so that there is still disagreement with CGLS at this energy.

The coplanar cross section $d\sigma/d\Omega_n d\Omega_p d\theta_\gamma$ at 200 MeV comparing the potentials HJ and BS and illustrating certain contributions explicitly is shown in Fig. 1. We use the convention that the photon angle θ_γ is measured from 0° in the beam direction to +180° (−180°) on the side of θ_p (θ_n). In Figure 1(a) a comparison between HJ and BS is made for the angular distribution of the photon corresponding to the 200 MeV (30, 30°) entry in Table I. As can be seen, there is very little difference in the results for the two potentials. This small difference between the two potentials holds true for all angles we have calculated.

As discussed earlier, $V_{\text{em}}^{(2)}$ as obtained by (2.25) includes momentum-dependent and spin-orbit effects as well as exchange (EXCH) only for the BS potential as calculated according to (3.8) and (3.9). The contribution of external radiation scattering alone (EXT) and the effect of exchange plus external radiation scattering (EXT + EXCH) is also included in Fig. 1(b). As can be seen[18] from Fig. 1(b) the interference of EXT and EXCH has an important effect on the cross section, increasing it by more than a factor of 2 over that for EXT alone. The main contribution to the cross section of Fig. 1(a) comes from EXT and EXCH. The rescattering term[19] increases the cross section about 10%, although for θ_γ forward or backward in the lab the effect is more like 25%. The terms designated as *NL* (from spin-orbit and momentum-dependent terms) contribute about 1% to the cross section.

Presented in Fig. 2 are the 200-MeV cross sections $d\sigma/d\Omega_n d\Omega_p d\theta_\gamma$ versus photon angle θ_γ for the remaining cases in Table I. A comparison is made between EXT and EXT + INT for the HJ potential. A comparison between the two potentials is not included because the results are so similar. In Fig. 3 similar results are presented at 130 MeV for four sets of exit angles from Table II, again for the HJ potential.

A preliminary investigation indicates that the higher order terms in $V_{\text{em}}^{(2)}$ will not alter our results significantly. We calculated the effects of some order K^2 terms using the replacement $\vec{r} \to \vec{r} - \vec{r}\,(\vec{K}\cdot\vec{r})^2/24$ (see Appendix B). This order-of-magnitude determination of K^2 terms is relatively easy to introduce in the existing code as compared to the order-K terms which require major modification because of the spin dependence. (The relationship between these order-K^2 terms and the order-K terms is discussed in Appendix B.) The percent increase in the $np\gamma$ cross section $d\sigma/d\Omega_n d\Omega_p d\theta_\gamma$ is maximum at the peaks where it is <2% for 200 MeV (30, 30°).

TABLE II. The coplanar $np\gamma$ cross section $d\sigma/d\Omega_n d\Omega_p$ in μb/sr^2 for incident laboratory energy $E = 130$ MeV and various neutron (θ_n) and proton (θ_p) exit angles. The results for the BS and HJ potentials calculated in the present work including EXT + INT are compared with the experimental results of Ref. 7.

θ_n (deg)	θ_p (deg)	BS (EXT + INT)	HJ (EXT + INT)	Experiment (Ref. 7)
23	20	38.5	40.1	47 ± 35
26		40.4	42.0	16 ± 29
29		42.8	44.4	35 ± 28
38		53.9	56.2	64 ± 24
23	32	23.3	23.5	17 ± 29
26		26.5	26.6	66 ± 29
29		30.6	30.6	77 ± 32
38		55.6	55.2	116 ± 21

V. CONCLUSIONS

In summary, we have introduced a gauge-invariant prescription for including exchange effects in $np\gamma$ which are induced by the nuclear potential. This prescription has the advantage of being formulated entirely within a potential framework. We have shown that to lowest order in the photon momentum this prescription is unambiguous. Terms higher order (through order K^2) in the photon momentum have been obtained for the OPE part of the nuclear potential only; these terms are expected to dominate the higher order-K terms because of the longer range in OPE.

Our calculations have been performed so that the individual contributions from the various terms can be examined. The major contributions to $np\gamma$ come from the external radiation and from the exchange part of the internal radiation. The contribution from the latter is very important (in contrast to $pp\gamma$, where it is mostly absent); this exchange part increases the $np\gamma$ cross section roughly by a factor of 2 and is required to obtain agreement with experiment. The next largest contribution comes from that part of the internal radiation called rescattering. The contribution from rescattering varies with the photon angle, and in our geometry reaches a maximum of about 25%. The nonlocal effects such as explicit momentum-dependence or spin-orbit effects contribute about 1% to the $np\gamma$ cross section. We have also looked at the order of magnitude of the K^2 terms that enter $V_{\rm em}^{(2)}$ by the method described in Sec. IV and Appendix B. These terms contribute less than 2%. Because the order-K terms cannot interfere with the dominant electric-pole amplitude in $np\gamma$ (without polarization), it is suggested that their effect might compare in magnitude with the order-K^2 terms we have looked at. It would, however, be interesting to see the effect of initial or final polarization in isolating the contribution of the order-K terms.

It is perhaps worthwhile to restate what we have learned thus far from the point of view that $np\gamma$ might provide a testing ground, in a simple two-nucleon system, for the importance of exchange effects in photonuclear reactions. If the lowest-order terms in $V_{\rm em}^{(2)}$ resulting from our prescription are combined with the corresponding terms of $V_{\rm em}^{(1)}$, obtained from the usual minimal coupling in the kinetic-energy part of the Hamiltonian, the result displays the familiar operator $\hat{\epsilon}\cdot[\frac{1}{2}(\tau_3^1$

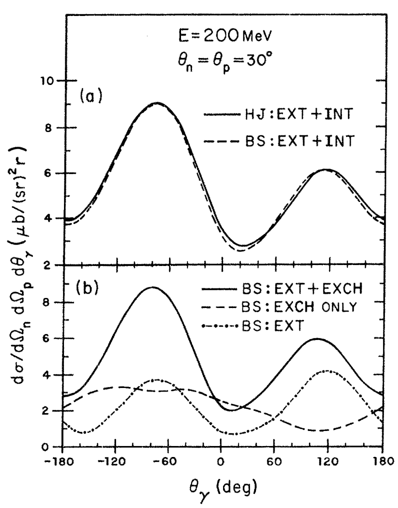

FIG. 1. The $np\gamma$ cross section $d\sigma/d\Omega_n\, d\Omega_p\, d\theta_\gamma$ with incident laboratory energy $E=200$ MeV and coplanar symmetric angles $\theta_n=\theta_p=30°$ for (a) a comparison between the Hamada-Johnston (HJ) and Bryan-Scott III (BS) potentials including external (EXT) plus internal (INT) radiation as described in the text, and (b) the contribution of EXT, the exchange (EXCH) part of INT, and the interference effect of these terms (EXT + EXCH) as calculated with BS.

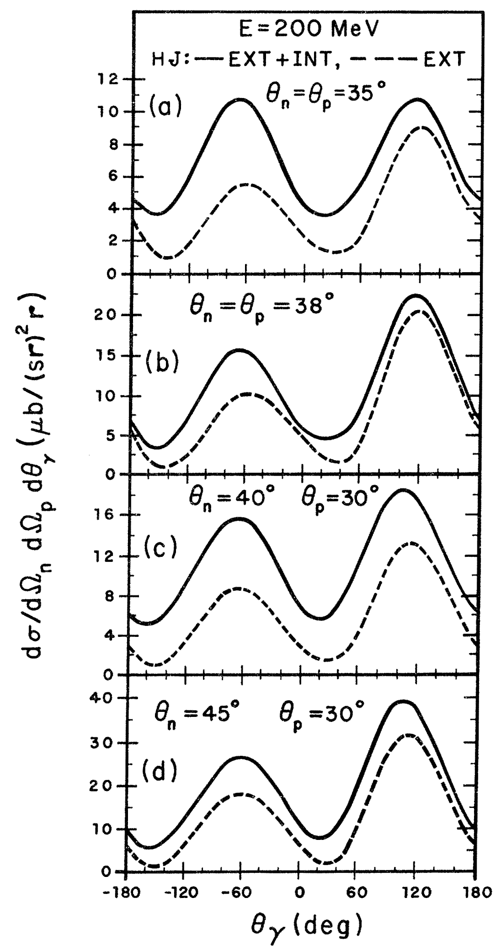

FIG. 2. The $np\gamma$ cross section $d\sigma/d\Omega_n\, d\Omega_p\, d\theta_\gamma$ with incident laboratory energy $E=200$ MeV and various sets of coplanar angles (θ_n, θ_p): (a) (35, 35°), (b) (38, 38°), (c) (40, 30°), and (d) (45, 30°). These results are calculated with the Hamada-Johnston (HJ) potential comparing the contribution from external (EXT) radiation alone to that including internal (INT) radiation as described in the text.

$-\tau_3^2)\vec{r}, H_n]$ which is associated with Siegert's theorem.[9] Application of this operator includes all electric-dipole contributions including exchange and any other nonlocal effects in the extreme dipole limit. For $np\gamma$ the kinetic-energy part of H_n in this operator must be treated separately because of the initial and final unbound states. We have demonstrated by explicit calculation that the potential-energy contribution from this operator is mainly exchange and that it is large, increasing the cross section by roughly a factor of 2 at 130 and 200 MeV. In the absence of the importance of magnetic terms involving spin changes, such as in polarization studies or np capture at threshold, we can conclude that exchange effects are, in fact, large at the energies we have considered, but that they would be mainly accounted for by the operator obtained in Siegert's theorem.

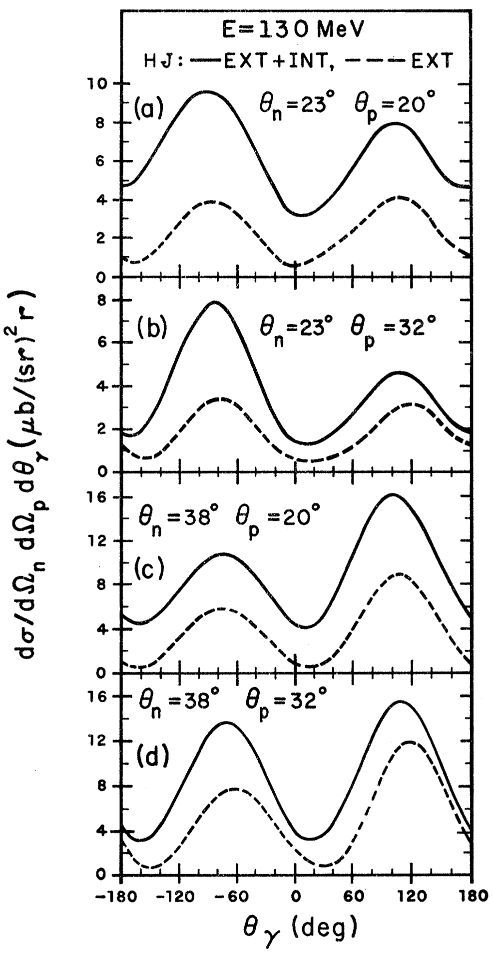

FIG. 3. The $np\gamma$ cross section $d\sigma/d\Omega_n\, d\Omega_p\, d\theta_\gamma$ with incident laboratory energy $E=130$ MeV and various sets of coplanar asymmetric angles (θ_n, θ_p): (a) (23, 20°), (b) (23, 32°), (c) (38, 20°), and (d) (38, 32°). These results are calculated with the Hamada-Johnston (HJ) potential comparing the contribution from external (EXT) radiation alone to that including internal (INT) radiation as described in the text.

A comparison of the HJ and BS potentials has been included in our $np\gamma$ calculations; the difference is less than was found for these same potentials applied to $pp\gamma$.[1] It may be that these results are due to the characteristics of the particular potentials under comparison. However, the small difference between potentials could be a general feature of $np\gamma$.

Finally, by comparing our calculation with that of CGLS,[12] who used the low-energy theorem[16] for the internal radiation, we come to the conclusion that use of the low-energy theorem is not a good approximation for $np\gamma$ at 130 MeV and higher.

Appendix A

The connection between a nonlocal potential and a momentum-dependent potential is well known, but, for completeness, we indicate their relationship in this appendix.

A nonlocal nucleon-nucleon potential is given by Eq. (2.18) in terms of center-of-mass coordinates as

$$V(\vec{r}_i) = \delta(\vec{R} - \vec{R}\,')V(\vec{r}\,', \vec{r}) . \qquad \text{(A1)}$$

Equation (2.1) for a matrix element of this potential then becomes

$$M_{\beta\alpha} = \int d\vec{r}\, d\vec{r}\,' \psi_\beta^*(\vec{r}\,')V(\vec{r}\,', \vec{r})\psi_\alpha(\vec{r}) . \qquad \text{(A2)}$$

Introducing the momentum representation leads to

$$M_{\beta\alpha} = \int d\vec{p}\, d\vec{p}\,' \phi_\beta^*(\vec{p}\,')U(\vec{p}\,', \vec{p})\phi_\alpha(\vec{p}) , \qquad \text{(A3)}$$

with

$$U(\vec{p}\,', \vec{p}) = (1/2\pi)^3 \int d\vec{r}\, d\vec{r}\,' e^{-i\vec{p}\,'\cdot\vec{r}\,'} V(\vec{r}\,', \vec{r}) e^{i\vec{p}\cdot\vec{r}} . \qquad \text{(A4)}$$

Reintroducing the original coordinate-space wave functions gives

$$M_{\beta\alpha} = (1/2\pi)^3 \int d\vec{p}\, d\vec{p}\,' d\vec{r}\, d\vec{r}\,' \psi_\beta^*(\vec{r}\,')e^{i\vec{p}\,'\cdot\vec{r}\,'} \times U(\vec{p}\,', \vec{p})e^{-i\vec{p}\cdot\vec{r}}\psi_\alpha(\vec{r}) . \qquad \text{(A5)}$$

We now use the variables

$$\vec{k} = \vec{p} - \vec{p}\,', \quad \vec{q} = \tfrac{1}{2}(\vec{p} + \vec{p}\,'), \quad \vec{x} = \vec{r} - \vec{r}\,', \quad \vec{\rho} = \tfrac{1}{2}(\vec{r} + \vec{r}\,') \qquad \text{(A6)}$$

so that Eq. (A5) can be written

$$M_{\beta\alpha} = (1/2\pi)^3 \int d\vec{p}\, d\vec{p}\,' d\vec{r}\, d\vec{r}\,' \psi_\beta^*(\vec{\rho} - \tfrac{1}{2}\vec{x})e^{-i\vec{k}\cdot\vec{\rho}} e^{-i\vec{q}\cdot\vec{x}} \times \overline{U}(\vec{k}, \vec{q})\psi_\alpha(\vec{\rho} + \tfrac{1}{2}\vec{x}) , \qquad \text{(A7)}$$

with $\bar{U}(\vec{k}, \vec{q}) = U(\vec{q} - \frac{1}{2}\vec{k}, \vec{q} + \frac{1}{2}\vec{k})$. If the original potential $V(\vec{r}', \vec{r})$ were in fact local $[V(\vec{r}', \vec{r}) = \delta(\vec{r}' - \vec{r})V(\vec{r})]$, Eq. (A4) would lead to a $\bar{U}(\vec{k}, \vec{q})$ that had no q dependence and Eq. (A7) would lead back to a local potential as expected. When $V(\vec{r}', \vec{r})$ is nonlocal, $\bar{U}(\vec{k}, \vec{q})$ will depend on q as well as k. In that case we could rewrite Eq. (A7) as

$$M_{\beta\alpha} = (1/2\pi)^3 \int d\vec{k}\, d\vec{q}\, d\vec{x}\, d\vec{\rho}\, \psi_\beta^*(\vec{\rho} - \tfrac{1}{2}\vec{x}) e^{-i\vec{k}\cdot\vec{\rho}} \times \psi_\alpha(\vec{\rho} + \tfrac{1}{2}\vec{x}) \bar{U}(\vec{k}, i\vec{\nabla}_x) e^{-i\vec{q}\cdot\vec{x}} . \tag{A8}$$

We can do the q integration leading to δ function derivatives, and then the x integration resulting in

$$M_{\beta\alpha} = \int d\vec{k}\, d\vec{\rho}\, e^{-i\vec{k}\cdot\vec{\rho}} \times \lim_{x\to 0} \{ \bar{U}(\vec{k}, -i\vec{\nabla}_x)[\psi_\beta^*(\vec{\rho} - \tfrac{1}{2}\vec{x})\, \psi_\alpha(\vec{\rho} + \tfrac{1}{2}\vec{x})]\} . \tag{A9}$$

We can now introduce a "local" potential with gradients to write

$$M_{\beta\alpha} = \int d\vec{\rho} \lim_{x\to 0} \{ \bar{V}(\vec{\rho}, -i\vec{\nabla}_x)[\psi_\beta^*(\vec{\rho} - \tfrac{1}{2}\vec{x})\psi_\alpha(\vec{\rho} + \tfrac{1}{2}\vec{x})]\} , \tag{A10}$$

with

$$\bar{V}(\vec{\rho}, -i\vec{\nabla}_x) = \int d\vec{k}\, e^{-i\vec{k}\cdot\vec{\rho}}\, \bar{U}(\vec{k}, -i\vec{\nabla}_x) . \tag{A11}$$

Equation (A10) could also be written

$$M_{\beta\alpha} = \int d\vec{\rho}\ \psi_\beta^*(\vec{\rho}) \bar{V}(\vec{\rho}, \vec{q}) \psi_\alpha(\vec{\rho}) \tag{A12}$$

where $\vec{q}$ is the operator

$$\vec{q} = (1/2i)(\vec{\nabla}_\rho - \overleftarrow{\nabla}_\rho) \tag{A13}$$

with the further understanding that $\vec{q}$ operates only on the wave functions and *not* on the ρ dependence of $\bar{V}(\vec{\rho}, \vec{q})$. The integral in (A12) usually allows integration by parts with vanishing surface terms and then we can write

$$M_{\beta\alpha} = \int d\vec{\rho}\ \psi_\beta^*(\vec{\rho}) \bar{V}\left(\vec{\rho}, \frac{1}{2i}(\vec{\nabla}_1 + \vec{\nabla}_2)\right) \psi_\alpha(\vec{\rho}) , \tag{A14}$$

where $\vec{\nabla}_1$ acts only on $\psi_\alpha(\vec{\rho})$ while $\vec{\nabla}_2$ acts on $\psi_\alpha(\vec{\rho})$ and on all the explicit ρ dependence of $\bar{V}$.

The spin-orbit interaction is a special case of nonlocality of the form $\bar{V}(\vec{\rho}, \vec{q}) = \bar{V}_{LS}(\vec{\rho}) \vec{S} \cdot \vec{\rho} \times \vec{q}$. If $\bar{V}_{LS}(\vec{\rho})$, by itself, is central (as is usually the case), then $\vec{\rho} \times \vec{\nabla}_\rho$ commutes with $\bar{V}_{LS}(\rho)$, and the spin-orbit potential can be written in its usual form

$$\bar{V}(\vec{\rho}, \vec{q}) = \bar{V}_{LS}(\rho) \vec{S} \cdot \vec{\rho} \times \vec{\nabla}_\rho / i = \bar{V}_{LS}(\rho) \vec{L} \cdot \vec{S} . \tag{A15}$$

Appendix B

The higher-order (in K) corrections to exchange (and nonlocal) bremsstrahlung are complicated in our approach by the fact that the differential operator $\vec{\nabla}$ does not commute with $\vec{A}(\vec{r})$ beyond zeroth order in K. This leads to ambiguity in the exchange bremsstrahlung (beyond zeroth order) because the result depends on the order in which the momentum operators are written in Eq. (2.8) and also on which explicit momentum operators are used. This ambiguity beyond zeroth order in K is inherent in any method that uses only the requirement of gauge invariance to induce exchange bremsstrahlung.[10]

The ambiguity cannot be resolved unless there is an explicit form for the momentum representation of the nuclear potential. For the case of a meson-exchange potential (e.g., the Bryan-Scott potential) a momentum representation does, of course, exist. For purely coordinate-space potentials there is generally a long-range part either given by one-pion exchange (OPE) or closely represented by one-pion exchange so that it can be used for the momentum representation of the long-range part of the potential. To the extent that the expansion in K can be considered an expansion in the dimensionless ratio K/μ, where μ is the mass of the meson exchanged (or the inverse of the characteristic range of the potential), OPE will be the dominant part of the higher order (in K) terms. We therefore use OPE to approximate these higher orders.

The form of OPE in the nucleon-nucleon potential is usually written

$$U_{\text{OPE}}(\vec{p}_i) = -\frac{(f/\mu)^2 \vec{\sigma}_1 \cdot \vec{k} \vec{\sigma}_2 \cdot \vec{k} \vec{\tau}^1 \cdot \vec{\tau}^2}{(2\pi)^3 (k^2 + \mu^2)} \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \tag{B1}$$

with

$$\vec{k} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_1' - \vec{p}_2 + \vec{p}_2') . \tag{B2}$$

The coupling constant f is the rationalized pion-nucleon coupling constant used for the OPE part of the potential ($f^2/4\pi = 0.08$). Because of the momentum-conservation δ function in Eq. (B1), there are a number of equivalent (for scattering) ways in which we could write Eq. (B1). But these could lead to different bremsstrahlung to higher orders in K. We guide ourselves by choosing a particular form for $U_{\text{OPE}}(\vec{p}_i)$ that leads to the same result as the Feynman-diagram prescription would give for bremsstrahlung due to pion exchange.

Such a form is

$$U_{\text{OPE}}(\vec{p}_i) = \frac{(f/\mu)^2}{(2\pi)^3} \vec{\sigma}_2 \cdot (\vec{p}_2 - \vec{p}_2') \frac{\vec{\tau}^1 \cdot \vec{\tau}^2}{(\vec{p}_2 - \vec{p}_2')^2 + \mu^2} \vec{\sigma}_1 \cdot (\vec{p}_1 - \vec{p}_1') \times \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') . \tag{B3}$$

The form (B3) is set up as if the ordering (not necessarily time ordering) was that nucleon 1 first emitted a pion which was then abosrbed by nucleon 2. The final bremsstrahlung matrix element turns out to be the same for the opposite order (1 and 2 interchanged). The form (B1) corresponds to instantaneous pion emission, transmission, and absorption, which is suitable for scattering in the adiabatic limit, but leads to inappropriate momentum dependence for bremsstrahlung. The explicit momentum dependence in (B3) is such that photon emission from the propagator $[(\vec{p}_2-\vec{p}_2')^2+\mu^2]^{-1}$ will lead to the appropriate modification of the momenta in the absorption factor $\vec{\sigma}_2\cdot(\vec{p}_2-\vec{p}_2')$ [cf. Eq. (B6)]. Because of the δ function, the form (B3) is equivalent to (B1) for scattering.

The result corresponding to using the form (B3) in Eq. (2.10) is

$$V_{\mathrm{em}+n}(\vec{r}_i)=(2\pi)^{-9}(f/\mu)^2\int d\vec{p}_i\vec{\sigma}_2\cdot[i\vec{\nabla}_2+i\vec{\nabla}_2'-q_2\vec{A}(\vec{r}_2)+q_2'\vec{A}(\vec{r}_2')]\frac{\vec{\tau}^1\cdot\vec{\tau}^2}{[i\vec{\nabla}_2+i\vec{\nabla}_2'-q_2\vec{A}(r_2)+q_2'\vec{A}(r_2')]^2+\mu^2}$$
$$\times\vec{\sigma}_1\cdot[i\vec{\nabla}_1+i\vec{\nabla}_1'-q_1\vec{A}(\vec{r}_1)+q_1'\vec{A}(\vec{r}_1')]\delta(\vec{p}_1+\vec{p}_2-\vec{p}_1'-\vec{p}_2')e^{i(\vec{p}_1'\cdot\vec{r}_1'+\vec{p}_2'\cdot\vec{r}_2'-\vec{p}_1\cdot\vec{r}_1-\vec{p}_2\cdot\vec{r}_2)}\,. \tag{B4}$$

We have not made the replacement $\vec{\nabla}\to\vec{\nabla}-iq\vec{A}$ in the δ function, because it does not lead to a contribution to $V_{\mathrm{em}}^{(2)}$.[20] The charges q_i (q_i') appearing in Eq. (B4) are the charges of the ith (i'th) nucleon. We now introduce the radiation vector potential of Eq. (2.12), perform the gradient operations, and subtract off V_n to give

$$V_{\mathrm{em}}^{(2)}(\vec{r}_i)=(2\pi)^{-9}(f/\mu)^2(2\pi/K)^{1/2}\vec{\tau}^1\cdot\vec{\tau}^2\int d\vec{p}_i\,\delta(\vec{p}_1+\vec{p}_2-\vec{p}_1'-\vec{p}_2')e^{i(\vec{p}_1'\cdot\vec{r}_1'+\vec{p}_2'\cdot\vec{r}_2'-\vec{p}_1\cdot\vec{r}_1-\vec{p}_2\cdot\vec{r}_2)}$$
$$\times\left\{\frac{\vec{\sigma}_2\cdot\hat{\epsilon}\,(q_2'e^{-i\vec{K}\cdot\vec{r}_2'}-q_2e^{-\vec{K}\cdot\vec{r}_2})\vec{\sigma}_1\cdot(\vec{p}_1-\vec{p}_1')}{(\vec{p}_2-\vec{p}_2')^2+\mu^2}\right.$$
$$+\frac{\vec{\sigma}_2\cdot(\vec{p}_2-\vec{p}_2')\vec{\sigma}_1\cdot\hat{\epsilon}(q_1'e^{-i\vec{K}\cdot\vec{r}_1'}-q_1e^{-i\vec{K}\cdot\vec{r}_1})}{(\vec{p}_2-\vec{p}_2')^2+\mu^2}$$
$$\left.-\frac{2\vec{\sigma}_2\cdot(\vec{p}_2-\vec{p}_2'+\vec{K})\vec{\sigma}_1\cdot(\vec{p}_1-\vec{p}_1')(\vec{p}_2-\vec{p}_2')\cdot\hat{\epsilon}(q_2'e^{-i\vec{K}\cdot\vec{r}_2'}-q_2e^{-i\vec{K}\cdot\vec{r}_2})}{[(\vec{p}_2-\vec{p}_2'+\vec{K})^2+\mu^2][(\vec{p}_2-\vec{p}_2')^2+\mu^2]}\right\}. \tag{B5}$$

We note that the $\vec{p}_i$ in Eq. (B5) are Fourier transform variables and are not the actual momenta of the nucleons in $np\gamma$. Three of the four momentum integrals in (B5) can be done immediately, leaving an integration over the variable $\vec{k}$ [defined in Eq. (B2)]

$$V_{\mathrm{em}}^{(2)}(\vec{r}\,',\vec{r})=(f/\mu)^2(2\pi)^{-3}(2\pi/K)^{1/2}\delta(\vec{r}\,'-\vec{r})\int d\vec{k}\,e^{-i\vec{k}\cdot\vec{r}}\left\{\frac{\vec{\sigma}_2\cdot\hat{\epsilon}\vec{\sigma}_1\cdot\vec{k}e^{i\vec{K}\cdot\vec{r}/2}[q_2,\vec{\tau}^1\cdot\vec{\tau}^2]}{k^2+\mu^2}-\frac{\vec{\sigma}_2\cdot\vec{k}\vec{\sigma}_1\cdot\hat{\epsilon}e^{-i\vec{K}\cdot\vec{r}/2}[q_1,\vec{\tau}^1\cdot\vec{\tau}^2]}{k^2+\mu^2}\right.$$
$$\left.-\frac{2\vec{\sigma}_2\cdot(\vec{k}-\vec{K})\vec{\sigma}_1\cdot\vec{k}\hat{\epsilon}\cdot\vec{k}e^{-i\vec{K}\cdot\vec{r}/2}[q_2,\vec{\tau}^1\cdot\vec{\tau}^2]}{[(\vec{k}-\vec{K})^2+\mu^2](k^2+\mu^2)}\right\}$$
$$=V_{\mathrm{em}}^{(2)}(\vec{r})\delta(\vec{r}\,'-\vec{r})\,. \tag{B6}$$

Now, the q_i are to be considered the isotopic spin operators defined in Eq. (2.21). We note from Eq. (B6) that the bremsstrahlung potential from OPE is local, except for its obvious exchange character.

The charge-operator commutators give

$$[q_1,\vec{\tau}^1\cdot\vec{\tau}^2]=-[q_2,\vec{\tau}^1\cdot\vec{\tau}^2]=ie(\vec{\tau}^1\times\vec{\tau}^2)_3\,. \tag{B7}$$

The operator $(\frac{1}{2}i)(\vec{\tau}^1\times\vec{\tau}^2)_3$ acting on eigenstates of total isotopic spin $|0\rangle$ and $|1\rangle$ with $I_3=0$ (np system) has the effect

$$(\tfrac{1}{2}i)(\vec{\tau}^1\times\vec{\tau}^2)_3|0\rangle=|1\rangle \tag{B8}$$

and

$$(\tfrac{1}{2}i)(\vec{\tau}^1\times\vec{\tau}^2)_3|1\rangle=-|0\rangle \tag{B8$'$}$$

so that it just changes the isotopic spin (up to a sign).

We can change the integration variable in (B6) and, using the fact that $\hat{\epsilon}\cdot\vec{K}=0$, put it in the form

$$V_{\rm em}^{(2)}(\vec{r}) = -(\tfrac{1}{2}ie)(f/\mu)^2(2\pi)^{-3}(2\pi/K)^{1/2}(\vec{\tau}^1\times\vec{\tau}^2)_3\int d\vec{k}\,e^{-i\vec{k}\cdot\vec{r}} \times\left\{\frac{\vec{\sigma}_1\cdot\hat{\epsilon}\vec{\sigma}_2\cdot(\vec{k}-\frac{1}{2}\vec{K})}{(\vec{k}-\frac{1}{2}\vec{K})^2+\mu^2}+\frac{\vec{\sigma}_1\cdot(\vec{k}+\frac{1}{2}\vec{K})\vec{\sigma}_2\cdot\hat{\epsilon}}{(\vec{k}+\frac{1}{2}\vec{K})^2+\mu^2} - \frac{2\hat{\epsilon}\cdot\vec{k}\vec{\sigma}_1\cdot(\vec{k}+\frac{1}{2}\vec{K})\vec{\sigma}_2\cdot(\vec{k}-\frac{1}{2}\vec{K})}{[(\vec{k}-\frac{1}{2}\vec{K})^2+\mu^2][(\vec{k}+\frac{1}{2}\vec{K})^2+\mu^2]}\right\}. \quad \text{(B9)}$$

Equation (B9) agrees with the result that would follow directly from the Feynman diagrams of Fig. 4 if $\vec{k}$ is defined by (B2) and the momentum conservation $(\vec{p}_1+\vec{p}_2=\vec{p}_1'+\vec{p}_2'+\vec{K})$ [cf. the discussion following Eq. (2.20)] appropriate to bremsstrahlung is used.[11,21]

The integral in Eq. (B9) cannot be done exactly, but it is useful to expand it in powers of K. This turns out to be an expansion in K/μ, either explicitly or implicitly, because the range of $V_{\rm em}^{(2)}$ is $1/\mu$. For photon energies for which the potential picture would be reasonable, this expansion is expected to converge rapidly. We write

$$V_{\rm em}^{(2)} = \sum_{i=0}^{\infty} V_i, \quad \text{(B10)}$$

where the subscript i denotes the order of K in the expansion. The V_i are given by

$$V_i = -i\,e(f/\mu)^2(2\pi)^{-3}(2\pi/K)^{1/2}(\vec{\tau}^1\times\vec{\tau}^2)_3 \times\int d\vec{k}\,e^{-i\vec{k}\cdot\vec{r}}\,v_i(\vec{k}), \quad \text{(B11)}$$

and the first three v_i are

$$v_0 = \hat{\epsilon}\cdot\vec{\nabla}_k\vec{\sigma}_1\cdot\vec{k}\vec{\sigma}_2\cdot\vec{k}/(k^2+\mu^2), \quad \text{(B12)}$$

$$v_1 = \frac{(\vec{\sigma}_1\times\vec{\sigma}_2)}{(k^2+\mu^2)}\cdot\left[\frac{\vec{k}\vec{k}-k^2\overleftrightarrow{I}}{k^2+\mu^2}+\tfrac{1}{2}\overleftrightarrow{I}\right]\cdot(\vec{K}\times\hat{\epsilon}), \quad \text{(B13)}$$

where $\overleftrightarrow{I}$ is the unit dyadic, and

$$v_2 = \frac{(\hat{\epsilon}\cdot\vec{k})(\vec{\sigma}_1\cdot\vec{K})(\vec{\sigma}_2\cdot\vec{K})}{2(k^2+\mu^2)^2} - \frac{(\vec{K}\cdot\vec{r})^2[(\vec{\sigma}_1\cdot\hat{\epsilon})(\vec{\sigma}_2\cdot\vec{k})+(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\hat{\epsilon})]}{8(k^2+\mu^2)} - \frac{2(\hat{\epsilon}\cdot\vec{k})(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k})}{(k^2+\mu^2)^4}[(\vec{k}\cdot\vec{K})^2-\tfrac{1}{2}K^2(k^2+\mu^2)]. \quad \text{(B14)}$$

The zeroth order potential V_0 is just what we have used when the commutator $[\vec{A},\vec{\nabla}]$ was neglected. The first order potential V_1 is the well known "exchange moment" contribution and can be put (after some algebra) in the form[11]

$$V_1(r) = \vec{M}\cdot\vec{H}, \quad \vec{H} = -i(2\pi/K)^{1/2}(\vec{K}\times\hat{\epsilon}), \quad \text{(B15)}$$

with

$$\vec{M} = -\tfrac{1}{2}e(f^2/4\pi\mu^2)(\vec{\tau}^1\times\vec{\tau}^2)_3(e^{-\mu r}/r)[(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot\hat{r}\hat{r}(1+\mu r) - (\vec{\sigma}_1\times\vec{\sigma}_2)\mu r]. \quad \text{(B16)}$$

After some algebra, the second-order potential can be written

$$V_2(\vec{r}) = \tfrac{1}{24}e(2\pi/K)^{1/2}(\vec{\tau}^1\times\vec{\tau}^2)_3\hat{\epsilon}\cdot\vec{r}(\vec{K}\cdot\vec{r})^2V^{\rm OPE} + \tfrac{1}{12}e(2\pi/K)^{1/2}(f^2/4\pi\mu^2)(\vec{\tau}^1\times\vec{\tau}^2)_3e^{-\mu r} \times\{(\vec{\sigma}_1\cdot\hat{\epsilon}\vec{\sigma}_2\cdot\hat{r}+\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{\epsilon})[(1+\mu r)(\vec{K}\cdot\hat{r})^2-K^2] -(1+\mu r)\vec{K}\cdot\hat{r}\,\hat{\epsilon}\cdot\hat{r}(\vec{\sigma}_1\cdot\vec{K}\vec{\sigma}_2\cdot\hat{r}+\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\vec{K}) +\vec{K}\cdot\hat{r}(\vec{\sigma}_1\cdot\vec{K}\vec{\sigma}_2\cdot\hat{\epsilon}+\vec{\sigma}_1\cdot\hat{\epsilon}\vec{\sigma}_2\cdot\vec{K}) - 2\hat{\epsilon}\cdot\hat{r}\vec{\sigma}_1\cdot\vec{K}\vec{\sigma}_2\cdot\vec{K} +K^2\hat{\epsilon}\cdot\hat{r}[(1+\mu r)\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}-\vec{\sigma}_1\cdot\vec{\sigma}_2]\}. \quad \text{(B17)}$$

The first term of Eq. (B17) comes from the modification of the meson propagator and would be the same (with the appropriate V_n) for any type of exchange. It can be taken into account in Eq. (2.25) by simply making the replacement $\vec{r}\rightarrow\vec{r}-\frac{1}{24}\vec{r}(\vec{K}\cdot\vec{r})^2$. This then includes some order-K^2 corrections for the entire potential (and not just OPE). We have included this term in some $np\gamma$ calculations and it has very little effect.

All the terms in $V_{\rm em}^{(2)}$ of even order in K (not counting the $K^{-1/2}$ in the photon "wave function") are of odd parity and, since the isospin must change, satisfy $\Delta S=0$. On the other hand, all terms of odd order in K are of even parity with $\Delta S=1$. For $V_{\rm em}^{(1)}$, the electric terms have $\Delta S=0$ while the magnetic terms (which are of order K) can have $\Delta S=1$ or 0. This means that, if nucleon polarizations are not measured, the order-K terms of $V_{\rm em}^{(2)}$ can only interfere with the order-K magnetic terms of $V_{\rm em}^{(1)}$ to give an order-K^2 correction to the $np\gamma$ cross section. Thus, a consistent treat-

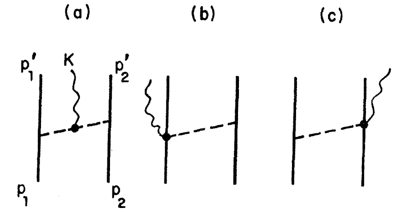

FIG. 4. Feynman graphs for $V_{\rm em}^{(2)}$ due to OPE.

ment of the higher orders in K of $V_{em}^{(2)}$ would require keeping both V_1 and V_2, which might be expected to contribute in the same order in the $np\gamma$ cross section.

We indicate this schematically below. We can write the $np\gamma$ amplitude as

$$A=[K^{-1}E+M+V_0+K\overline{V}_1+K^2\overline{V}_2]\,, \tag{B18}$$

where E and M are the electric and magnetic parts, respectively, of $V_{em}^{(1)}$, and $K^i\overline{V}_i$ are the ith order (in K) parts of $V_{em}^{(2)}$ [the V_i of Eq. (B10)]. The V_i are independent of K, and the terms E and M approach finite values as $K\to 0$. The rescattering contribution of $V_{em}^{(1)}$ starts out with one power of K higher than the corresponding electric and magnetic-pole contributions and can be included with E and M for this discussion. The unpolarized $np\gamma$ cross section is proportional to $K|A|^2$ summed over spins, which can be written

$$\begin{aligned}K|A|^2=K^{-1}|E|^2&+2\,\mathrm{Re}E^*(V_0+M_{\Delta S=0})+K(|M|^2+|V_0|^2\\&+2\,\mathrm{Re}M^*_{\Delta S=0}V_0)+K^2(2\,\mathrm{Re}M^*_{\Delta S=1}\overline{V}_1+2\,\mathrm{Re}E^*\overline{V}_2)\\&+O(K^3)\,.\end{aligned} \tag{B19}$$

Other cross terms are absent from (B19) as seen in our previous discussion. It is clear from (B19) that V_0 can have a large effect because of its interference with the electric pole term E and that V_0 enters *two* orders in K before either $\overline{V}_1$ or $\overline{V}_2$, which both first appear to order K^2.

It has been suggested that the order-K term in $V_{em}^{(2)}$ will be important because it introduces new spin transitions to the $np\gamma$ amplitude.[22] This is indeed the case for threshold radiative np capture[23] because it is the leading contribution to the transition ${}^1S_0\to{}^3D_1$ (of the deuteron) which interferes with the dominant ${}^1S_0\to{}^3S_1$ transition. However, for n-p threshold capture, initial S states are all that need be considered and the magnetic part of $V_{em}^{(1)}$ is the leading contribution. While for $np\gamma$ at moderate energies, the S states do not play a dominant role and the electric part of $V_{em}^{(1)}$ is the leading contribution. This could diminish the importance of V_1 and make its effect comparable to V_2 as discussed above. If nucleon polarization data were obtained, then V_1 would be expected to play a significant role.

We have indicated in this appendix how the higher-order terms in K can be generated. We limited our discussion to the case of OPE because the effective expansion in K/μ indicates that the higher orders will be dominated by the long-range force, well fitted by OPE. Even two-pion exchange might be expected to contribute more to these higher orders than heavy-meson exchange.

The prescription that worked for OPE (in the sense of reproducing the field-theory result) is a reasonable one which is readily extended to more general uses, if desired. For any potential whose Fourier transform can be put in the form

$$U(p_i)=V(\vec\sigma_2,\vec p_2,\vec p_2{}')G(\vec p_2,\vec p_2{}')V(\vec\sigma_1,\vec p_1,\vec p_1{}')\delta(\vec p_1+\vec p_2-\vec p_1{}'-\vec p_2{}') \tag{B20}$$

with V and G arbitrary, the same development will work. The momenta have to be arranged in the order shown (or 1 and 2 everywhere interchanged) so that photon emission at one point will have the appropriate effect on the other momenta, consistent with momentum conservation everywhere.

Appendix C

The Bryan-Scott III potential is a momentum-dependent, one-boson-exchange potential that satisfies the Schrödinger equation

$$(-\nabla^2/m+\tilde V_i)\tilde\psi_i=E\tilde\psi_i\,, \tag{C1}$$

where

$$\tilde V_i=V_i-(1/m)(\nabla^2\phi_i+\phi_i\nabla^2)\,. \tag{C2}$$

The subscript i in Eqs. (C1) and (C2) represents isospin; the fact that there are two separate isospin equations is a consequence of the customary assumption of charge independence. If we introduce the usual transformation[24]

$$\tilde\psi_i=\psi_i/(1+2\phi_i)^{1/2}\,, \tag{C3}$$

then we obtain the familiar Schrödinger equation for a static potential

$$(-\nabla^2/m+W_i)\psi_i=E\psi_i\,, \tag{C4}$$

where

$$W_i=(V_i+2\phi_iE)/(1+2\phi_i)-[\phi_i'/(1+2\phi_i)]^2/m. \tag{C5}$$

The effect of the exchange potential (3.13) acting on $\tilde\psi_i$ in terms of the second derivative of ϕ_i and the first derivative of $\tilde\psi_i$, can be obtained using Eqs. (C2) and (C3) and some operator manipulations as

$$\begin{aligned}\tfrac12(\tilde V_1-\tilde V_0)\tilde\psi_1=\tfrac12(1+2\phi_0)&\\ \times\Big\{(W_1-W_0)\tilde\psi_1&+\left(\frac{2\phi_0'}{1+2\phi_0}-\frac{2\phi_1'}{1+2\phi_1}\right)\left(\frac{\tilde\psi_1}{r}+\tilde\psi_1'\right)\\&+\left[\frac{\phi_0''}{1+2\phi_0}-\frac{\phi_1''}{1+2\phi_1}-\left(\frac{\phi_0'}{1+2\phi_0}\right)^2\right.\\&\left.+\left(\frac{\phi_1'}{1+2\phi_1}\right)^2\right]\tilde\psi_1\Big\}\end{aligned} \tag{C6}$$

The primes in Eqs. (C5) and (C6) denote radial derivatives. To obtain the corresponding equation for the exchange potential acting on $\tilde\psi_0$ just interchange the subscripts 0 and 1 in Eq. (C6).

*This work was performed in part under the auspices of the U.S. Atomic Energy Commission.

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[17]It is of interest to notice from Tables I and II that the cross section does not necessarily increase with total inclusive angle, but depends separately on θ_n and θ_p, and within our geometry the predicted cross section seems to increase with θ_n but decrease with θ_p.

[18]The result in Fig. 1(b) includes a minor modification in the exchange contribution (on the proton side of the beam) in our calculation for 200 MeV (30, 30°) presented at the Gull-Lake Conference. (See Ref. 6.)

[19]This represents a modification of the preliminary results for the rescattering contributions as presented in Ref. 4.

[20]This can be seen by using an explicit representation of the δ function such as

$$\delta(x) = (1/\pi)\lim_{\alpha\to 0} \alpha/(\alpha^2 + x^2).$$

[21]The "seagull" diagrams of Fig. 4(b) and 4(c) which correspond to the first two terms in Eq. (B6), are the nonrelativistic limit of the nucleon-pair terms, which would be present with a γ_5 pion-nucleon interaction. The reduction from pair diagrams to seagull diagrams parallels the well-known nonrelativistic reduction $\gamma_5 \rightarrow \vec{\sigma}\cdot\overrightarrow{\nabla}/2m$. For a discussion of this reduction, see Ref. 23.

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