PHYSICAL REVIEW C VOLUME 8, NUMBER 5 NOVEMBER 1973

Study of the Sasakawa Approach to Two-Particle Scattering

David Blasczak and Michael G. Fuda
Department of Physics and Astronomy, State University of New York, Buffalo, New York 14214
(Received 25 June 1973)

It is shown that the Kowalski formulation of the Sasakawa approach to potential scattering can be used as the basis for a momentum-space formulation of the theory of the Jost function. Two examples are presented for which the Kowalski equations can be solved in closed form. One example is a separable potential, and the other is the exponential potential. The separable potential illustrates the fact that the series obtained by iterating Kowalski's equations does not always converge. The exponential potential provides a verification of Coester's proof that the iteration series does converge for a certain class of local potentials of arbitrary strength. The practicality of Kowalski's equations are demonstrated by using them to calculate the phase shifts and half-off-shell T matrix that are produced by the Reid potential in some of the uncoupled states of the two-nucleon system.

I. INTRODUCTION

In the usual integral formulation of the Schrödinger equation, the inhomogeneous term is a spherical Bessel function. Iteration of this integral equation leads to the well-known Born series for the scattering amplitude. In general, this series does not converge for arbitrary values of the potential strength. Several years ago Sasakawa[1] developed a new integral equation for describing nonrelativistic potential scattering. In his integral equation, the inhomogeneous term is the sum of a spherical Bessel function and a term, which is the product of the scattering amplitude and a spherical Hankel function. The Sasakawa[1] equation is an integral equation of the Volterra type. Coester[2] has shown that the iterative solution of this equation converges for all local potentials for which the function $rV(r)$ is absolutely integrable. Sasakawa's[1] integral equation has been generalized to nonlocal potentials and to a broader class of inhomogeneous terms.[2,3] Austern[3] has used the Sasakawa approach to develop a theory of inelastic scattering and rearrangement collisions. It has also been shown that this approach can be applied to the Faddeev equations for three-particle scattering.[4] Kowalski[5] has shown that the Sasakawa[1] approach can be formulated as a technique for removing the singularities in the kernels of the standard integral equations for the T matrix.

In this paper we shall show how Kowalski's[5] equations can be used as the basis for a momentum-space formulation of the theory of the Jost[6] function. As is well known, the Jost[6] function provides a convenient framework for the development of a rigorous treatment of potential scattering.[7,8] We shall also present two examples for which the iterative solution of Kowalski's[5] equations can be obtained in closed form. The first example is a separable potential, and illustrates that the sequence of iterations does not converge for a nonlocal potential of arbitrary strength. The second example is provided by the exponential potential. For the exponential potential, it is found that the terms in the series obtained by iterating Kowalski's[5] equations correspond to the terms in the infinite-series representation of a generalized hypergeometric function. We shall demonstrate the practicality of Kowalski's[5] equations by using them to calculate the phase shifts and half-off-shell T matrix produced by the Reid[9] potential in some of the uncoupled two-nucleon partial waves.

In Sec. II we summarize Kowalski's[5] equations and show how they relate to the theory of the Jost function. The analysis of the separable potential and the exponential potential are presented in Sec. III. The work on the Reid potential is given in Sec. IV. Section V is a brief discussion.

II. GENERAL METHOD

In general, the fully off-shell T matrix can be obtained as the solution of either of the equations given below,

$$T_l(p,p';s) = V_l(p,p') + \int_0^\infty V_l(p,q)\,\frac{q^2dq}{s-q^2}\,T_l(q,p';s) \tag{2.1a}$$

$$= V_l(p,p') + \int_0^\infty T_l(p,q;s) \times \frac{q^2dq}{s-q^2}\,V_l(q,p'), \tag{2.1b}$$

where $V_l(p,p')$ is the matrix element of the potential taken with respect to the states whose space representation is

$$\langle \vec{r}\,|plm\rangle = \sqrt{2/\pi}\; j_l(pr)Y_{lm}(\hat{r}). \tag{2.2}$$

The states (2.2) are normalized so that

$$\langle plm|p'l'm'\rangle = \frac{\delta(p-p')}{p^2}\,\delta_{ll'}\delta_{mm'}. \tag{2.3}$$

When $s = k^2 \pm i\epsilon$ $(0 < \epsilon \ll 1)$, the Eqs. (2.1) are difficult to solve numerically, since the kernel has a singularity adjacent to the path of integration. In his treatment of the Sasakawa[1] method, Kowalski[5] has shown how to remove the singularity. He has shown that the half-off-shell T matrix can be obtained from the relation

$$T_l(p,k;k^2+i\epsilon) = \frac{\Gamma_l(p,k)}{\Gamma_l(k,k)}\,T_l(k), \tag{2.4}$$

where the on-shell T matrix $T_l(k)$ is obtained from

$$T_l(k) = \frac{\Gamma_l(k,k)}{1 - \int_0^\infty \gamma_l(k,q)\,\frac{q^2dq}{k^2+i\epsilon-q^2}\,\Gamma_l(q,k)}. \tag{2.5}$$

Here γ_l is a function which has the property

$$\gamma_l(k,k) = 1 \tag{2.6}$$

and Γ_l is the solution of the nonsingular integral equation

$$\Gamma_l(p,k) = V_l(p,k) + \int_0^\infty A_l(p,q;k^2)dq\,\Gamma_l(q,k), \tag{2.7}$$

whose kernel is given by

$$A_l(p,q;k^2) = [V_l(p,q) - V_l(p,k)\gamma_l(k,q)]q^2/(k^2-q^2). \tag{2.8}$$

It is easy to show that $T_l(k)$ as given by (2.5) satisfies the unitarity relation

$$\mathrm{Im}T_l(k) = -\frac{\pi k}{2}\,|T_l(k)|^2 \tag{2.9}$$

for any real Γ_l. In particular the iteration solution of (2.7) to any order, will yield an on-shell T matrix that satisfies (2.9). It follows from (2.9) that our on-shell T matrix has the normalization

$$T_l(k) = -\frac{2}{\pi k}\,e^{i\delta_l(k)}\sin\delta_l(k), \tag{2.10}$$

where $\delta_l(k)$ is the phase shift for the lth partial wave.

In this paper, we shall consider the following iteration scheme for solving (2.7):

$$\Gamma_l^{(0)}(p,k) = V_l(p,k), \tag{2.11a}$$

$$\Gamma_l^{(n+1)}(p,k) = V_l(p,k) + \int_0^\infty A_l(p,q;k^2)dq\Gamma_l^{(n)}(q,k). \tag{2.11b}$$

The nth order approximation for the on-shell T matrix is given by

$$T_l^{(n)}(k) = \frac{\Gamma_l^{(n)}(k,k)}{1 - \int_0^\infty \gamma_l(k,q)\,\frac{q^2dq\Gamma_l^{(n)}(q,k)}{k^2+i\epsilon-q^2}}. \tag{2.12}$$

It is easy to show from Coester's[2] work on the Sasakawa[1] method that this iteration scheme always converges if the potential is local and γ_l is taken to be

$$\gamma_l(k,q) = (q/k)^l. \tag{2.13}$$

We shall now show that if γ_l is taken to be

$$\gamma_l(k,q) = T_l(k,q;k^2+i\epsilon)/T_l(k), \tag{2.14}$$

then each order of iteration gives the exact value for the on-shell T matrix. From (2.1), (2.8), and (2.11b), it follows that

$$\int_0^\infty \gamma_l(k,q)\,\frac{q^2dq}{k^2+i\epsilon-q^2}\,\Gamma_l^{(n)}(q,k) = 1 - \frac{\Gamma_l^{(n)}(k,k)}{T(k)} \tag{2.15}$$

Putting (2.15) into (2.12), we find

$$T_l^{(n)}(k) = T_l(k). \tag{2.16}$$

This result can be used as an aid in determining some of the properties that γ_l should have so that the iteration scheme (2.11a)–(2.12) converges as rapidly as possible. In particular, it follows from the results of Ref. 10 and (2.14) that

$$\gamma_l(k,q) \underset{k,q\to 0}{\longrightarrow} (q/k)^l. \tag{2.17}$$

The choice (2.13) satisfied this relation.

The choice (2.13) leads to a connection between the approach to the T matrix being considered here and the Jost[6] function. The Jost[6] function is a function which plays an essential role in the rigorous treatment of potential scattering.[7,8] The phase of the Jost function is the negative of the phase shift, i.e.,

$$f_l(k) = |f_l(k)|\,e^{-i\delta_l(k)}, \tag{2.18}$$

and the zeroes of the Jost function in the upper half of the k plane fall on the imaginary axis and correspond to the energies of the bound states.[8] It is well known that the Schrödinger wave function labeled by the wave number k, and the angular

momentum quantum numbers l and m, can be written in the form

$$|\Psi_{klm}\rangle = |klm\rangle + (k^2 + i\epsilon - H_0)^{-1} T(k^2 + i\epsilon)|klm\rangle , \tag{2.19}$$

where H_0 is the kinetic energy operator and T is the transition operator whose matrix elements taken with respect to the states (2.2) are the solutions of (2.1). It is not hard to show from (2.2), (2.10), and (2.19) that

$$\langle \vec{r}\,|\Psi_{klm}\rangle = \sqrt{2/\pi}\ \psi_l(k,r) Y_{lm}(\hat{r}) , \tag{2.20}$$

where ψ_l is the solution of the radial Schrödinger equation with the normalization

$$\psi_l(k,r) \underset{r\to\infty}{\sim} (kr)^{-1} e^{i\delta_l(k)} \sin(kr - \tfrac{1}{2} l\pi + \delta_l) . \tag{2.21}$$

It follows from (2.21) and Eqs. (11.5) and (12.145) of Ref. 8 that

$$\psi_l(k,r) = \frac{k^l \phi_l(k,r)}{f_l(k)(2l+1)!!\, r} . \tag{2.22}$$

Here ϕ_l is a regular solution of the Schrödinger equation with the normalization

$$\lim_{r\to 0} r^{-l-1}\phi_l(k,r) = 1 . \tag{2.23}$$

It is shown in Ref. 5 that

$$T_l(p,k;k^2+i\epsilon) = \Gamma_l(p,k)[1 + I_l(k)] , \tag{2.24}$$

where

$$I_l(k) = \int_0^\infty \gamma_l(k,q) \frac{q^2 dq}{k^2 + i\epsilon - q^2} T_l(q,k;k^2+i\epsilon) . \tag{2.25}$$

It follows from (2.2), (2.3), (2.6), (2.19), (2.20), and (2.25) that

$$1 + I_l(k) = \frac{2}{\pi} \int_0^\infty r^2 dr \psi_l(k,r) \int_0^\infty q^2 dq \gamma_l(k,q) j_l(qr) . \tag{2.26}$$

The completeness relation for the spherical Bessel functions is given by

$$(2/\pi) \int_0^\infty q^2 dq\ j_l(qr) j_l(qr') = \frac{\delta(r-r')}{r^2} . \tag{2.27}$$

Using this, (2.22), (2.23), (2.26), and making the choice (2.13), it is not hard to see that

$$1 + I_l(k) = f_l^{-1}(k) . \tag{2.28}$$

It now follows from (2.5), (2.24), and (2.28) that if $\gamma_l(k,q) = (q/k)^l$ then

$$\Gamma_l(p,k) = f_l(k) T_l(p,k;k^2+i\epsilon) \tag{2.29}$$

and

$$f_l(k) = 1 - \int_0^\infty \gamma_l(k,q) \frac{q^2 dq}{k^2 + i\epsilon - q^2} \Gamma_l(q,k) . \tag{2.30}$$

These relations provide the basis for a momentum-space formulation of the Jost function, which, as far as we know, has not been given before.

We shall illustrate the results of this section in the next section by considering some exactly solvable examples.

III. EXACTLY SOLVABLE EXAMPLES

A. Separable Potential

As our first example, we shall consider a potential of the form

$$V_l(p,q) = g_l(p)\lambda_l g_l(q) , \tag{3.1}$$

where g_l is a real function and λ_l determines the strength of the potential. Inserting (3.1) into (2.11a) and (2.11b) and using (2.8), we find that

$$\Gamma_l^{(n)}(p,k) = g_l(p)\lambda_l F_l^{(n)}(k) , \tag{3.2}$$

where $F_l^{(n)}$ is the solution of the system of equations

$$F_l^{(0)}(k) = g_l(k) , \tag{3.3a}$$

$$F_l^{(n+1)}(k) = g_l(k) + J_l(k) F_l^{(n)}(k) . \tag{3.3b}$$

Here

$$J_l(k) = \lambda_l \int_0^\infty g_l(q)[g_l(q) - g_l(k)\gamma_l(k,q)] \frac{q^2 dq}{k^2 - q^2} . \tag{3.4}$$

It is easy to verify that the solution of (3.3a) and (3.3b) is

$$F_l^{(n)}(k) = g_l(k) \frac{1 - J_l^{n+1}(k)}{1 - J_l(k)} , \quad n = 0, 1, 2, \ldots . \tag{3.5}$$

Obviously this sequence converges if

$$|J_l(k)| < 1 . \tag{3.6}$$

The parameter J_l is the eigenvalue of the equation

$$\int_0^\infty A_l(p,q;k^2) dq \Gamma_l(q,k) = J_l(k)\Gamma_l(p,k); \tag{3.7}$$

thus the condition (3.6) is an example of the well-known result that the eigenvalues of the kernel of an integral equation must be less than one in magnitude if the Neumann or Born series is to converge.[11]

The sequence (3.5) converges immediately if γ_l is taken to be

$$\gamma_l(k,q) = g_l(q)/g_l(k) . \tag{3.8}$$

This follows from (3.4) and can be shown to be equivalent to the choice (2.14). From (2.4) and (2.10), we have

$$k\cot\delta_l(k)=\frac{2}{\pi\Gamma_l(k,k)}\times\left[P\int_0^\infty\frac{\gamma_l(k,q)q^2da\Gamma_l(q,k)}{k^2-q^2}-1\right], \tag{3.9}$$

where P stands for the Cauchy principal value. The exact value of $k\cot\delta_l$ arising from the potential (3.1) can be obtained by substituting (3.8) into (3.9), and by replacing Γ_l with the potential V_l [see (2.11a)]. Using (2.12), (3.2), (3.5), and (3.9) it is not hard to show that

$$k\cot\delta_l^{(n)}(k)=k\cot\delta_l(k)-\frac{2}{\pi}\frac{J_l^{n+1}(k)[1-J_l(k)]}{V_l(k,k)[1-J_l^{n+1}(k)]}. \tag{3.10}$$

This relation shows that one can get false convergence if $|J_l(k)|>1$, since for values of $|J_l(k)|$ in this range, the second term on the right-hand side of (3.10) approaches a well-defined limit, which is different from zero. It is not hard to detect this false convergence, since under these circumstances it follows from (3.2) and (3.5) that $\Gamma_l^{(n)}$ approaches infinity as n does.

B. Exponential Potential

An example for which Coester's[2] convergence proof holds is provided by the potential

$$V(r)=-V_0e^{-r/a}. \tag{3.11}$$

The s-wave Jost function for this potential is known,[8] and is given by

$$f_0(k)=(V_0a^2)^{ika}\Gamma(1-2ika)J_{-2ika}(2V_0^{1/2}a) \tag{3.12a}$$

$$=\sum_{n=0}^\infty\frac{(-V_0a^2)^n}{(1-2ika)_n\,n!}. \tag{3.12b}$$

Here $\Gamma(z)$ is the γ function and $(z)_n$ is the Pochhammer symbol, which is defined by

$$(z)_n=\Gamma(z+n)/\Gamma(z). \tag{3.13}$$

The series (3.12b) converges for all values of the potential strength V_0.

The exact s-wave T matrix for the potential (3.11) has been worked out previously.[12] Using the expressions given in Ref. 12, as well as (2.29) and (3.12a), it can be shown that

$$\Gamma_0(p,k)=(\pi ip)^{-1}\sum_{n=0}^\infty\left[\frac{1}{(1-ipa+ika)_n(1-ipa-ika)_n}-\frac{1}{(1+ipa-ika)_n(1+ipa+ika)_n}\right](-V_0a^2)^n. \tag{3.14}$$

TABLE I. Phase shifts (in rad) for the $^1S_0$ state. These calculations were done with the 32-point Gauss-Legnedre quadrature rule with $c=5.0$ [see (4.8)].

Iteration \ c.m. Energy (MeV)	12	24	48	72	104	152	176
0	−0.040	−0.065	−0.106	−0.141	−0.180	−0.227	−0.247
1	−0.016	−0.050	−0.114	−0.170	−0.236	−0.318	−0.353
2	0.055	0.018	−0.064	−0.140	−0.229	−0.339	−0.387
3	0.173	0.130	0.025	−0.072	−0.183	−0.320	−0.379
4	0.335	0.273	0.136	0.016	−0.117	−0.278	−0.346
5	0.515	0.421	0.246	0.105	−0.048	−0.229	−0.305
6	0.672	0.542	0.335	0.176	0.009	−0.187	−0.268
7	0.777	0.621	0.393	0.223	0.047	−0.157	−0.241
8	0.833	0.663	0.424	0.249	0.068	−0.140	−0.226
9	0.855	0.580	0.436	0.260	0.077	−0.132	−0.219
10	0.861	0.685	0.440	0.263	0.080	−0.130	−0.217
11	0.861	0.685	0.441		0.081	−0.129	−0.216
12	0.861	0.685	0.440		0.080	−0.130	
13	0.860	0.684					
14	0.860	0.684					
15	0.860	0.684					
16	0.861	0.685					
Reid's values	0.862	0.696	0.454	0.277	0.093	−0.118	−0.205

This Γ_0 arises from the choice (2.18) for γ_0; i.e., $\gamma_0 = 1$. The series in (3.14) also converges for all values of V_0. Each term in the series corresponds to an iteration of (2.7). The relations (2.5), (2.30), (3.12b), and (3.14) can be combined to give an expression for the on-shell T matrix, which is the ratio of two convergent power series in the potential strength. The convergence of these power series confirms Coester's[8] result that the Sasakawa iteration scheme always converges if the potential is local and the choice (2.13) is made for γ_l.

In the next section, we shall consider a realistic example for which the results cannot be obtained analytically.

IV. REID POTENTIAL

In this section, we shall use the relations given in Sec. II to calculate the on-shell, as well as the half-off-shell T matrix for the Reid[9] potential in some of the uncoupled two-nucleon states. The choice (2.13) will be used for all of the calculations. The states we shall treat are the $^1S_0$, $^1D_2$, and the $^3P_1$. The $^1S_0$ and $^1D_2$ potentials are simply superpositions of Yukawa potentials. For a single term of the form

$$V(r) = -V_0 \frac{e^{-\mu r}}{\mu r} \tag{4.1}$$

the matrix element taken with respect to the states given by (2.2) turns out to be

$$\langle plm \,|\, V \,|\, qlm \rangle = V_l(p, q),$$
$$= -\frac{V_0}{\pi \mu p q} Q_l\left[\frac{p^2 + q^2 + \mu^2}{2pq}\right], \tag{4.2}$$

where Q_l is the Legendre function, whose integral representation is

$$Q_l(z) = \frac{1}{2}\int_{-1}^{1} \frac{P_l(t)\,dt}{z - t}. \tag{4.3}$$

P_l is a Legendre polynomial. The $^3P_1$ potential contains a sum of Yukawa shapes plus a term of the form

$$U(r) = h[(2/x + 2/x^2)e^{-x} - (8/x + 2/x^2)e^{-4x}]/x, \tag{4.4}$$
$$x = \mu r.$$

The matrix element of this potential can be obtained by integrating the relation (4.2) with respect to the parameter μ, and by using (4.3) and the identity

$$Q_l^{-1}(z) = -(z^2 - 1)^{-1/2}\int_z^{\infty} Q_l(z')\,dz'. \tag{4.5}$$

The result is

$$\langle plm \,|\, U \,|\, qlm \rangle = U_1(p, q)$$
$$= \frac{2h}{\mu^3 \pi}[(b^2 - 1)^{1/2} Q_1^{-1}(b) - (a^2 - 1)^{1/2} Q_1^{-1}(a)], \tag{4.6}$$

where

$$a = \frac{p^2 + q^2 + \mu^2}{2pq},$$
$$b = \frac{p^2 + q^2 + 16\mu^2}{2pq}. \tag{4.7}$$

In order to carry out numerically the iteration scheme given by (2.11a) and (2.11b), we have replaced the integral in (2.11b) with a sum by using a quadrature rule which was obtained by mapping the Gauss-Legendre points and weights from the interval $(-1, 1)$ to the interval $(0, \infty)$ by means of the transformation

$$q = c\frac{1 + x}{1 - x}. \tag{4.8}$$

The phase shifts were obtained from (3.9) by carrying out the integral using the same points

TABLE II. Phase shifts (in rad) for the $^3P_1$ state. These calculations were done with the 32-point Gauss-Lengendre quadrature rule with $c = 2.5$ [see (4.8)].

Iteration \ c.m. Energy (MeV)	12	24	48	72	104	152	176
0	−0.026	−0.049	−0.088	−0.120	−0.154	−0.195	−0.211
1	−0.048	−0.088	−0.154	−0.208	−0.267	−0.334	−0.362
2	−0.064	−0.115	−0.198	−0.265	−0.337	−0.421	−0.455
3	−0.073	−0.130	−0.221	−0.294	−0.372	−0.463	−0.500
4	−0.076	−0.135	−0.230	−0.305	−0.385	−0.478	−0.516
5	−0.077	−0.137	−0.232	−0.308	−0.389	−0.482	−0.520
6	−0.078		−0.232		−0.390	−0.483	−0.521
7			−0.233				
Reid's values	−0.074	−0.133	−0.228	−0.304	−0.386	−0.479	−0.518

TABLE III. Phase shifts (in rad) for the $^1D_2$ state. These calculations were done with the 32-point Gauss-Legendre quadrature rule with $r=0.5$ [see (4.8)].

Iteration \ c.m. Energy (MeV)	12	24	48	72	104	152	176
0	0.002	0.004	0.008	0.011	0.016	0.020	0.021
1	0.004	0.008	0.018	0.026	0.036	0.045	0.047
2	0.006	0.014	0.029	0.043	0.059	0.074	0.078
3	0.008	0.018	0.039	0.058	0.080	0.100	0.105
4	0.009	0.022	0.046	0.070	0.095	0.120	0.126
5	0.010	0.024	0.052	0.078	0.106	0.134	0.141
6	0.011	0.026	0.055	0.083	0.114	0.144	0.151
7	0.011	0.027	0.058	0.087	0.119	0.150	0.158
8	0.012	0.027	0.059	0.089	0.122	0.154	0.163
9		0.028	0.060	0.090	0.124	0.156	0.165
10				0.091	0.125	0.158	0.166
11				0.091	0.125	0.158	0.166
12				0.091	0.125	0.158	0.166
13				0.091	0.125	0.157	0.166
14				0.091	0.125	0.157	0.166
15				0.091	0.124	0.157	0.165
16				0.090		0.157	
17						0.156	
Reid's values	0.011	0.027	0.059	0.089	0.123	0.156	0.165

and weights as were used for the iterations. The principal value integral was replaced by an integral with a well-behaved integrand by using the identity

$$P\int_0^\infty \frac{\gamma_l(k,q)q^2dq\Gamma_l(q,k)}{k^2-q^2} = \int_0^\infty \frac{dq}{k^2-q^2}\left[\gamma_l(k,q)q^2\Gamma_l(q,k) - k^2\Gamma_l(k,k)\right]. \tag{4.9}$$

The results for the phase shifts for the $^1S_0$, $^3P_1$, and $^1D_2$ states are given in Tables I, II, and III, respectively. The energies correspond to those given in Reid's paper.[9] Most of our converged phase shifts are in reasonable agreement with Reid's.[9] We have also compared our phase shifts with those calculated by Picker, Redish, and Stephenson[13] and found perfect agreement to the number of places they give, which is three. The convergence of the half-off-shell T matrix elements is illustrated in Figs. 1, 2, and 3, where we have plotted the ratio of the half-off-shell T matrix to the on-shell T matrix. This ratio was calculated by using (2.4). The iterations converge fast enough, and the matrices that we have to deal with are small enough, that we conclude that this approach is a practical one for solving the two-nucleon problem with a realistic local potential.

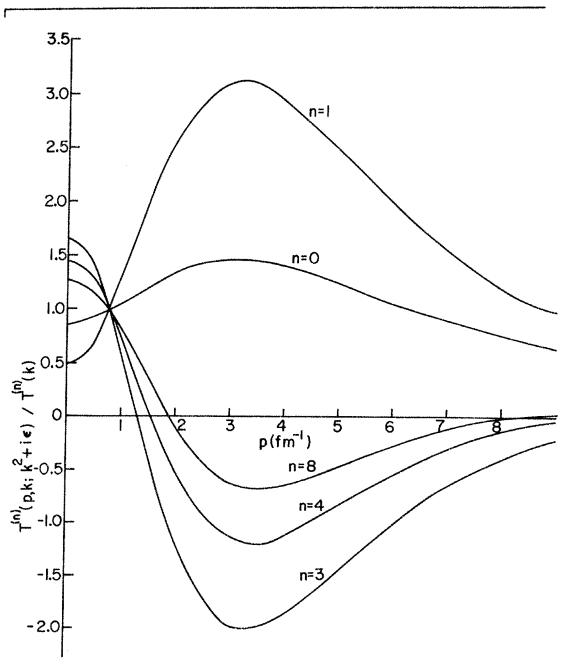

FIG. 1. Convergence of the ratio of the half-off-shell T matrix to the on-shell T matrix for the 24-MeV (c.m.) $^1S_0$ state.

V. SUMMARY AND DISCUSSION

We have shown how Kowalski's[5] formulation of the Sasakawa[1] approach to scattering theory can be used as a basis for a momentum-space formulation for the theory of the Jost function. This formalism has the advantage that it treats all

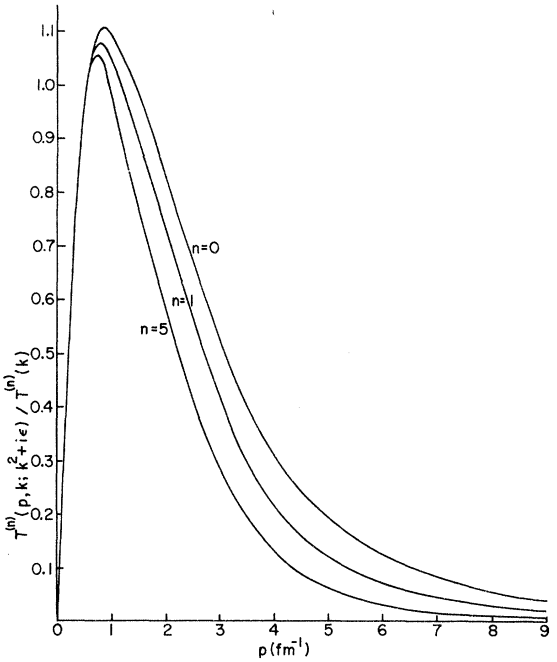

FIG. 2. Convergence of the ratio of the half-off-shell T matrix to the on-shell T matrix for the 12-MeV (c.m.) $^3P_1$ state.

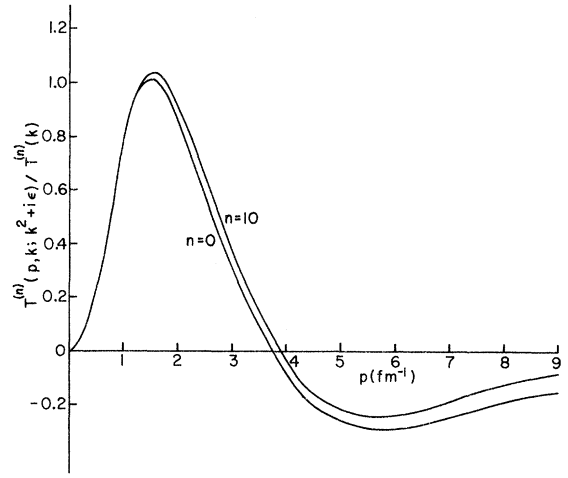

FIG. 3. Convergence of the ratio of the half-off-shell T matrix to the on-shell T matrix for the 72-MeV (c.m.) $^1D_2$ state.

types of potentials on the same footing. It applies equally well to nonlocal potentials as to local potentials. One can easily show using this formalism that the Fredholm determinant for a separable potential is not identical to its Jost function. Other applications of these equations, we are sure, will arise.

The two exactly solvable examples we have presented illustrate an important feature of the Sasakawa approach; namely, the iteration scheme proposed by him does not converge for all potentials, but it does converge for local potentials of arbitrary strength for a particular choice of $\gamma_l(k, q)$. It would be useful if some results could be obtained which would allow one to choose γ_l so as to insure convergence of the iteration scheme for all potentials. The false convergence we found in our separable-potential example is important to keep in mind, since it suggests that one can be misled in numerical work into believing that the iterations are converging to the true result, whereas in fact they are not. As was pointed out, the false convergence can be detected by looking at the function $\Gamma_l(p, k)$.

Our work on the Reid potential, we feel, demonstrates the practicality of the iteration scheme presented in Sec. II. It might be possible to make the method even more practical by developing quadrature rules that are tailored to the potential being used in the calculation. We are looking into this possibility. We also plan to test the method by applying it to the coupled states of the two-nucleon system. The general equations that one needs to do this have already been given by Kowalski.[5]

[1]T. Sasakawa, Prog. Theor. Phys. (Kyoto) Suppl. 27, 1 (1963).
[2]F. Coester, Phys. Rev. C 3, 525 (1971).
[3]N. Austern, Phys. Rev. 188, 1595 (1969).
[4]M. G. Fuda, Nuovo Cimento 11A, 701 (1972).
[5]K. L. Kowalski, Nucl. Phys. A190, 645 (1972).
[6]R. Jost, Helv. Phys. Acta 20, 256 (1947).
[7]V. De Alfaro and T. Regge, *Potential Scattering* (North-Holland, Amsterdam, 1965).
[8]R. G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill, New York, 1966).
[9]R. V. Reid, Jr., Ann. Phys. (N.Y.) 50, 411 (1968).
[10]M. G. Fuda, Phys. Rev. C 1, 1910 (1970).
[11]S. Weinberg, Phys. Rev. 131, 440 (1963).
[12]M. G. Fuda, J. Math. Phys. 12, 1163 (1971).
[13]H. S. Picker, E. F. Redish, and G. J. Stephenson, Jr., Phys. Rev. C 4, 287 (1971).