Lifetimes of the Lowest $\frac{5}{2}^+$ and $\frac{9}{2}^+$ States of the Mirror Pair ^{23}Na-^{23}Mg†

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The lifetimes of the $\frac{5}{2}^+$ and $\frac{9}{2}^+$ members of the $K^\pi = \frac{3}{2}^+$ ground-state rotational band in the mirror pair ^{23}Na-^{23}Mg have been measured via the Doppler-shift attenuation method. The reactions $^{12}C + {}^{12}C = {}^{23}Na + p$ or $^{23}Mg + n$ were used and line shapes of the decay γ rays were observed in singles for recoil into Al and Ni backings. The results are 1.63 ± 0.20 and 2.00 ± 0.24 psec for the ^{23}Na and ^{23}Mg $\frac{5}{2}^+$ first excited states and 0.11 ± 0.02 and 0.08 ± 0.02 psec for the ^{23}Na and ^{23}Mg $\frac{9}{2}^+$ states. These results are compared to the predictions of the Nilsson model. The separation of the isoscalar and isotensor components of the $M1$ rates is discussed as well as the separation of the spin and orbital contributions to the isotensor component. Incidental to this work, the energies of the first excited states were determined as 439.80 ± 0.15 keV(^{23}Na) and 450.70 ± 0.15 keV (^{23}Mg), and the $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$ transition energies were measured with comparable accuracy.

I. INTRODUCTION

Electromagnetic transitions between bound states of nuclei provide a most stringent and widely applicable testing ground between experiment and theory. In order to obtain the highest possible sensitivity and meaning from this comparison of experiment with nuclear models it is desirable to isolate, insofar as is possible, the various contributions to the transition matrix elements. One such obvious separation is between the contributions of the protons and neutrons

$$B(L) = (\lambda_p + \lambda_n)^2 , \tag{1}$$

where $B(L)$ is the transition strength of an electric or magnetic multipole of order L (i.e., EL or ML radiation), and λ_p and λ_n are the proton and neutron transition amplitudes. In the light nuclei where isospin is well conserved, another natural and straightforward separation is into isoscalar and isovector amplitudes, e.g., for $\Delta T = 0$ transitions,

$$B(L, T_3) = (\lambda_0 + T_3\lambda_1)^2 , \tag{2}$$

where $T_3 = (N - Z)/2$ designates one nucleus from the members of the isospin multiplet and λ_0 and $T_3\lambda_1$ are, respectively, the isoscalar and isovector amplitudes with λ_0 and λ_1 otherwise independent of T_3.[1]

In the case of Eq. (2) the experimental procedure used to extract λ_0 and λ_1 is to measure analogous transition strengths $B(L, T_3)$ in an isospin multiplet for two or more different T_3. The simplest situation for which this is possible is analogous transitions in mirror nuclei ($T_3 = \pm\frac{1}{2}$).

For the case of ML transitions in mirror nuclei a third separation is possible; namely, the orbital and spin contributions to the isovector amplitude can be determined separately from the ML transitions in the $T_3 = \pm\frac{1}{2}$ nuclei and from the unique n-forbidden β decay (with $n = L - 1$) connecting the initial state in one nucleus with its analog in the other.[1-3]

Recently, the University of Florence group has separated the isoscalar and isovector amplitudes of several $E2$[4-6] and $M1$[7,8] transitions in $(2s, 1d)$ nuclei, while Bardin and Becker[3] have presented a detailed treatment of the comparison of mirror $\Delta T = 0$ $M1$ transitions with Gamow-Teller β-decay ($n = 0$) matrix elements together with a compilation of the relevant experimental data for the $(2s, 1d)$ shell.

This is a report of the measurement of the lifetimes of the lowest $\frac{5}{2}^+$ and $\frac{9}{2}^+$ states of ^{23}Na and ^{23}Mg.[9-11] These states are presumed to belong to the $K^\pi = \frac{3}{2}^+$ ground-state rotational band[9,10] illustrated in Fig. 1. Results for the ^{23}Na states have been published, but lifetimes have not been previously reported for ^{23}Mg. The motivation of the measurements was to obtain the data necessary to separate the contributions to the transition matrix elements as outlined above. To this end the experiment was designed to obtain the lifetimes of these analog states, especially the first excited states, as accurately as possible.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The lifetimes to be measured were predicted or known to be in the range accessible to the Doppler-shift attenuation method (DSAM). Reactions suitable for use with this method, namely, ^{12}C-(^{12}C, $n\gamma$)^{23}Mg ($Q = -2.60$ MeV) and ^{12}C(^{12}C, $p\gamma$)^{23}Na ($Q = +2.24$ MeV), were used to populate these levels. γ rays were observed at 66° to the beam with a 60-cm^3 Ge(Li) detector at 14 cm from the target. Three different targets were used, consisting of a self-supporting 50-μg/cm^2 C foil, a 50-μg/cm^2

C target on a thick Al backing, and a 100-μg/cm^2 C target on a thick Ni backing. The beam energy, 19 MeV, was chosen to maximize the peak to background ratio for the ^{23}Mg $1\rightarrow 0$ full-energy peak when observed with the Al-backed targets; this value corresponds approximately to the Coulomb barrier for ^{12}C on ^{27}Al. The beam energy loss in the 100-μg/cm^2 C layer was ~250 keV, which resulted in a center-of-mass velocity for ^{12}C on ^{12}C of $\beta_{\text{c.m.}}=0.029$ (in units of c). The recoiling ^{23}Mg nuclei were emitted within 10° of the beam axis with a velocity spread confined within ±17% of $\beta_{\text{c.m.}}$. These conditions are adequate for the DSAM so that it was not necessary to employ a coincidence condition between the outgoing nucleons and the deexcitation γ rays in order to further define the recoil axis. Use of a singles measurement had the great advantage that the ^{23}Na and ^{23}Mg lifetimes could be obtained simultaneously. However, it also meant that the angle θ_γ of the γ-ray detector to the beam axis had to be chosen large enough so that the Doppler-broadened and -shifted lines of the ^{23}Na and ^{23}Mg $1\rightarrow 0$ transitions did not significantly overlap, and yet small enough so that the sensitivity of the line shapes to the lifetimes of these transitions was retained. A compromise between these two factors dictated our choice of $\theta_\gamma=66°$.

The region of γ-ray energy containing the full-energy-loss peaks of the ^{23}Na and ^{23}Mg $1\rightarrow 0$ transitions is displayed in Fig. 2 for the Ge(Li) spectra recorded for the three different targets. The actual unattenuated Doppler shifts and Doppler line shapes were determined from the spectrum observed for decay in vacuum. The detector angle θ_γ was actually calculated from

$$\cos\theta_\gamma=\frac{E_K(\theta_\gamma)-E_0}{E_K(0°)-E_0}, \tag{3}$$

where E_0 is the energy of γ rays emitted by nuclei at rest. $E_K(\theta_\gamma)$ is the full Doppler-shifted γ-ray energy obtained from

$$E_\gamma(\theta_\gamma,t)=E_0[1+\frac{1}{c}v(t)\cos\theta_\gamma] \tag{4}$$

with $t=0$, i.e., $v(0)$ is the z component of nuclear

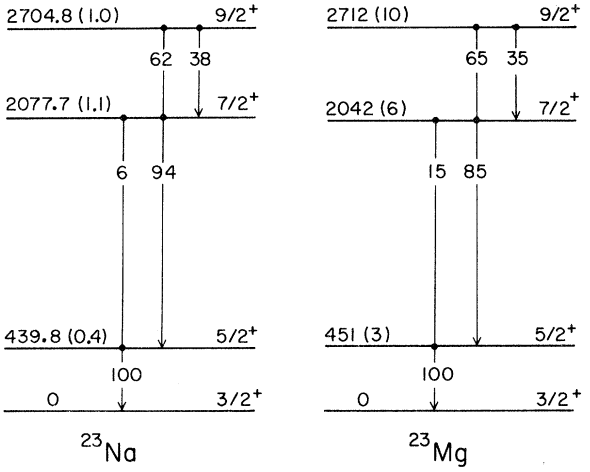

FIG. 1. The first four levels of the ground-state rotational bands of the $T_z=\pm\frac{1}{2}$ mirror pair ^{23}Na-^{23}Mg. The excitation energies and γ-ray branching ratios are from Ref. 9 (^{23}Mg) and Refs. 10 and 11 (^{23}Na). The spin-parity of the ^{23}Mg 2.71-MeV level is not known. It is assumed to be $\frac{9}{2}^+$ because its properties are those expected for the analog of the ^{23}Na $\frac{9}{2}^+$ state.

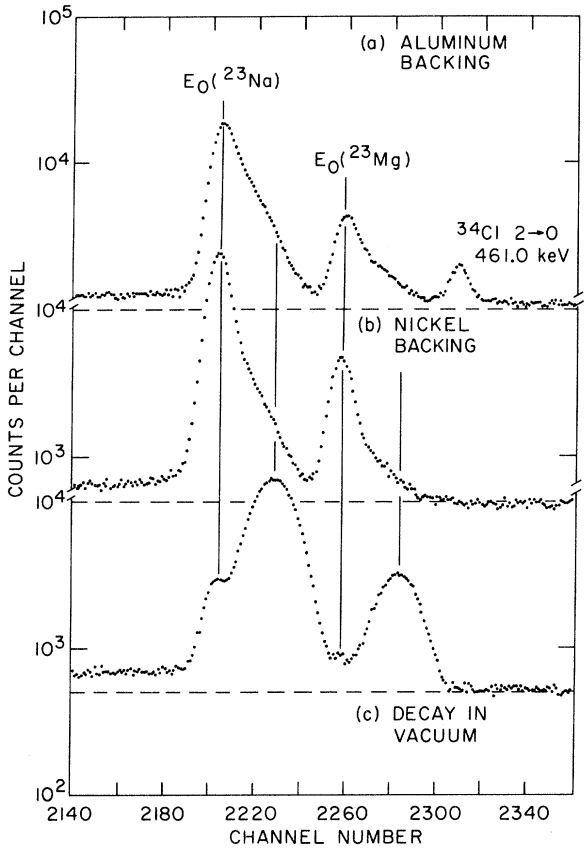

FIG. 2. (a)–(c) Partial γ-ray spectra illustrating the lineshapes of the γ rays corresponding to the decay of the first excited states of ^{23}Na and ^{23}Mg at 440 and 451 keV, respectively. The detector resolution in this γ-ray energy region is 1.95 keV FWHM. The symbol E_0 refers to the γ-ray energy for nuclei decaying at rest. The ^{34}Cl peak in Fig. 2(a) arises from the ^{27}Al(^{12}C, αn)-^{34}Cl reaction induced in the aluminum backing. For the decay in vacuum [Fig. 2(c)] the observed Doppler shifts were 1.16% which corresponds to a γ-ray detector angle of 66°. The observed Doppler broadening in Fig. 2(c) corresponds to a spread in the angle subtended by the detector of ±9°. Other details are given in the text.

recoil velocity at $t=0$ (in this case at the moment of emission from the C target) and $E_K(\theta_\gamma) \equiv E_\gamma(\theta_\gamma,0)$. The unattenuated Doppler line shapes for decay in vacuum were found to be represented very well by Gaussians. After unfolding the detector response of 1.95 keV FWHM, the full widths at half maximum (FWHM) were found to be 3.57 and 3.94 keV, respectively, for the ^{23}Mg and ^{23}Na $1\rightarrow 0$ transitions. Apparent in Fig. 2(c) is the shoulder on the low-energy side of the ^{23}Na peak due to unshifted ^{23}Na $1\rightarrow 0$ γ rays following ^{23}Mg$(\beta^+)^{23}$Na. This peak amounted to 6.7% of the ^{23}Na $1\rightarrow 0$ intensity and the correction for it in the spectra of Figs. 2(a) and 2(b) was easily made. Also apparent in Fig. 2(c) is a small peak in the ^{23}Mg $1\rightarrow 0$ line shape at E_0, which is due to reactions induced in ^{12}C on the beam stop and slit systems. The line shape was also easily corrected for contributions from such reactions.

The ^{34}Cl $2\rightarrow 0$ peak in Fig. 2 arises from ^{12}C + ^{27}Al. The response of the Ge(Li) detector to these γ rays was determined from a spectrum recorded for an Al target (no ^{12}C), and the spectrum due to the 461-keV γ ray was stripped from the spectrum of Fig. 2(a). The background under the composite ^{23}Na-^{23}Mg lines was determined by interpolating [the dashed curves of Figs. 2(a) and 2(b)] an exponential least-squares fit made to a region above the lines with the condition that the intensity of the line shapes in the region just below the main peaks [channels 2140–2180 in Figs. 2(a) and 2(b)] be in agreement with the known response of the Ge(Li) detector to monoenergetic γ rays. The ^{23}Na and ^{23}Mg line shapes are seen to overlap slightly. They were separated iteratively using the theoretical line shapes which we will now describe.

The procedure used to extract the mean lifetime τ from a Doppler line shape has been described fully.[12] Briefly, from Eq. (4) we see that the distribution of γ-ray energies is equivalent to a distribution of the nuclear recoil velocities at the moment of γ-ray emission. This velocity distribution, dN/dv, can be related to the radioactive decay law, dN/dt, and the energy loss in the target,

$$\frac{dE}{dx} = M\frac{dv}{dt} ,$$

through

$$\frac{dN}{dv} = \frac{dN/dv}{dt/dt} . \tag{5}$$

Thus a knowledge of dE/dx, or more accurately dv_z/dt, and dN/dv leads to a determination of dN/dt and thus τ.

In the present case we parametrize the energy loss by

$$M\frac{dv_z}{dt} = K_n\left(\frac{v}{v_0}\right)^{-1} + K_e\left(\frac{v}{v_0}\right) - K_3\left(\frac{v}{v_0}\right)^3 , \tag{6}$$

TABLE I. Parameters for the energy loss in Al and Ni. The parametrization is given in the text.

Ion	Stopping material	K_n	K_e	K_3
^{23}Na	Al	0.3342	2.7713	0.0281
	Ni	0.2723	1.7201	0.0154
^{23}Mg	Al	0.3871	2.8368	0.0221
	Ni	0.3140	1.8081	0.0144

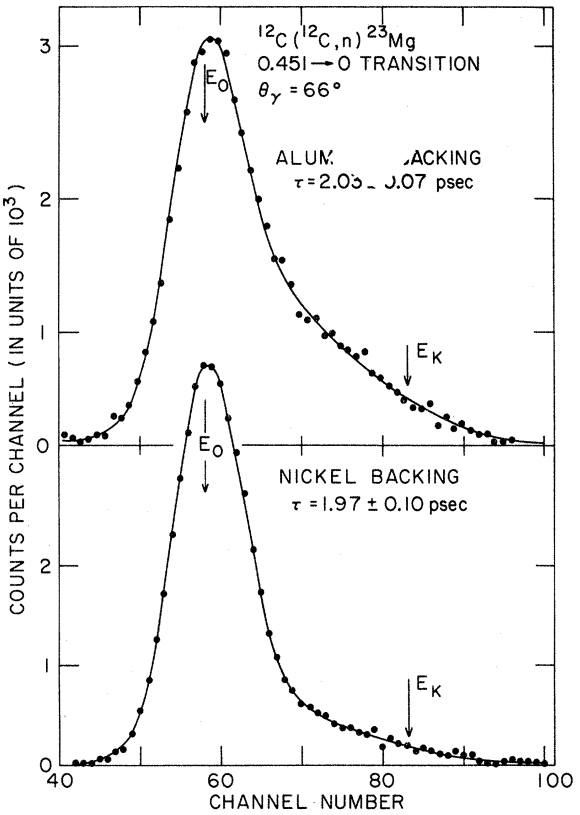

FIG. 3. Doppler line shape of the ^{23}Mg $0.45\rightarrow 0$ transition observed at 66° to the beam in the ^{12}C(^{12}C,n)^{23}Mg reaction. The data are the same as those of Fig. 2. Background has been subtracted, as has the high-energy portion of the ^{23}Na $0.44\rightarrow 0$ Doppler line shape and, for the aluminum backing, the low-energy portion of the ^{34}Cl $0.46\rightarrow 0$ transition. The solid curves are least-squares fits, as explained in the text, with resulting values of χ^2 (normalized to the number of degrees of freedom) of 1.10 (aluminum) and 1.38 (nickel) and the indicated mean lifetimes. The uncertainties in τ are statistical only, and do not include any contribution from that in dE/dx. The symbol E_0 designates the γ-ray energies for emission by nuclei at rest while E_K represents the γ-ray energies for those decaying in vacuum [i.e., the centroid energies of the Doppler-shifted peaks of Fig. 2(c)].

where $v_0 = c/137$. The parameters K_n, K_e, K_3 were determined from experimental data and systematics[13] utilizing the effective charge concept. The parameters used are listed in Table I. The fitting procedure was to determine and fix the detector response, kinematical, and dE/dx parameters and to determine χ^2 as a function of τ. Results for the ^{23}Mg $1 \rightarrow 0$ transitions are illustrated in Fig. 3 for the minima in χ^2. The uncertainties are from the width of the χ^2 minima.

The fit to the Doppler line shape for the ^{23}Na $0.44 \rightarrow 0$ transition was made in exactly the same manner, with the exception that the contribution to the "stopped" portion of the line shape from ^{23}Mg$(\beta^+)^{23}$Na$(1 \rightarrow 0)$ was subtracted before the fit was made. The results for the Al and Ni backings were identical: $\tau = 1.63 \pm 0.07$ psec where, again, the uncertainties are statistical only.

The final values of τ for the ^{23}Mg 0.45- and ^{23}Na 0.44-MeV levels are 2.00 ± 0.24 and 1.63 ± 0.20 psec, respectively, where the assigned 12% errors arise from the uncertainty assumed in dE/dx for ^{23}Mg and ^{23}Na ions slowing down in the target-backing combinations and from other possible systematic errors such as cascade feeding from long-lived states.

Since cascade feeding from states with lifetimes not negligibly short compared to the ones in question is a source of possible error, the γ-ray spectra were carefully examined for evidence of formation of such states. We found for both nuclei that members of the ground-state rotational band were populated much more strongly than other states, and thus the only states which could significantly influence our results are the $\frac{7}{2}^+$ and $\frac{9}{2}^+$ states of this band. From the relative γ-ray intensities and the known branching ratios[11,9] in ^{23}Na and ^{23}Mg it was determined that the $\frac{9}{2}^+$, $\frac{7}{2}^+$, $\frac{5}{2}^+$ states were directly formed in the ratio 50:60:10 in ^{23}Na and 40:30:30 in ^{23}Mg with uncertainties of 15–20%. The small corrections to the $\frac{5}{2}^+$ lifetimes for the feeding from the $\frac{9}{2}^+$ and $\frac{7}{2}^+$ states were made using the lifetimes determined in the present work for the $\frac{9}{2}^+$ states and theoretical estimates for the $\frac{7}{2}^+$ states. These estimates are 0.015 and 0.031 psec for ^{23}Na and ^{23}Mg, respectively, corresponding to rotational model predictions for $\frac{7}{2}^+ \rightarrow \frac{5}{2}^+$ $M1$ transitions as discussed in Sec. III.

The mean lifetimes of the $\frac{9}{2}^+$ states were determined from the centroid shifts of the $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$ cascades as illustrated in Fig. 4. The DSAM attenuation factor $F(\tau)$ was obtained as the ratio of the mean Doppler shifts (centroid energy minus rest energy) for decay in nickel or aluminum to that for decay in vacuum. The results for the ^{23}Na, $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$, $2.70 \rightarrow 2.08$ transition were $F(\tau) = 0.79 \pm 0.025$ and 0.86 ± 0.027 for the Ni and Al backings, respectively. For the ^{23}Mg, $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$, $2.71 \rightarrow 2.05$ transition the result for Ni is 0.81 ± 0.03. Due to interference from ^{12}C + ^{27}Al γ rays, no result was obtained for the aluminum backing. These $F(\tau)$ values were converted to mean lifetimes using both the parametrization for dE/dx given here and also a computer calculation based on the method of Blaugrund.[15] The two methods, which differ in the treatment of the nuclear stopping and scattering, represented in our treatment by $K_n(v/v_0)^{-1}$, gave mean lifetimes 2.5% different when the same value of K_e was used. The final results are 0.11 ± 0.02 and 0.08 ± 0.02 psec, respectively, for the ^{23}Na and ^{23}Mg $\frac{9}{2}^+$ lifetimes.

The energies of the γ rays corresponding to the ^{23}Na, $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$, $2705 \rightarrow 2078$, and $\frac{5}{2}^+ \rightarrow \frac{3}{2}^+$, $440 \rightarrow 0$ transitions and the ^{23}Mg, $\frac{9}{2}^+ \rightarrow \frac{7}{2}^+$, $2712 \rightarrow 2042$, and $\frac{5}{2}^+ \rightarrow \frac{3}{2}^+$, $451 \rightarrow 0$ transitions were accurately determined as a consequence of these measurements. The results are 626.80 ± 0.4 and 439.80 ± 0.15 keV for ^{23}Na and 662.04 ± 0.4 and 450.70 ± 0.15 keV for ^{23}Mg. The results for ^{23}Na are in fair agreement with previous results.[10] There are no published accurate energies for the ^{23}Mg γ rays.

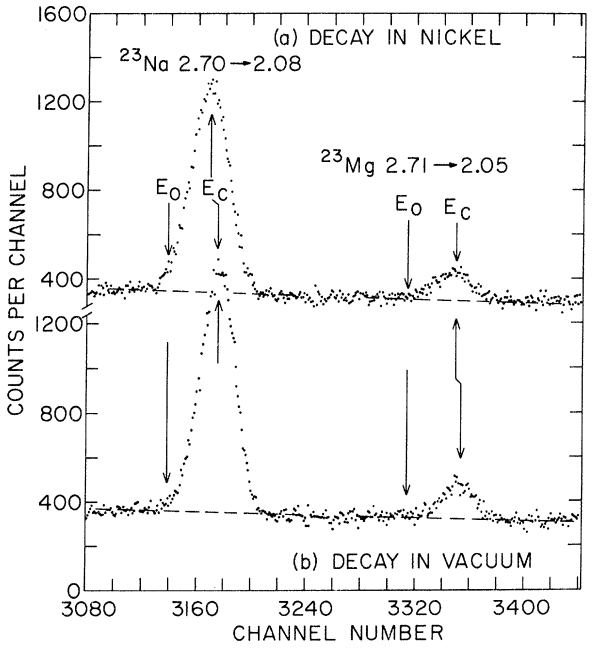

FIG. 4. Partial γ-ray spectra illustrating the Doppler shifts of the γ rays corresponding to the ^{23}Na $2.70 \rightarrow 2.08$ and ^{23}Mg $2.71 \rightarrow 2.04$ transitions. The symbol E_0 refers to the γ-ray energy for nuclei decaying at rest and E_c is the energy of the centroid of the counts in the peaks with exponential backgrounds (denoted by the dashed curves) subtracted. The lifetimes were deduced from the difference of E_c for decay in vacuum and in Ni as explained in the text.

TABLE II. Mass 23 transition strengths (in Weisskopf units). The experimental (expt) values are from the present lifetime measurements and the branching ratios of Refs. 11 (Na) and 16 (Mg). The theoretical (Nilsson) values are discussed in the text.

J_i^π	J_f^π	Multipolarity	$B(L)_{\text{expt}}$ ^{23}Na	$B(L)_{\text{expt}}$ ^{23}Mg	B(Mg)/B(Na) (expt)	$B(L)_{\text{Nilsson}}$ ^{23}Na	$B(L)_{\text{Nilsson}}$ ^{23}Mg	B(Mg)/B(Na) (Nilsson)
$\frac{5}{2}^+$	$\frac{3}{2}^+$	$M1$	0.226 ± 0.027	0.171 ± 0.020	0.76 ± 0.07	0.329	0.159	0.482
$\frac{9}{2}^+$	$\frac{7}{2}^+$	$M1$	0.44 ± 0.09	0.47 ± 0.18	1.07 ± 0.46	0.493	0.237	0.482
$\frac{9}{2}^+$	$\frac{5}{2}^+$	$E2$	19.5 ± 3.8	28.0 ± 8.2	1.44 ± 0.50	...	...	...

III. DISCUSSION

The transition strengths determined by the present results are listed in Table II. For the decay of the $\frac{7}{2}^+$ levels we have used the branching ratios[11,16] given in Fig. 1. The strengths are given in Weisskopf units (W.u.) corresponding to the single-particle estimate quoted in the review article of Wilkinson.[17]

Also listed in Table II are the Nilsson[18] model predictions for the $M1$ rates. For transitions between the states of the $K^\pi = \frac{3}{2}^+$ ground-state band these are given by

$$B(M1; J_i \to J_f) = \tfrac{1}{30}(J_i\, \tfrac{3}{2}\, |0|\, J_f\, \tfrac{3}{2})^2\, G_{M1}{}^2 \text{ (W.u.)},$$

where

$$G_{M1} = 3g_l - 3g_R + (g_s - g_l)(a_{21}{}^2 - a_{22}{}^2)$$

as described by Nilsson and by Howard, Allen, and Bromley.[19] Following Poletti *et al.*[11] we have used a deformation parameter $\eta = +4$ and a core gyromagnetic ratio $g_R = 0.3$ in the Nilsson predictions.

In the extreme Nilsson model, which we use here, the extra-core nucleon making the transition is in orbit No. 7. This is a proton in ^{23}Na and a neutron in ^{23}Mg; therefore the separation of Eq. (1) is a natural choice since it corresponds to the separation implicit in the calculation. In particular we have $\lambda_p \gg \lambda_n$ in ^{23}Na, and $\lambda_n \gg \lambda_p$ in ^{23}Mg. In fact, the contribution of the lesser term (λ_p or λ_n) would vanish in this model were it not for the contribution of the core parametrized by g_R.

With respect to Table II, if we compare the Nilsson model with experiment for either ^{23}Na or ^{23}Mg (but not both) the agreement could be considered very good, and could obviously be improved if η were allowed to vary independently. The advantage of extending the comparison to both nuclei is clearly seen when we compare the experimental and theoretical values for the ratio B(Mg)/B(Na). First of all, the experimental ratio may be obtained with smaller uncertainty than the individual $B(M1)$'s. For example, the uncertainties in dE/dx involved in the lifetime measurements cancel out to first order in taking the ratio. More important is the fact that the theoretical ratio is quite insensitive to the parameters of the model (the ratio varies from 0.468 to 0.489 for $\eta = +2$ to $+6$). Thus, a comparison of the model prediction for this ratio with experiment clearly shows that the simple model is deficient and a more complex model is needed.

We now consider the separation of the $M1$ radiative widths into isoscalar and isotensor components. Using Eq. (2) we find $|\lambda_0/T_3\lambda_1| = 0.070 \pm 0.023$ for the $\frac{5}{2}^+ \to \frac{3}{2}^+$ transition and 0.02 ± 0.11 for the $\frac{9}{2}^+ \to \frac{5}{2}^+$ transition. We reject the other solution to the quadratic, namely the reciprocal of these values, since we expect[1] $|\lambda_1| > |\lambda_0|$. These ratios are within the range delimited by local systematics.[3]

For the $\frac{5}{2}^+ \to \frac{3}{2}^+$ transition, the Gamow-Teller β decay of ^{23}Mg to the ^{23}Na $\frac{5}{2}^+$ state can be combined with the present result to separate the spin and orbital contributions to the $M1$ decay. This has already been done by Bardin and Becker[3] for the assumption $|\lambda_1| \gg |\lambda_0|$, which aside from the choice of the relative phase of λ_0 and λ_1, we have now verified. Schematically, we have a separation into spin and orbital contributions to the $M1$ isovector matrix element, $\lambda_1 = \lambda_s + \lambda_l$, and the β-decay matrix element is $\lambda_\beta = \lambda_s$. We obtain λ_1 using Eq. (2), i.e.,

$$|\lambda_1| = [(\lambda_0 + \tfrac{1}{2}\lambda_1) - (\lambda_0 - \tfrac{1}{2}\lambda_1)].$$

Actually, our choice of relative phases to give $|\lambda_0| \ll |\lambda_1|$ means

$$|T_3\lambda_1| = \tfrac{1}{2}[B(\text{Mg})^{1/2} + B(\text{Na})^{1/2}]$$

which gives, using the evaluation for λ_β of Bardin and Becker,[3]

$$\frac{(\lambda_s + \lambda_l)}{\lambda_s} = \tfrac{1}{2}[3.22^{1/2} + 2.42^{1/2}] = 1.68\,.$$

In summary, the last relation shows that the spin and orbital contributions are in phase, with the orbital contribution 68% of the spin contribution, a result well within the range expected from local systematics. The result is also in reasonable accord with that expected[3] for nuclei for which $d_{5/2} \rightarrow d_{5/2}$ transitions should give the dominant contribution to the matrix element.

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