Calculation of Nuclear Reactions in ^{16}O in the Random-Phase Approximation*

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We present a practical formalism of low-energy nuclear reactions in which the ground-state correlations are treated in the random-phase approximation. The building blocks of the method are the matrix elements of two weakly energy-dependent effective interactions describing particle-hole pair scattering and pair creation or annnihilation in a correlated system. The corresponding transition operators, which give complete information on any nuclear reaction, are then introduced. We apply the method to a study of nucleon scattering, photonuclear, muonuclear reactions proceeding through the 1^- states of ^{16}O. The single-particle basis states are generated by a local, real Woods-Saxon potential, while the residual interaction has a zero range. In the relatively high excitation energy being studied, the correlations retained in the random-phase approximation affect very little nucleon scattering but reduce transition rates in photonuclear reactions and muon capture by about 8%.

I. INTRODUCTION

Presently the most satisfactory microscopic description of low-energy nuclear reactions is based on a specialization of the formal scattering theory[1] in which ideas and methods proper to the nuclear shell model for bound states are systematically applied. In its best-known version,[2] which we refer to as the Tamm-Dancoff treatment, it rests on two main hypotheses; in one the independent-particle picture is used to represent the ground state of the system, and in the other the excited states of interest are assumed to result from the particle-hole excitations of that ground state. The Tamm-Dancoff approximation (TDA) has been the object of many studies, both analytical[2] and numerical.[3-6] It was thus possible to acquire a deeper understanding of the resonance phenomena, the doorway states, and other concepts of the theory of nuclear reactions. The success of past numerical results indicated that the model was not far from reality for the simple nuclei so far considered, and, by extension, could provide a good first-order approximate representation of nuclei having more complicated structure. At the same time these calculations and analyses revealed the limitations of the model, as, for example, in its oversimplified description of the ground state.

We already know from bound-state studies via the Hartree-Fock procedure[7] or shell-model calculations in an extended basis[8] that the ground states of nuclei are highly correlated. Part of the long-range correlations can be treated in a simple manner by the random-phase approximation (RPA) method.[9] This method, in spite of some of its formal difficulties, still is the only existing method that can provide a practical microscopic description of the collective excitations. The extension of the RPA to the continuum was achieved by Dietrich and Hara,[10] Lemmer and Vénéroni,[11] and Fliessbach[12]; properties of the scattering matrix were subsequently discussed by Ginocchio, Schucan, and Weidenmüller[13] as well as by Hahne and Dover.[14] A numerical calculation in the ^{208}Pb system, using a separable interaction and neglecting exchange effects, was performed by Dover and Dietrich.[15] The work reported in this paper places itself along this line of endeavor. The object was to investigate the formalism from the practical point of view and apply the experience gained to study some reactions involving the 1^- states of the ^{16}O system.

In Sec. II the solutions of the RPA equations are written in a form easily accessible to numerical calculation. To this purpose we define two energy-dependent effective interactions and two corresponding transition operators, the matrix elements of which satisfy a system of coupled integral equations. The results are used to derive formulas for elastic and inelastic scattering cross sections, photonuclear reaction cross sections, and muon-capture rates (Sec. III). Section IV contains a description of the numerical methods, gives other practical details, and presents the results. Finally in Sec. V we give our conclusions.

II. FORMALISM

In this section we first define the particle-hole (ph) basis states, write down the RPA equations and their solutions, and finally introduce the transition operators T and $\tilde{T}$ in terms of which the continuum nuclear wave functions can be expressed.

A. TDA and RPA

We restrict ourselves to even-even doubly magic compound systems, and, following the traditional

approach, include only the nuclear interaction explicitly. Excitations induced by other means are treated as perturbations. The starting point is a single-particle Hartree-Fock (HF) Hamiltonian h_0 which generates a finite spectrum of bound states and a continuum of scattering states:

$$h_0|n\rangle = \epsilon_n|n\rangle . \tag{1}$$

If the system is spherically symmetric as it is here assumed, n is the set of quantum numbers of single-particle states (ϵ_n, l, j, m, q) with l the orbital angular momentum, j the total angular momentum, m its third component, and q the charge. The energy ϵ_n is negative and discrete for a bound state, positive and continuous for a scattering state. In the coordinate representation $|n\rangle$ is $R_n(r)Y^l_{jm}(\hat{r})X_q$, where X_q stands for an isospinor, Y^l_{jm} an angle-spin function

$$Y^l_{jm}(\hat{r}) = \sum_{m_l} \langle jm | l m_l \tfrac{1}{2} m_s \rangle Y_{lm_l}(\hat{r}) X_{m_s} . \tag{2}$$

The radial part $R_n(r)$ is real, normalized to a δ function in energy for both positive and negative energies; for scattering states the function $R_n(r)$ is regular at the origin and behaves asymtotically, $r \to \infty$, as

$$R_n(r) \sim \left(\frac{2\mu}{\pi k \hbar^2}\right)^{1/2} \frac{1}{r} \sin\left(kr - \frac{l\pi}{2} + \delta_n\right), \tag{3}$$

where δ_n is a real phase shift, μ the nucleon reduced mass, and $k = (2\mu\epsilon_n/\hbar^2)^{1/2}$. To introduce a second-quantized formalism we define the particle creation and annihilation operators $a_n^\dagger$, a_n and a particle vacuum $|0\rangle$ such that

$$\begin{aligned} a_n^\dagger|0\rangle &= |n\rangle , \\ a_n|0\rangle &= 0 . \end{aligned} \tag{4}$$

In the HF approximation the ground state of a doubly magic nucleus is given uniquely, except for a phase, by a determinantal wave function $|\Phi_0\rangle$ constructed by filling the lowest bound levels of h_0. We shall henceforth denote the filled orbitals by a subscript h, e.g. in $a_h^\dagger, a_h$, and the unfilled orbitals by a subscript a, e.g. in $a_a^\dagger, a_a$, unless we want to be more specific in which case an unoccupied bound state is represented by $|b\rangle$, and an unoccupied unbound state by $|c\rangle$. With this notation we then have

$$a_h^\dagger|\Phi_0\rangle = a_a|\Phi_0\rangle = 0 . \tag{5}$$

As is well known the HF procedure determines only the occupied orbitals $|h\rangle$, leaving undefined the unoccupied orbitals $|a\rangle$. Any unitary transformation of the particle states $|a\rangle$ would leave the HF results invariant and would provide an equally acceptable basis whenever one wishes to use the HF results as a starting point.

Let us introduce the particle-hole operators:

$$A_\alpha^\dagger = a_a^\dagger a_h \delta_{q_a q_h} , \tag{6}$$

$$A_\alpha = a_h^\dagger a_a \delta_{q_a q_h} , \tag{7}$$

We shall consistently call $\alpha \equiv (ah)$ any particle-hole configuration, while reserving the symbol $\beta \equiv (bh)$ for bound configurations, and $\gamma \equiv (ch)$ for unbound configurations.

In both the TDA and the RPA the excited states are formed by the particle-hole excitations of the ground state. Whereas in the first approximation the ground state is identical to the HF state $|\Phi_0\rangle$ since the latter remains stationary with respect to such excitations, in the second it is by assumption a correlated state $|\Psi_0\rangle$, which contains, besides $|\Phi_0\rangle$, 2p-2h, 4p-4h, ..., components. In practice, however, no explicit specification of the particle correlations is ever needed; their existence is simply implied by the definition of the excitation operator

$$\begin{aligned} Q_\mu^\dagger = &\sum_\beta (X_\beta^\mu A_\beta^\dagger - \bar{X}_\beta^\mu A_\beta) \\ &+ \sum_\gamma \int_{|\epsilon_h|}^\infty dE_\gamma (X_\gamma^\mu A_\gamma^\dagger - \bar{X}_\gamma^\mu A_\gamma) , \end{aligned} \tag{8}$$

which when operating on $|\Psi_0\rangle$ yields the excited state $|\Psi_\mu\rangle$. Together with the requirement that the particle-hole operators satisfy, in violation of the Pauli principle, commutation relations, Eq. (8) completely defines the RPA.

It follows that in the Hamiltonian H, written in the HF basis as

$$\begin{aligned} H &\equiv H_0 + \hat{V} \\ &= \sum_n \epsilon_n : a_n^\dagger a_n : + \frac{1}{4} \sum_{\substack{n\,m \\ n'm'}} \langle nm|v|n'm'\rangle : a_n^\dagger a_m^\dagger a_{m'} a_{n'} : \end{aligned} \tag{9}$$

only part of the residual interaction $\hat{V}$ may contribute. It is represented on the one hand by matrix elements of the type $\langle ah|v|a'h'\rangle$, typical of the TDA, and on the other hand by matrix elements such as $\langle aa'|v|hh'\rangle$, characteristic of the RPA. In graphic terms the former correspond to the forward-going graphs, and the latter to the backward-going graphs, Fig. 1. Thus of all the possible interaction vertices, the RPA keeps those containing the bubbles intact, i.e., in which the particle-hole pairs remain inert and act as bosons.

B. RPA Equations

Since states of good total angular momentum are needed we define the angular momentum coupled

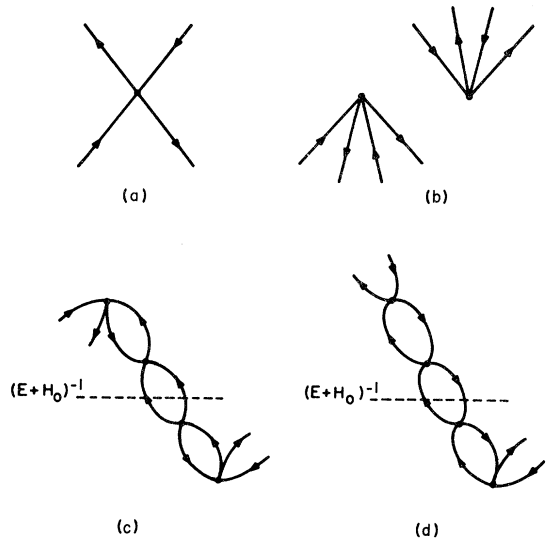

FIG. 1. Basic interactions in the formalism: (a), particle-hole scattering; (b), particle-hole creation or annihilation; (c) and (d), typical expansion terms of the effective interactions v and $\tilde{v}$; the energy factor for the intermediate states is $(E+H_0)^{-1}$.

particle-hole operators:

$$A_\alpha^\dagger(JM)=\sum_{m_a m_h}\langle j_a m_a j_h - m_h | JM\rangle (-)^{j_h+m_h} a_a^\dagger a_h , \tag{10a}$$

$$A_\alpha(JM)=[A_\alpha^\dagger(JM)]^\dagger , \tag{10b}$$

$$\bar{A}_\alpha(JM)=(-)^{J-M}A_\alpha(J,-M) . \tag{10c}$$

The symbol α in the coupled representation should of course be understood as containing all single-particle quantum numbers except the components of the angular momenta. The operators $A_\alpha^\dagger(JM)$, $A_\alpha(JM)$ in the RPA satisfy the commutation relations

$$[A_\alpha(JM), A_{\alpha'}^\dagger(J'M')]=\delta_{\alpha\alpha'}\delta_{JJ'}\delta_{MM'} ,$$
$$[A_\alpha(JM), A_{\alpha'}(J'M')]=[A_\alpha^\dagger(JM), A_{\alpha'}^\dagger(J'M')]=0 , \tag{11}$$

and the eigenequation

$$[H_0, A_\alpha^\dagger(JM)]=E_\alpha A_\alpha^\dagger(JM) , \tag{12}$$

where $E_\alpha=\epsilon_a-\epsilon_h$ stands for the particle-hole energy.

The excitation operator is expanded in the p-h basis

$$Q_\mu^\dagger(JM)=\sum_\alpha [X_{\alpha J}^\mu A_\alpha^\dagger(JM)-\bar{X}_{\alpha J}^\mu \bar{A}_\alpha(JM)] , \tag{13}$$

where here and in the following the summation over α means summation over bound states and integration over scattering states. The operator $Q_\mu^\dagger$ satisfies the eigenequation

$$[H, Q_\mu^\dagger(JM)]=E_\mu Q_\mu^\dagger(JM) , \tag{14}$$

and is subject to the orthonormality condition

$$[Q_\mu(JM), Q_{\mu'}^\dagger(J'M')]=\delta_{\mu\mu'}\delta_{JJ'}\delta_{MM'} , \tag{15}$$

and a requirement consistent with the definition of the ground state

$$Q_\mu(JM)|\Psi_0\rangle=0 . \tag{16}$$

Equation (14) yields both bound and unbound states. The unbound solutions, which are of particular interest to us, represent the situations resulting from the scattering of a particle initially in state c by an odd nucleus of configuration $a_h|\Psi_0\rangle$. The boundary condition then requires that the scattering function $|\Psi_\mu\rangle$ may have asymptotically either an incoming or an outgoing wave only in the channel $\gamma\equiv(ch)$. Equations (13) and (14) together with this boundary condition lead to the solution

$$Q_\gamma^{(\pm)\dagger}(JM)=e^{\pm i\delta_c}A_\gamma^\dagger(JM)+\sum_{\alpha'}A_{\alpha'}^\dagger(JM)\frac{1}{E^\pm-E_{\alpha'}}[A_{\alpha'}(JM),[V,Q_\gamma^{(\pm)\dagger}(JM)]]$$
$$-\sum_{\alpha'}\bar{A}_{\alpha'}(JM)\frac{1}{E+E_{\alpha'}}[\bar{A}_{\alpha'}^\dagger(JM),[V,Q_\gamma^{(\pm)\dagger}(JM)]] . \tag{17}$$

The scattering energy is $E=E_\gamma=\hbar^2k^2/2\mu-\epsilon_h$. The scattering wave functions are

$$|\Psi_\gamma^{(\pm)}(JM)\rangle=Q_\gamma^{(\pm)\dagger}(JM)|\Psi_0\rangle . \tag{18}$$

With the help of Eq. (17) one readily shows that the orthonormality relation (15) is satisfied, and that the scattering matrix can be calculated from its definition:

$$\delta(E-E')S_{\gamma\gamma'}^J\equiv\langle\Psi_{\gamma'}^{(-)}(JM)|\Psi_\gamma^{(+)}(JM)\rangle ,$$
$$=[Q_{\gamma'}^{(-)}(JM), Q_\gamma^{(+)\dagger}(JM)] . \tag{19}$$

C. Transition Operators

If Eq. (17) is solved up to the first order of the interaction, the excitation operators will depend on the following quantities:

$$V_{\alpha\alpha'}^J=[A_\alpha(JM),[V,A_{\alpha'}^\dagger(JM)]] , \tag{20}$$

$$\tilde{V}_{\alpha\alpha'}^J=[A_\alpha^\dagger(JM),[V,A_{\alpha'}^\dagger(JM)]] . \tag{21}$$

These are none other than the matrix elements characteristic of the RPA and which represent, respectively, the particle-hole pair scattering and

pair creation (or annihilation) processes. They are independent of M for a rotationally invariant interaction, and are real since the Condon-Shortley phase convention for the spherical harmonics and stationary wave functions are being used.

The processes described in the Born approximation by Eqs. (20) and (21) are given exactly by the following matrix elements:

$$T^J_{\alpha\gamma} = [A_\alpha(JM), [V, Q^{(+)\dagger}_\gamma(JM)]]\, e^{-i\delta_c}, \tag{22}$$

$$\tilde{T}^J_{\alpha\gamma} = [\bar{A}^\dagger_\alpha(JM), [V, Q^{(+)\dagger}_\gamma(JM)]]\, e^{-i\delta_c}. \tag{23}$$

From Eq. (17), it is seen that they satisfy a system of coupled integral equations

$$T^J_{\alpha\gamma} = V^J_{\alpha\gamma} + \sum_{\alpha'} \left(\frac{V^J_{\alpha\alpha'} T^J_{\alpha'\gamma}}{E^+ - E_{\alpha'}} - \frac{\tilde{V}^J_{\alpha\alpha'} \tilde{T}^J_{\alpha'\gamma}}{E + E_{\alpha'}} \right), \tag{24}$$

$$\tilde{T}^J_{\alpha\gamma} = \tilde{V}^J_{\alpha\gamma} + \sum_{\alpha'} \left(\frac{\tilde{V}^J_{\alpha\alpha'} T^J_{\alpha'\gamma}}{E^+ - E_{\alpha'}} - \frac{V^J_{\alpha\alpha'} \tilde{T}^J_{\alpha'\gamma}}{E + E_{\alpha'}} \right). \tag{25}$$

To simplify the notation we introduce the operators V, $\tilde{V}$, T, and $\tilde{T}$ which are defined by their matrix elements, Eqs. (20)–(23). It is of course understood that they are to be calculated in the ph basis defined previously, Eq. (10), and that T and $\tilde{T}$ operate on the right on unbound configurations. Let us introduce two effective interactions, both real and smooth functions of energy. The first represents ph pair scattering in a correlated system

$$\mathcal{U} = V - \tilde{V} \frac{1}{E + H_0 + V} \tilde{V}, \tag{26}$$

the other describes pair creation and annihilation lation

$$\tilde{\mathcal{U}} = \tilde{V} - V \frac{1}{E + H_0 + V} \tilde{V}. \tag{27}$$

Note that in both the second terms originate from the ground-state correlations and decreases smoothly with energy. Typical terms in the expansion series of $\mathcal{U}$ and $\tilde{\mathcal{U}}$ are illustrated in Fig. 1. We further define a wave operator F in terms of the interaction $\mathcal{U}$ by the integral equation

$$F = 1 + \frac{1}{E^+ - H_0} \mathcal{U} F, \tag{28}$$

which may formally be solved to give

$$F = \frac{1}{\mathcal{U} - \mathcal{U}[1/(E^+ - H_0)]\mathcal{U}}\, \mathcal{U}. \tag{29}$$

As we insert $\mathcal{U}$, $\tilde{\mathcal{U}}$, and F in Eqs. (24) and (25) the transition operators become

$$T = \mathcal{U} F, \tag{30}$$

and

$$\tilde{T} = \tilde{\mathcal{U}} F. \tag{31}$$

To summarize, the problem has been reduced essentially to the definition of two transition operators T and $\tilde{T}$. The economy in our formulation has been achieved by the introduction of two effective interactions, $\mathcal{U}$ and $\tilde{\mathcal{U}}$, which differ from the "bare" interactions, V and $\tilde{V}$, by a term normally both small in magnitude and weakly energy-dependent. Thus the main energy features of the transition operators, in particular the resonances they may exhibit, appear in F, their common factor. Formally the expression for the T matrix is identical to its analog in the TDA, and may be written in the form of the Lippman-Schwinger equation for a particle scattering with a potential $\mathcal{U}$:

$$T = \mathcal{U} + \mathcal{U} \frac{1}{E^+ - H_0} T. \tag{32}$$

III. EXCITATION MECHANISMS

To test the wave function, Eq. (18), we consider several processes: nucleon scattering,[3] photonuclear reactions,[4,5] and muon capture.[6] Since the derivations of their cross sections have appeared in many places, we will merely state the results.

A. Nucleon Scattering

As is clear from Eq. (17) only the first two terms of the wave function $\Psi^{(+)}_\gamma$ contain particle scattering waves at large distances. This can be shown by calculating the S matrix, Eq. (19),

$$S^J_{\gamma'\gamma} = e^{i(\delta_c + \delta_{c'})} [\delta_{\gamma\gamma'} - 2\pi i\, T^J_{\gamma'\gamma}(E)]. \tag{33}$$

The energy argument in T indicates that T is calculated on shell. We recall our notational convention: $\gamma = (ch)$, $\gamma' = (c'h')$ with $E_\gamma = E_{\gamma'} = E$. The total cross section for particle scattering leading from the initial target state h to the final state h' is:

$$\sigma_{h'h} = \sum_J \sigma^J_{h'h}, \tag{34}$$

$$\sigma^J_{h'h} = \frac{2\pi^3(2J+1)}{k^2(2j_h+1)} \sum_{l_c j_c l'_c j'_c} |\hat{T}^J_{\gamma'\gamma}|^2, \tag{35}$$

where $\hat{T}$ includes the potential scattering

$$\hat{T}^J_{\gamma'\gamma} = -\frac{1}{\pi}(e^{i\delta_c} \sin\delta_c)\delta_{\gamma'\gamma} + e^{i(\delta_c + \delta_{c'})} T^J_{\gamma'\gamma}(E). \tag{36}$$

B. Photonuclear Reactions and Muon Capture

As is well known the photonuclear reactions and the capture of stopped negative muon in nuclei proceed predominantly through the giant resonance. Because they complement each other in giving information about this important feature of the nucleus they should be studied together; in the former only the isospin components of the resonance

are excited, in the latter, because a higher momentum transfer is involved (80 MeV/c), magnetic and electric spin-flip components are also enhanced.[16,17]

We treat the electromagnetic and weak interactions in first-order approximation. The photon and muon channels are not included from the outset in the calculation of the total wave function. This function is simply taken to be a product of the nuclear wave function and the photon or muon wave function. The photon is described by a plane wave, the muon by a bound wave function, $\phi_\mu(\vec{r})$. The nuclear wave function to be used represents a particle of linear momentum $\vec{k}$ and spin projection m_s being scattered by a target in the state h. Normalized in a box of unit volume, it is given by

$$|\Psi^{(+)}(h\vec{k}m_s)\rangle = \frac{1}{\sqrt{\rho_k}} \sum \langle l m_l \tfrac{1}{2} m_s | jm\rangle \langle jm j_h m_h | JM\rangle \times i^{l-l_h} Y^*_{lm_l}(\hat{k}) |\Psi^{(+)}_{ch}(JM)\rangle , \quad (37)$$

where $\rho_k = [\mu k/(2\pi)^3\hbar^2]$.

The differential cross section for photoabsorption by an even-even nucleus leading to the emission of a nucleon of momentum $\vec{k}$ and spin orientation m_s and leaving the residual nucleus in a state h is given by

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar c}\rho_k \sum_{m_s m_h} |\langle \Psi^{(-)}(h\vec{k}m_s)| H_{\rm em}|\Psi_0\rangle|^2 , \quad (38)$$

where $H_{\rm em}$ is the electromagnetic interaction. For electric dipole excitations it is given by

$$\hat{\omega}_1 = \left(\frac{4\pi}{3}\right)^{1/2} r Y_1(\hat{r})\tau_3 . \quad (39)$$

The excitation function of the neutron emitted following muon capture is given by the following well-known relation[16]

$$\frac{d\Lambda}{d\epsilon_n} = \frac{m_\mu{}^2}{2\pi} |\phi_\mu|^2_{\rm av} [G_V{}^2 M_V{}^2 + 3G_A{}^2 M_A{}^2 + (G_P{}^2 - 2G_P G_A) M_P{}^2] , \quad (40)$$

where standard assumptions about the weak interaction, including the absence of exchange currents, have been made. The values of the constants are the same as in Ref. 16, the notation is also unchanged except of course for the inclusion of the continuum and a proper normalization of the scattering states. The muon-capture matrix elements $|M_{V,A,P}|^2$ are given by

$$|M_{V,A,P}|^2 = \sum_{m_s m_h} \left(\frac{\nu}{m_\mu}\right)^2 \rho_k \int \frac{d\hat{\nu}}{4\pi} \left| \left\langle \Psi^{(-)}(h\vec{k}m_s) \right| \sum_{i=1}^{A} 0^i_{VAP} \tau^i_3 e^{-i\vec{\nu}\cdot\vec{r}_i} \left| \Psi_0 \right\rangle \right|^2 , \quad (41)$$

where

$$0_V = 1, \quad 0_A = \frac{\vec{\sigma}}{\sqrt{3}}, \quad 0_P = \vec{\sigma}\cdot\hat{\nu} . \quad (42)$$

The interactions responsible for the two processes under study, Eqs. (38) and (41), are both given by sums of one-particle operators of rank λ to a spin tensor of rank s:

$$\hat{\Omega}^{(\lambda s)}_J = \sum_{mn} \langle m|\hat{\omega}^{\lambda s}_J|n\rangle a^\dagger_m a_n . \quad (43)$$

We have then to calculate the matrix elements

$$\langle \Psi_0|\hat{\Omega}^{(\lambda s)}_{J,-M}|\Psi^{(+)}_\gamma(JM)\rangle = \Omega_\gamma(\lambda s JM) + \sum_{\alpha'} \left[\Omega_{\alpha'}(\lambda s JM) \frac{1}{E^+ - E_{\alpha'}} T^J_{\alpha'\gamma} - \tilde{\Omega}_{\alpha'}(\lambda s JM) \frac{1}{E + E_{\alpha'}} \tilde{T}^J_{\alpha'\gamma} \right] e^{i\delta_c} , \quad (44)$$

where

$$\Omega_\alpha(\lambda s JM) = \langle \Psi_0| [\hat{\Omega}^{(\lambda s)}_{J,-M}, A^\dagger_\alpha(JM)] |\Psi_0\rangle = \frac{(-)^{J-M}}{\sqrt{2J+1}} (-)^{j_a + j_h - J} (h \| \hat{\omega}^{\lambda s}_J \| a) , \quad (45)$$

$$\tilde{\Omega}_\alpha(\lambda s JM) = \langle \Psi_0| [\hat{\Omega}^{(\lambda s)}_{J,-M}, \bar{A}_\alpha(JM)] |\Psi_0\rangle = \frac{(-)^{J-M}}{\sqrt{2J+1}} (a \| \hat{\omega}^{\lambda s}_J \| h) . \quad (46)$$

The reduced single-particle matrix elements are calculated in the usual manner.[18]

IV. NUMERICAL CALCULATION

We have applied the above formalism to a study of the 1^- states of ^{16}O. In some sense this may not be an ideal system to test the effectiveness of the ground-state correlations because of their expected small effects in this excitation energy region. But on the other hand the giant dipole resonance in ^{16}O has a structure well known from both bound-state[19] and continuum[3-6] calculations so so that correlation effects, even small, may be easily identified; furthermore the simplicity of the system makes an accurate calculation possible with the least number of additional approximations.

A. Treatment of the Continuum

The difficulty in applying the shell model to the continuum resides in the correct treatment of the

continuous energy variable. A method appealing for its practical value is the separation method[20-22] which factorizes the energy and radial dependence of the single-particle scattering wave function. It proceeds from the observation that in calculating the nuclear wave function the continuum states of the basis are operated upon by a short-range nuclear two-body force; hence there exists a radius somewhat larger than the range of the force beyond which contributions are negligible. It is then possible in the internal region of the configuration space to expand the single-particle continuum functions in terms of a finite set of discrete basis states; the amplitudes of the expansion contain the full energy dependence. It has been found that the series converges rapidly.[22] In most practical calculations, for the degree of error we normally tolerate, a single term suffices; the energy-dependent factor is then simply related to the Jost function. Consequently in most of our calculations we fix the single-particle continuum function at its resonance energy if it exhibits a sharp resonance, or at the scattering energy otherwise. However an exact calculation of the continuum is always performed wherever a Green's function appears since this is crucial for getting the resonances at their correct positions:

$$G_0^{(+)}(E)=\sum_\beta \frac{|\beta\rangle\langle\beta|}{E-E_\beta}+\sum_\gamma \int_{|\epsilon_h|}^{\infty} dE_\gamma \frac{|\gamma\rangle\langle\gamma|}{E^+-E_\gamma}\,. \tag{47}$$

The principal-value integral in general causes no difficulty unless the configuration $|\gamma\rangle$ exhibits a sharp resonance, in which case the resonance should first be removed (for example, by subtracting from the integrand a resonance of Lorentzian shape) and integrate the background in the usual way.[5]

In the expressions for T and $\tilde{T}$, Eqs. (30) and (31), there appears the inverse of a matrix A:

$$A_{\alpha'\alpha}\equiv\langle\alpha'|\,\upsilon-\upsilon G_0^{(+)}\upsilon|\,\alpha\rangle\,. \tag{48}$$

Because the matrix A is of finite dimension in our approximation scheme the matrix elements of the transition operators may be written in the familiar ND^{-1} form:

$$T_{\alpha\gamma}=\sum_{\alpha'\alpha''}\upsilon_{\alpha\alpha'}A_{\alpha'\alpha''}{}^{-1}\upsilon_{\alpha''\gamma}=-\frac{1}{\det A}\begin{vmatrix} A_{\alpha'\alpha''} & \upsilon_{\alpha\alpha'} \\ \upsilon_{\alpha''\gamma} & 0\end{vmatrix}, \tag{49}$$

$$\tilde{T}_{\alpha\gamma}=\sum_{\alpha'\alpha''}\tilde{\upsilon}_{\alpha\alpha'}A_{\alpha'\alpha''}{}^{-1}\upsilon_{\alpha''\gamma}=-\frac{1}{\det A}\begin{vmatrix} A_{\alpha'\alpha''} & \tilde{\upsilon}_{\alpha\alpha'} \\ \upsilon_{\alpha''\gamma} & 0\end{vmatrix}. \tag{50}$$

The numerators N, which differ from one element to the other, are given by determinants of the arrays obtained by adding a row and a column to the matrix A as indicated. The same denominator $D=\det A$ appears in all elements of T and $\tilde{T}$. It can be considered as a generalized Jost function for nuclear reactions; its zeroes in energy lead to observable resonances.[2]

B. Parameters of the Calculation

We assumed the single-particle potential to have a local Woods-Saxon radial form:

$$U=-V_N\frac{1}{1+e^x}-V_{so}\frac{\hbar^2}{m_\pi c}\frac{1}{ar}\frac{e^x}{(1+e^x)^2}\vec{l}\cdot\vec{\sigma}+V_c(r)\delta_{q\,1/2}\,, \tag{51}$$

$$x=\frac{(r-R)}{a}\,,$$

$$R=r_0(A-1)^{1/3}\,,$$

$$V_c=\begin{cases}\dfrac{(Z-1)e^2}{r}\,, & r\geq R\\[2ex] \dfrac{Z-1}{2R}e^2\left(3-\dfrac{r^2}{R^2}\right), & r\leq R\,.\end{cases}$$

Values of the parameters are given in Table I.

The residual two-body interaction was taken to be

$$V(r)=-V_0\delta(\vec{r})\,(0.7+0.3P_\sigma)\,, \tag{52}$$

where $V_0=650$ MeV fm^3 chosen to locate the giant resonance around 22 MeV. The symbol P_σ stands for a spin-exchange operator.

We assumed, to a good approximation,[23] that the $(\frac{1}{2}^-)$ ground state and the $(\frac{3}{2}^-)$ excited state of ^{15}N or ^{15}O could be described by the single-hole configurations $(1p_{1/2})^{-1}$ and $(1p_{3/2})^{-1}$, respectively. We built our single-particle basis space from the bound states $1d_{5/2}$ and $2s_{1/2}$ and the unbound states $d_{3/2}$, $s_{1/2}$, and $d_{5/2}$, of which the $d_{3/2}$ wave exhibits a resonance at 0.94 MeV for a neutron and 4.5 MeV for a proton. In all we used 16-ph configurations of angular momentum $J=1$ of negative parity

TABLE I. Single-particle energies (MeV) and potential parameters (MeV) in ^{16}O. We also used $r_0=1.25$ fm for the reduced radius and $a=0.53$ fm for the difference parameter.

	Proton			Neutron		
State	ϵ	V_N	V_{so}	ϵ	V_N	V_{so}
$1p_{3/2}$	−18.4	57.95	9.89	−21.8	57.4	9.64
$1p_{1/2}$	−12.1	57.95	9.89	−15.6	57.4	9.64
$1d_{5/2}$	−0.6	54.4	5.3	−3.3	54.9	5.3
$2s_{1/2}$	−0.1	55.9	0	−4.14	56.8	0
$1d_{3/2}$	4.5	54.4	5.3	0.94	54.9	5.3

leading to one of the particle channels:

$$^{15}N_{g.s.}+p,\ ^{15}O_{g.s.}+n,\ ^{15}N_{6.33}+p,\ ^{15}O_{6.16}+n.$$

The energy integration which yielded the Green function was performed by applying in succession three 10-point Gaussian formulas over an energy range extending from thresholds up to a maximum fixed at 30 MeV. The transition rates, to be presented later on, were calculated between 12.5 and 25.5 MeV by intervals of 0.25 MeV. The numerical scheme was fairly simple: It started from the calculation of all relevant elements of the real matrices $\mathcal{V}$ and $\tilde{\mathcal{V}}$, and constructed the elements of the complex matrices T and $\tilde{T}$ using the complex matrix A via Eqs. (49) and (50).

C. Results

Since we limited our calculations to processes going through the 1^- intermediate states no agreement with experimental data is to be expected, except possibly for photonuclear reactions. However a comparison of the TDA and RPA results is instructive because it clarifies the role of the ground-state correlations in the amplitudes, positions, and widths of the resonances. Here by TDA we mean the additional assumption $\tilde{V}=0$ to be introduced in the formalism of Sec. II.

We first calculated proton-induced reaction on ^{15}N. The elastic scattering cross section is shown in Fig. 2; the structure is dominated by a resonance at the proton energy $E_p=4.7$ MeV which can be traced to the single-particle $d_{3/2}$ resonance; the small peak at $E_p=10$ MeV signals the presence of

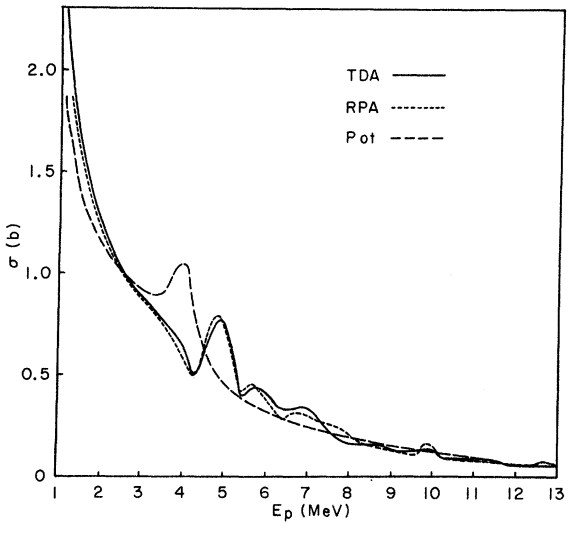

FIG. 2. Cross sections of proton elastic scattering proceeding through the 1^- states in ^{16}O in the TDA and RPA. The potential scattering cross section is also shown, curve labeled pot.

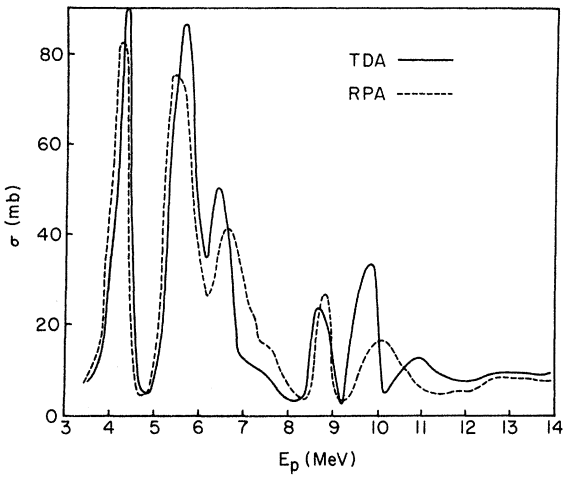

FIG. 3. Cross section of the charge-exchange reaction $^{15}N(p,n_0)^{15}O$.

a $T=1$ dipole resonance. The cross sections for inelastic scattering and charge-exchange reactions leading to the ground state and the $\frac{3}{2}^-$ excited state of ^{15}N or ^{15}O are shown in Figs. 3 and 4. The results from the TDA and RPA calculations are in

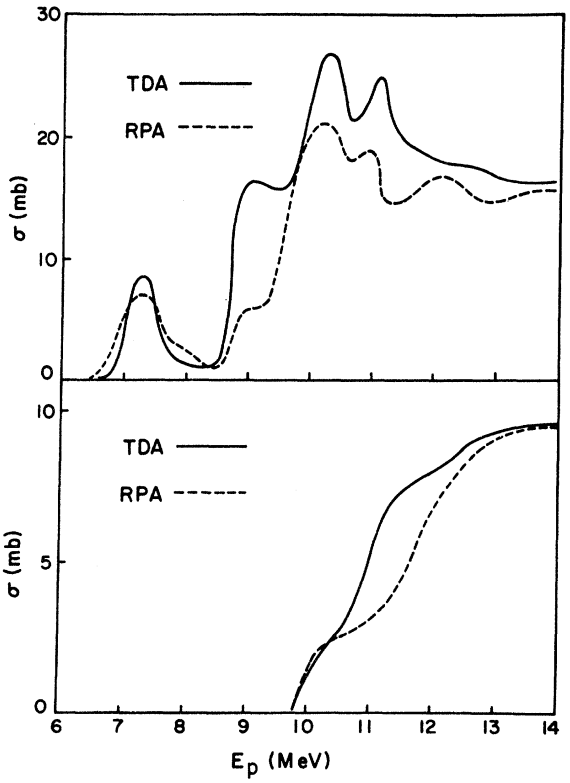

FIG. 4. Cross sections of the reaction $^{15}N(p,p')^{15}N_{6.33}$ in the upper part of the figure, and of the reaction $^{15}N(p,n')^{15}O_{6.16}$ in the lower part.

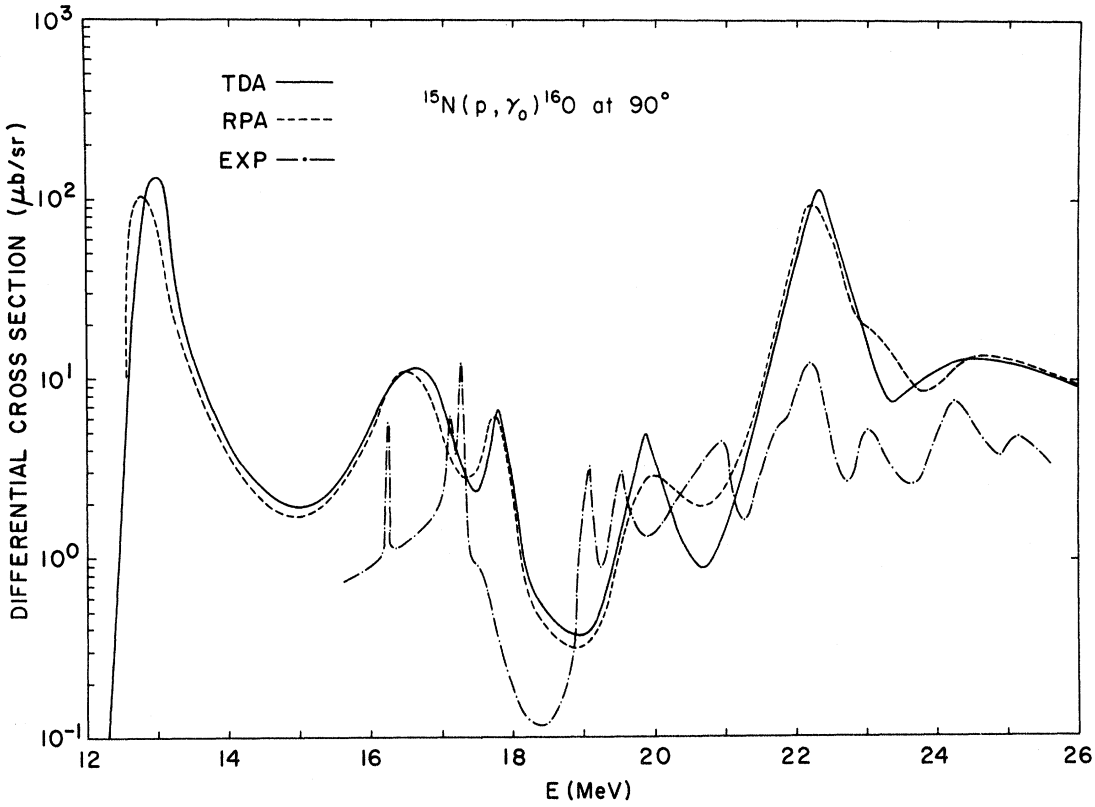

FIG. 5. Differential cross section of the capture reaction $^{15}N(p,\gamma_0)^{16}O$ at $\theta = 90°$. The experimental curve is drawn from the data given by Tanner, Thomas, and Earle (Ref. 24).

near mutual agreement; no systematic changes in amplitudes or shifts in resonance energies can be deduced. Unless we look for small details which might not be resolved in actual experiments, we should expect a certain indifference of the results to the two approximations being considered because the T matrix, the only to contribute to the present processes, differs only by the use of the ph interactions, either V or $\mathcal{V}$, which are generally of the same order of magnitude for any energy.

In photonuclear and muonuclear processes both T and $\tilde{T}$ contribute. Although $\tilde{T}$, which in itself is of the order $(\tilde{\mathcal{V}}/\mathcal{V})T$, cf. Eqs. (30) and (31), is considerably reduced in magnitude by the factor $(E+H_0)^{-1}$ which accompanies it, under favorable conditions it may play an appreciable role by its interference with T and the potential scattering term. Because of this effect and because of the factor $(E-H_0)^{-1}$ in front of T one may expect the resonances to be shifted from their positions observed in the particle scattering processes. The theoretical excitation functions for the reaction $^{15}N(p,\gamma_0)^{16}O$ at 90° are shown in Fig. 5 together with the experimental data.[24] Since this reaction has been extensively discussed in the past[4,5] we restrict our remarks on the following points. Compared with the TDA, the RPA leads to slightly reduced transition rates especially at the main peaks and shifts the resonances somewhat but does not seem to affect the widths. Compared with the data[24] both approximations give overestimated rates by several factors at the main peak; thus the most outstanding difficulty of the past calculations has not been resolved. The lowest peak, at $E = 13$ MeV, originates from the s-wave resonance. The resonances at $E = 16.5$ and 17.8 MeV are broad $E1$ states underlying the sharp structures experimentally observed in this energy region. The observed peak at 19 MeV would correspond to the one predicted at 19.8 MeV; however, the peak observed nearby is not reproduced in this calculation but appears in the (p,n_0) cross section; this is condistent with its interpretation as being a predominantly $T = 0$ state.[4] Finally, the main resonance appears at $E = 22.4$ MeV, and a secondary broad resonance at 24.5 MeV.

In Fig. 6 we present the total cross section for the $^{16}O(\gamma,p_0)^{15}N$ reaction. Ground-state correlations reduce the main peak by about 20% and shift its position by 0.2 MeV. The cross section inte-

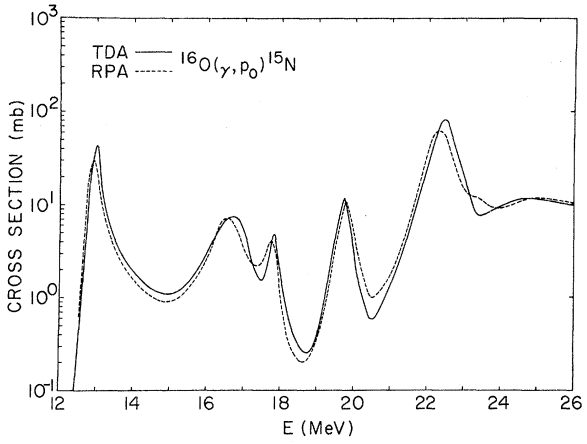

FIG. 6. Cross section of the reaction $^{16}O(\gamma,p_0)^{15}N$.

grated from threshold to 25.5 MeV is reduced by the same effects from 120 to 110 MeV mb, i.e., by about 10%.

In Fig. 7 we plot the theoretical cross sections together with the experimental cross section[25] for the reaction $^{16}O(\gamma,n_0)^{15}O$. In contrast to the previous results, the theoretical curves in the present case are relatively featureless although, above 21 MeV, at least five well separated peaks are experimentally observed. The predicted amplitudes are again larger than the measured data by a factor of 2 in the main peak region. There is also a reduction of 8% in the integrated cross section by ground-state correlations.

The spectrum of neutrons emitted following muon capture in ^{16}O has been previously calculated in the TDA.[6] The results of our calculation of the capture through the 1^- states are presented in Fig. 8 where the total contributions from both vector and axial vector excitations are shown. The vector part yields a neutron spectrum similar to that observed in the (γ,n_0) process from which it differs only by a retardation factor[16] and, consequently, is not shown separately.

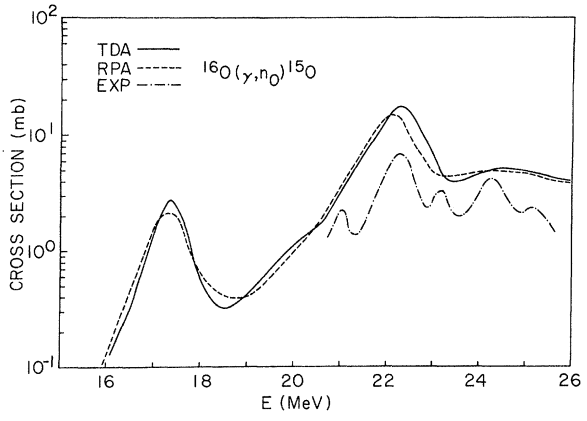

FIG. 7. Cross section of the reaction $^{16}O(\gamma,n_0)^{15}O$. The experimental curve is drawn from the data given by Bramblett *et al.* (Ref. 25).

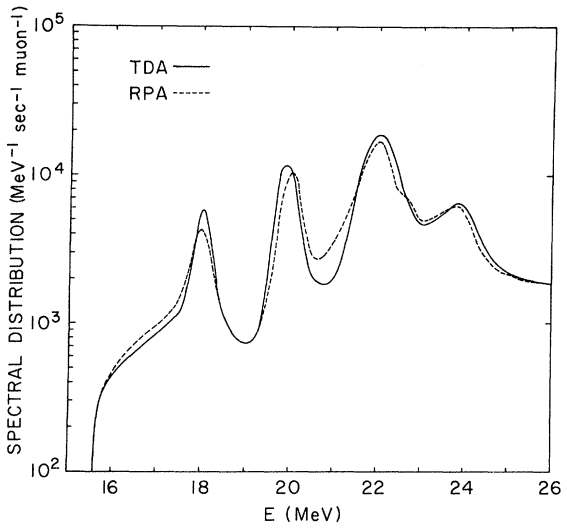

FIG. 8. Spectral distribution of neutron emitted following muon capture proceeding via the 1^- states in ^{16}O.

Ground-state correlations decrease the total capture rate

$$\Lambda = \int^{E_0} dE \frac{d\Lambda}{dE}$$

from 4.30×10^4 sec^{-1} to 3.94×10^4 sec^{-1}, i.e., by about 8%. The energy cutoff we have used, $E_0 = 25.5$ MeV, may be too small for our result to be of much signification; however, to have a point of reference, we recall that Balashov *et al.*[6] obtained 4.89×10^4 sec^{-1} in their own continuum calculation.

V. CONCLUSIONS

From the formalism presented in this paper one realizes that the ground-state correlations, at least those retained in the RPA, can be treated without difficulty in the continuum. One first defines two weakly energy-dependent effective interactions $\mathfrak{v}$ and $\tilde{\mathfrak{v}}$ and calculates the transition operators T and $\tilde{T}$. The calculation of the T matrix is particularly simple; as defined here it differs from the TDA scattering matrix only by the replacement of the ph interaction V by the effective interaction $\mathfrak{v}$. Since the latter is a smooth function of the energy it acquires no new features in its structure; it contains all information on elastic and inelastic scattering the parameters of which can be easily extracted, cf. Eq. (49). However, for the same energy, nucleon scattering may not

be as sensitive a test of the existence of correlations as reactions via channels other than particle channels. Here both T and $\tilde{T}$ contribute and their interference may affect the parameters of the resonances in an appreciable way.

From our expressions of the transition operators it is simple enough to derive formulas of the resonance parameters in terms of the effective interactions; however it is difficult to deduce the effects of correlations from these results without performing an explicit calculation. From our numerical work we can draw the following conclusions. Firstly, in nucleon scattering in this fairly high excitation energy region only slight changes are to be expected; no general rules on the relative magnitudes of the TDA and RPA cross sections can be inferred from our results. Secondly, in reactions proceeding through photon or muon channels, resonances are displaced but widths are minimally enhanced. The 8% reduction in the transition rates, though larger than predicted by bound-state calculations, is still insufficient to bring the theoretical results to agree with the experimental data. In several previous calculations[4] an imaginary single-particle potential was found necessary to mock up the absorptive effects and reduce the transition rates. This result is a clear indication that the nature of the excited states is more complicated than assumed. To drive towards a more realistic model, one may, for example, follow essentially the traditional TDA approach, but enlarge the basis space of expansion of the excitation operator Q_μ beyond the 1p-1h states.[26] Another approach, more consistent in spirit with the present formalism, is to keep the 1p-1h basis for the excitation operator but to introduce instead richer ground-state correlations, which can be done in various ways. One- and two-body density matrices can be explicitly defined,[27] or, within the framework of the quasiparticle formalism,[28] a renormalized interaction between the quasiparticles can be introduced.[29,30] Another promising approach lies in the equation-of-motion method[31] which manages to preserve the economy in language and the structural simplicity of the RPA, yet which satisfies the Pauli principle and is not subject to the early objections of the RPA. In this approach one works not with operators but rather with the ground-state expectation of operators; after such a transposition to the language of expectation values and a preliminary orthonormalization of the basis states, much of the present formulation remains unchanged and should lead to results more satisfactory (and correspondingly requiring more efforts) than those obtained in the present work.

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