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$E2/M1$ Multipole Mixing Ratios of γ Transitions in Even-Even Deformed Nuclei

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A summary is presented of the magnitudes and phases of previously measured $E2/M1$ multipole mixing ratios of γ transitions deexciting levels of the β- and γ-vibrational bands to the ground-state band in even-even deformed nuclei. A uniform phase, with few exceptions, is characteristic of transitions depopulating the γ band, while no systematic behavior is apparent for transitions from the β band; the magnitudes, while uniformly large, show little apparent systematic behavior among the nuclei in this region. Although none of the previously proposed theoretical interpretations is sufficient to explain both the magnitudes and relative phases of these mixing ratios, a phenomenological interpretation in terms of $\Delta K = 1$ band mixing through the intermediary of a $K = 1^+$ excitation is successful in predicting the relative magnitudes and phases in a number of cases.

I. INTRODUCTION

In the model of adiabatic vibrations of an ellipsoidally deformed nucleus, magnetic dipole ($M1$) transitions are forbidden to exist in γ transitions connecting rotational levels built on the vibrational excitations with those of the ground-state band; such transitions are expected to be pure electric quadrupole radiation ($E2$). However, nonvanishing $M1$ admixtures are found in such transitions in even-even nuclei throughout the mass region $150 < A < 190$; the $M1$ intensity generally comprises 0.5–2% of the total transition intensity.

The measurement of γ-ray angular distributions or correlations is sensitive to interference effects between the $M1$ and $E2$ amplitudes, and thus depends on the relative phase of the $M1$ and $E2$ matrix elements. A number of different conventions have been used in the literature to relate this phase to the observed angular distribution. This situation results from the various formalisms which have been proposed for interpreting angular correlation data. In the present work, the phase convention proposed by Krane and Steffen (KS)[1] is used; in that convention, emission matrix elements are consistently employed for the multipole operators, and the mixing ratio δ is defined as

$$\delta = \frac{\langle I_f \| \vec{j}_N \vec{A}(E2) \| I_i \rangle}{\langle I_f \| \vec{j}_N \vec{A}(M1) \| I_i \rangle}. \tag{1}$$

Here the interaction is written between the nuclear current $\vec{j}_N$ and the electromagnetic vector field $\vec{A}(\pi L)$. The convention then specifies that the expression describing the angular distribution of a γ ray which depopulates an oriented nuclear level (the orientation achieved by observation of a preceding radiation, nuclear reaction, Coulomb excitation, cryogenic orientation, etc.) is given by

$$A_k = \frac{F_k(11 I_f I_i) + 2\delta F_k(12 I_f I_i) + \delta^2 F_k(22 I_f I_i)}{1+\delta^2}. \tag{2}$$

If the first transition in a cascade is studied using angular correlation methods, the expression describing that transition is written with a negative interference term. With this choice, the phase of the mixing ratio is independent of its position in the cascade. This choice is related to the Biedenharn-Rose (BR)[2] and Rose-Brink (RB)[3] conventions for a γ_1–γ_2 cascade as:

$$\begin{aligned} \delta(\gamma_1)_{BR} &= -\delta(\gamma_1)_{KS}, \quad & \delta(\gamma_1)_{RB} &= -\delta(\gamma_1)_{KS}, \\ \delta(\gamma_2)_{BR} &= \delta(\gamma_2)_{KS}, \quad & \delta(\gamma_2)_{RB} &= -\delta(\gamma_2)_{KS}. \end{aligned} \tag{3}$$

Theoretical calculations are generally performed in terms of matrix elements of the Bohr-Mottelson multipole operators,[4] in terms of which the mixing ratio δ may be written

$$\frac{\delta}{E_\gamma(\mathrm{MeV})} = 0.835 \frac{\langle I_f \| \mathfrak{M}(E2) \| I_i \rangle}{\langle I_f \| \mathfrak{M}(M1) \| I_i \rangle}, \tag{4}$$

with the $E2$ matrix element in units of electron barns (eb) and the $M1$ matrix element in units of nuclear magnetons (μ_N); E_γ is the energy of the transition in MeV.

A comprehensive discussion of the properties of the electromagnetic transition operators and their matrix elements is given in the work of Alder and Steffen.[5]

II. RESULTS AND COMPARISON WITH THEORY

Table I presents a summary of the results obtained from an analysis of the angular correlation literature in terms of the present phase convention. The tabulated value is the "reduced" mixing ratio δ/E_γ given in Eq. (4). The quoted uncertainties are those arising from 1 standard deviation of the measured angular distribution coefficients. Transitions depopulating states of the β and γ bands with $I \leq 4$ have been analyzed; the identification of the γ band is usually obvious, and the β band has gen-

TABLE I. Reduced $E2/M1$ mixing ratios δ/E_γ (MeV) of transitions from levels in β and γ bands to levels in ground-state bands. The subscripts γ, β, and g refer to states of the γ, β, and ground-state bands, respectively. The experimental uncertainty of the last place is given in parentheses following each entry. The two numbers under each entry give, respectively, the prediction for the magnitude of δ as calculated according to the methods of Bès *et al.* (Ref. 11) and Greiner (Ref. 13), except for transitions from the β band, where only the predictions of Greiner are given.

Nucleus	$2_\gamma-2_g$	$3_\gamma-2_g$	$3_\gamma-4_g$	$4_\gamma-4_g$	$2_\beta-2_g$	$4_\beta-4_g$
^{152}Sm	−9.5(2) [a] (8.5; 12.5)	−8.0(9) [b] (7.4; 11.7)	−7.0(3) [a] (6.5; 8.5)	−2.8(3) [a] (5.1; 6.5)	$+(25^{+7}_{-4})$ [a] (6.6)	+4.7(21) [a] (3.4)
^{154}Gd	−11.6(11) [b] (7.3; 12.4)	−6.6(7) [b] (6.4; 11.6)	−7.5(2) [c] (5.6; 8.4)	−4.9(6) [c] (4.4; 6.4)	+16(4) [c] (6.4)	+9(3) [c] (3.4)
^{156}Gd	−17(3) [d] (9.5; 14.1)				$-(5.7^{+2.7}_{-1.4})$ [d] (7.3)	
^{160}Dy	−12.5(19) [e] (8.5; 13.6)	−9.4(25) [f] (7.4; 12.7)	$-(6^{+6}_{-2})$ [f] (6.5; 9.3)			
^{162}Dy	$-(9^{+\infty}_{-7})$ [g] (10.0; 14.9)			$-(3^{+6}_{-1})$ [h] (6.0; 7.8)		
^{164}Dy	$-(12^{+\infty}_{-9})$ [g] (12.0; 15.6)					
^{166}Er	$-(27^{+54}_{-13})$ [h] (9.0; 15.7)			$-(5^{+4}_{-2})$ [h] (5.4; 8.1)		
^{168}Er	$-(39^{+30}_{-12})$ [i] (10.8; 15.9)	+20(3) [i] (9.4; 14.8)	−7.7(5) [i] (8.3; 10.8)	$-(8^{+8}_{-5})$ [h] (6.5; 8.3)		
^{170}Er	$-(67^{+\infty}_{-48})$ [h] (11.2; 16.1)			$-(45^{+\infty}_{-26})$ [h] (6.7; 8.5)		
^{172}Yb	$-(7^{+4}_{-2})$ [f] (28; 16.5)	$-(4^{+2}_{-1})$ [f] (24; 15.4)				
^{174}Hf					<−4 [j] (8.2)	−3(1) [j] (4.2)
^{178}Hf	$-(30^{+\infty}_{-19})$ [k] (3.0; 14.3)					
^{182}W	$+(19^{+17}_{-5})$ [l] (4.2; 13.4)	$-(49^{+81}_{-16})$ [l] (3.6; 12.5)	−9(2) [l] (3.2; 9.1)		−0.51(5) [m] (6.7)	
^{184}W	−20(1) [n] (5.2; 13.0)	−14.7(10) [n] (4.5; 12.1)	−13.2(12) [n] (4.0; 8.8)	$-(8^{+4}_{-3})$ [n] (3.1; 6.8)	+2.3(6) [n] (6.5)	
^{186}W	$-(18^{+6}_{-5})$ [m] (5.2; 13.2)				$+(15^{+80}_{-7})$ [m] (6.6)	
^{186}Os	$-(16^{+24}_{-6})$ [o] (5.2; 14.0)	$-(17^{+12}_{-8})$ [o] (4.5; 13.1)				
^{188}Os	−26(6) [o] (; 13.5)	−11(5) [o] (; 12.6)				
^{190}Os	−23(3) [o] (; 13.7)	−16(3) [o] (; 12.8)				

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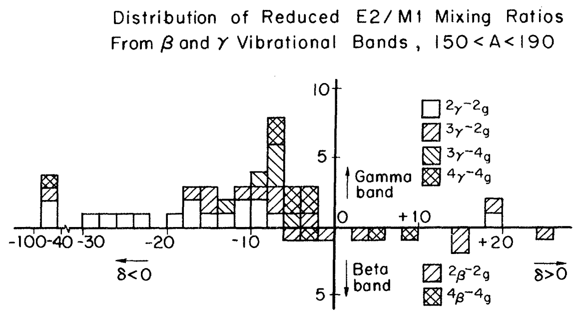

FIG. 1. Histogram of reduced $E2/M1$ mixing ratio, δ/E_γ (MeV), of selected γ transitions in even-even deformed nuclei. The ordinate above (below) the axis indicates the number of cases in which transitions depopulating the γ (β) band have reduced mixing ratios which fall within the range of values indicated on the abscissa.

erally been assigned as a $K=0^+$ excitation showing, for example, a large $E2$ excitation probability in a Coulomb excitation measurement. The experimental results are summarized graphically in Fig. 1. A number of similar compilations of $E2/M1$ mixing ratios have been done previously; the most recent is that of Hamilton.[6]

The systematic behavior of the phase of the mixing ratio is apparent from an inspection of the table. With minor exceptions, transitions from the γ band have negative phase, while a majority of the transitions from the β band seem to show the opposite phase

The magnitudes and phases of the mixing ratios may be predicted from a variety of different models.

a. $\Delta K=2$ band mixing. This type of analysis takes into account mutual mixing of the ground state, β, and γ bands, and has been widely used with reasonable success to interpret deviations of the relative reduced transition probabilities of transitions from the γ band from the predictions of the adiabatic rotational model. The interpretation of transitions from the β band has met with considerably less success. The present notation for the band-mixing parameters is that of Marshalek[7] and of the Oak Ridge-Vanderbilt group.[8] A similar analysis has been done by Rud and Bonde Nielsen.[9] The $M1$ matrix elements are now given in terms of the static magnetic moments of the admixed intrinsic states, and the mixing ratios are given by

$$I_\gamma - I_g:\ \frac{\delta}{E} = \frac{-AQ_0}{Z_\gamma(g_K-g_R)+Bg_R Z_\beta Z_{\beta\gamma}}, \qquad (5)$$

$$I_\beta - I_g:\ \frac{\delta}{E} = \frac{-AQ_0}{(Q_\gamma/Q_\beta)^2 Z_\gamma Z_{\beta\gamma}[g_R+4(g_K-g_R)/I(I+1)]}, \qquad (6)$$

where A and B have the following values:

	$2_\gamma-2_g$	$3_\gamma-2_g$	$3_\gamma-4_g$	$4_\gamma-4_g$	$2_\beta-2_g$	$4_\beta-4_g$
$A=$	0.176	0.330	0.048	0.092	0.029	0.001
$B=$	18	0	0	200		

This calculation assumes that the intrinsic quadrupole moment Q_0 and rotational g factor g_R are constant for the three bands; g_K is the intrinsic g factor evaluated for the γ band; Q_β and Q_γ are the intrinsic $E2$ excitation moments of the β and γ bands.[8] The band-mixing parameters are in the notation of Ref. 8. With $Z_\gamma \simeq 5\times10^{-2}$, this model gives δ values for γ-band transitions too large by at least an order of magnitude; i.e., the predicted $M1$ amplitudes are too small. Independent of Z_γ, the relative magnitudes of δ for the γ-band deexcitations are not in agreement with experiment [the ratio $\delta(3_\gamma-2_g)/\delta(3_\gamma-4_g)$ is predicted to have a value of 7, while the experimental values are generally in the range 1–2]. The phases for the γ-band transitions are not easily calculable, depending on the values of (g_K-g_R) and $Z_{\beta\gamma}$, which are not known for most of the nuclei considered. The ratio of the mixing ratios of the two transitions from the β band is not in agreement with the predictions based on this model.

b. $\Delta K=1$ band mixing. The first-order Coriolis interaction can mix $K^\pi=1^+$ states into the $K^\pi=0^+$ (ground state) and $K^\pi=2^+$ bands. The $M1$ matrix element resulting from such mixing is given by[10]

$$\langle I_f K=0\,\|\mathfrak{M}(M1)\|\,I_i K=2\rangle = (-1)^{I_i+I_f+1}[I_f(I_f+1)(2I_f+1)]^{1/2}\langle I_f 1 1 1\,|\,I_i 2\rangle M_1, \qquad (7)$$

where

$$M_1=\sqrt{2}\,\langle K=2\,|[\epsilon_{+1},\mathfrak{M}'(M1,\nu=1)]|\,K=0\rangle$$
$$= -\left\{\frac{\langle K=2\,|h_{+1}|\,K=1\rangle}{E_{K=1}-E_{K=2}}\langle 00\,\|\mathfrak{M}(M1)\|\,11\rangle + \left(\frac{2}{3}\right)^{1/2}\frac{\langle K=1\,|h_{+1}|\,K=0\rangle}{E_{K=1}-E_{K=0}}\langle 22\,\|\mathfrak{M}(M1)\|\,11\rangle\right\}, \qquad (8)$$

where $\mathfrak{M}'$ refers to the intrinsic system and h_{+1} is the operator associated with $\Delta K=1$ Coriolis mixing. The energies E_K in Eq. (8) refer to the excitation energy of the intrinsic states.

A similar calculation for the mixing of $K^\pi=1^+$ states into the ground state and β bands yields (setting $I_i=I_f=I$)

$$\langle IK=0\,\|\mathfrak{M}(M1)\|\,IK=0'\rangle = [I(I+1)(2I+1)]^{1/2}\langle I011\,|\,I1\rangle M_1', \qquad (9)$$

where

$$M_1' = \sqrt{2}\left\{\frac{\langle K=1|h_{+1}|K=0\rangle}{E_{K=1}-E_{K=0}}\langle 00\|\mathfrak{M}(M1)\|11\rangle - \frac{\langle K=1|h_{+1}|K=0'\rangle}{E_{K=1}-E_{K=0'}}\langle 11\|\mathfrak{M}(M1)00'\rangle\right\}. \tag{10}$$

The $K=0$ state indicates the ground-state band, while the $K=0'$ state refers to the β band.

The reduced mixing ratios may then be written as

$$I_\gamma - I_g:\ \frac{\delta}{E} = A(M_2/M_1)\,, \tag{11}$$

$$I_\beta - I_g:\ \frac{\delta}{E} = A(M_2'/M_1')\,, \tag{12}$$

where A has the following values:

	$2_\gamma-2_g$	$3_\gamma-2_g$	$3_\gamma-4_g$	$4_\gamma-4_g$	$2_\beta-2_g$	$4_\beta-4_g$
$A=$	0.446	0.418	0.305	0.233	0.258	0.135

and where M_2 and M_2' are the intrinsic $E2$ transition moments:

$$\begin{aligned} M_2 &= \langle K=0|\mathfrak{M}'(E2,-2)|K=2\rangle\,,\\ M_2' &= \langle K=0|\mathfrak{M}'(E2,0)|K=0'\rangle\,. \end{aligned} \tag{13}$$

At present there exists insufficient knowledge of $K^\pi = 1^+$ excitation to predict either the coupling or $M1$ matrix elements of Eqs. (8) and (10). However, conclusions are possible regarding the relative phases and magnitudes of the mixing ratios. The relative magnitudes are as follows:

$$\frac{\delta}{E}(2_\gamma-2_g):\frac{\delta}{E}(3_\gamma-2_g):\frac{\delta}{E}(3_\gamma-4_g):\frac{\delta}{E}(4_\gamma-4_g) = 1:0.94:0.68:0.52\,,$$

$$\frac{\delta}{E}(2_\beta-2_g):\frac{\delta}{E}(4_\beta-4_g) = 1:0.52\,.$$

These relationships are in better agreement with the observed values than are the relationships deduced above the $\Delta K=2$ mixing. The relative phases of the mixing ratios are predicted to be the same, which is likewise in agreement with experiment.

An estimate of the magnitude of the required coupling strength indicates that the observed magnitudes of the mixing ratios require, for $\langle 00\|\mathfrak{M}(M1)\|11\rangle \sim$ one single-particle unit, a coupling matrix element $\langle K+1|h_{+1}|K\rangle \simeq 10$ keV, which is not an unreasonably large value.

c. Microscopic theory of the γ band. Bès *et al.*[11] have considered the microscopic structure of the γ-vibrational state, in which the intrinsic state is treated as a superposition of quasiparticle pairs. The $M1$ amplitudes are obtained through Coriolis band mixing of the γ band and ground-state band. The predictions of Bès *et al.* for the magnitudes of the $E2/M1$ mixing ratios are given in Table I. The phase of the mixing ratio is not uniquely determined in this model, but rather depends on the competition between the rotational motion and the orbital motion of the protons. If, as concluded by Bès *et al.*,[11] the contribution from the rotational motion dominates, this model predicts $\delta > 0$, in disagreement with experiment, although the predicted magnitudes seem to be in good agreement with experimental values.

A microscopic calculation was also done by Tamura and Yoshida,[12] who considered the magnitudes and phase of the $M1$ matrix element in terms of the lowest-lying $K=2$ two-quasiparticle states which can mix with both the γ and ground-state bands. They estimated $|\delta| \sim 10$, in reasonable agreement with observed values, and also $\delta > 0$. However, their δ was defined in terms of absorption matrix elements, and the transformation to the presently employed emission matrix elements requires a knowledge of the spatial and temporal symmetry properties of the nuclear wave functions and multipole operators used. (A complete discussion of this problem is given by Alder and Steffen.[5]) If we assume the convention of Biedenharn-Rose[2] was used, then in terms of the present convention, $\delta < 0$, in agreement with experiment.

d. g-factor variation. In the $\Delta K=2$ band-mixing analysis given above, it was assumed that the g_R factors were identical for the β, γ, and ground-state bands. Relaxing this requirement gives rise to $M1$ transitions which depend on the variation of g_R; however, this additional contribution to the $M1$ matrix element occurs only for $\Delta I=0$ transitions. This contribution may be taken into account by introducing the following additional terms into the appropriate denominator of Eqs. (5) and (6):

$$\begin{aligned} 2_\gamma - 2_g&:\ -\tfrac{3}{2}Z_\gamma[g_R(g) - g_R(\gamma)],\\ 4_\gamma - 4_g&:\ -5Z_\gamma[g_R(g) - g_R(\gamma)],\\ 2_\beta - 2_g&:\ \tfrac{3}{2}Z_\beta[g_R(g) - g_R(\beta)],\\ 4_\beta - 4_g&:\ \tfrac{1}{3}Z_\beta[g_R(g) - g_R(\beta)]. \end{aligned} \tag{14}$$

The indices g, β, and γ refer to the ground state, β, and γ bands, respectively. The $2_\gamma - 2_g$ and $2_\beta - 2_g$ mixing ratios both require that the g_R-factor difference between the ground-state and vibrational bands be

$$\Delta g = g_R(g) - g_R(\beta,\gamma) \approx -0.5\,,$$

which implies an increase in g_R by a factor of $2\frac{1}{2}$ in the excited bands. Such an increase seems highly unlikely.

Greiner[13] has discussed the lowering of g_R factors from the value Z/A in terms of a model in which the proton distribution is characterized by a somewhat smaller deformation than that of the neutrons. The $M1$ transition operator then obtains a tensor character dependent on the collective variables, and thus has nonvanishing matrix elements between the collective bands. The predictions of this model for the magnitudes of the $E2/M1$ mixing ratios are given in Table I. This model is characterized by a smooth variation of δ from nucleus to nucleus, and thus is unable to account for the apparent sudden changes in δ in the Er and Yb nuclei. The phase of the mixing ratio appears in this model to be positive for transitions from both the β and γ bands; however, as discussed above, the absorption matrix elements experience a change of phase when converted to emission matrix elements. Consequently, although the predicted phase of the γ-band-mixing ratio agrees with experiment, the identical phase predicted for the β band does not agree.

e. Pairing-plus-quadrupole model. The apparent increase in δ/E for the osmium nuclei comes about through a decrease in the energy of the $K = 2^+$ level associated with the γ vibration, rather than through an increase in δ. For these nuclei, which are in a region of transition from deformed to spherical equilibrium shapes, Kumar and Baranger[14] have employed the pairing-plus-quadrupole model to predict energy levels and electromagnetic multipole moments. The $E2$ and $M1$ moments were calculated by Kumar,[15] and were found to be in good agreement with experimental $E2/M1$ mixing ratios (magnitude as well as phase) for Os nuclei, although the agreement is somewhat poorer for the (more deformed) W nuclei (see Refs. 1–o of Table I).

III. CONCLUSIONS

It can be concluded from this investigation that at present there is no satisfactory interpretation of both the magnitudes and phases of $M1$ admixtures in collective transitions in even-even deformed nuclei, although the $\Delta K = 1$ coupling through $K = 1^+$ excitations seems to hold the most promise for a successful theory. Further insight into this problem must await studies of $K = 1^+$ excitations, in order that the matrix elements entering into Eqs. (8) and (10) may be evaluated. Additionally, the agreement between the various theories and experiment seems to be poorest for the Er, Yb, and Hf nuclei, and it would thus be of great interest to reduce the experimental uncertainty for the Er results and to obtain additional results for Yb and Hf nuclei. Most of the results quoted in Table I result from radioactive-decay studies using high-resolution Ge(Li) detectors and otherwise conventional spectrometer systems. Since the subset of nuclei amenable to such studies has been nearly exhausted, other means must be employed to obtain the needed results. In particular, a number of nuclei have levels of the γ and β bands populated by short-lived radioactive decays (half-lives the order of a few minutes) which would require special techniques of sample handling and data accumulation. Additional cases could be studied by angular distributions following multiple Coulomb excitation or angular correlations of secondary γ rays following neutron capture or other nuclear reactions.

Finally, we note that, while most reasonable theories predict a unique phase for all mixing ratios of transitions depopulating the γ-vibrational band, $\delta(3_\gamma - 2_g)$ in ^{168}Er and $\delta(2_\gamma - 2_g)$ in ^{182}W are at variance with the remainder of the cases studied. While no explanation for the former case is apparent, ^{182}W also shows an anomalous phase and magnitude of $\delta(2_\beta - 2_g)$. (While Refs. l and m of Table I chose the larger root for δ, the directional correlation data of Herzog, Canty, and Killig[16] are more consistent with the smaller root.) Although the $K = 0$ excitation of ^{182}W is not a good β vibration, it is coupled rather strongly to the γ vibration, owing primarily to the small energy spacing.[17] In the $\Delta K = 2$ formalism, the anomalous $2_\gamma - 2_g$ value could arise from a contribution from the second term of the denominator of Eq. (2), and the $2_\beta - 2_g$ phase (compared with ^{184}W) is consistent with the sign change of the $Z_{\beta\gamma}$ mixing parameter[17] between ^{182}W and ^{184}W. A measurement of $\delta(4_\gamma - 4_g)$ in ^{182}W would shed considerable light on this problem.

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