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^{86,88}Sr(d, ^{3}He)^{85,87}Rb Reactions and a Possible $Z = 38$ Magic Number*

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The ^{86,88}Sr(d,^{3}He)^{85,87}Rb reactions were studied at a bombarding energy of 28 MeV and angular distributions were obtained for all observed states. Spectroscopic factors were extracted from distorted-wave Born-approximation calculations of the cross sections. These spectroscopic factors, and those from the ^{86,88}Sr(^{3}He, d)^{87,89}Y reactions, were renormalized to satisfy sum-rule constraints. Some configuration mixing in the ground state of ^{88}Sr is inferred. The two $g_{9/2}$ neutron holes in ^{86}Sr produce very sizable rearrangements of the proton orbital populations in ^{86}Sr.

I. INTRODUCTION

Within the framework of the shell model, the presence of the magic numbers $Z=40$ and $N=50$ and 56 makes nuclei in the mass-90 region very attractive for study. Discussions of such nuclei frequently consider the doubly magic nucleus ^{90}Zr as an inert core, even though it is known[1,2] that its ground state has sizable proton configuration mixing between the $p_{1/2}$ and $g_{9/2}$ orbitals.

It is sometimes suggested[3,4] that ^{88}Sr may be a better doubly-closed-shell nucleus than ^{90}Zr. Indeed, calculations with this assumption have been reasonably successful,[5,6] as have also been those of the Sr isotopes with a proton closed shell at $Z=38$.[7] Nevertheless, there are indications that this viewpoint also is not acceptable. The proton stripping[8] and pickup[9] reactions on ^{88}Sr are inconsistent with the assumption of filled proton $p_{3/2}$ and $f_{5/2}$ orbitals, but are in excellent agreement with the more configuration-mixed wave function of the ground state calculated by Hughes.[10]

The validity of $Z=38$ as a possible magic number is investigated here by a study of the proton-pickup (d, ^{3}He) reaction on the two strontium isotopes ^{86,88}Sr. The spectroscopic factors for the observed transitions are a measure of the proton orbital populations, even though the absolute values are subject to many uncertainties in the reaction mechanism. However, the proton-stripping (^{3}He, d) reaction has also been studied on both isotopes.[8,11] The results of both the stripping and pickup reactions for each nucleus may be directly compared and renormalized to satisfy sum-rule constraints (assuming a direct, one-step reaction mechanism).

The comparison between the spectroscopic information for the ^{86}Sr and ^{88}Sr nuclei is additionally interesting in terms of the effects on the proton distribution in the $1f$-$2p$ orbitals caused by the presence of two holes in the $1g_{9/2}$ neutron orbital. A rather sizable influence has already been noted,[11] especially for the $f_{5/2}$ orbital. The information obtained here is important for establishing a firm basis for the much needed detailed shell-model calculations of mass-90 nuclei.

II. EXPERIMENTAL PROCEDURES

A. Data Acquisition

The strontium targets were prepared by evaporating $Sr(NO_3)_2$, enriched in the appropriate isotope, from a Ta boat onto a target covered by a carbon and Formvar backing. The nitrate was reduced by the Ta so that only pure Sr was evaporated onto the backing. Only trace amounts of nitrogen were observed in elastic scattering data from these targets. Some oxygen is present in the Formvar, but this presented no difficulties for the experiment. Target thickness, as determined by the elastic scattering measurements, varied between 30 and 50 μg/cm^2. The isotopic abundances in the various targets are given in Table I.

The targets were transferred, under vacuum, to a 150-cm-diam scattering chamber and bombarded with deuterons of about 28-MeV energy from the Princeton University azimuthally-varying-field (AVF) cyclotron. The emerging particles passed through a double-slit collimator in front of the detector housing and into a silicon surface-barrier charged-particle telescope. The second of the two slits defined the solid angle while a U magnet between the slits swept away from the detectors those electrons which were accepted by the larger first slit. Data were taken on several different occasions and the details of the experimental configuration varied slightly in each case. However, the detector telescope always had a 98-μm ΔE detector, a 1500- or 2000-μm E detector, and a thick veto detector. It was cooled to about −40° C. The collimator subtended an

TABLE I. The isotope abundance of the Sr isotopes for each of the enriched and the natural Sr targets.

	Abundance (%)[a]			
Target	^{84}Sr	^{86}Sr	^{87}Sr	^{88}Sr
^{86}Sr	<0.05	97.6	0.68	1.73
^{88}Sr	<0.01	0.05	0.11	99.84
Nat. Sr	0.56	9.68	7.02	82.56

[a] Isotope Sales Division, Oak Ridge National Laboratory.

angle of about 1° in the scattering plane and defined a solid angle of about 0.7 msr.

Signals from the detectors were processed by conventional electronics circuitry and transmitted to an on-line computer which computed the particle-identification function and stored the event in an appropriate array. For most of the runs, the circuit included Sherman-Roddick preamplifiers[12] and a pileup-rejection subcircuit, and resulted in an over-all resolution of about 35 keV. Further details about this circuit are given in a separate paper.[13] Most of the data from ^{88}Sr and part of the ^{86}Sr data, however, were obtained with an electronics circuit without these features. The resolution was about 50 keV. Where comparisons could be made, the data obtained with the two different circuits were in excellent agreement.

The data for each spectrum were analyzed by a set of computer programs which included the peak-stripping program AUTOFIT.[14] Representative spectra for the reactions ^{86,88}Sr(d, ^{3}He)^{85,87}Rb are shown in Fig. 1. Arrows mark the positions of states in the residual nuclei which were observed in these experiments and for which angular distributions were extracted. Except for the strong $p_{3/2}$ transitions to states near zero excitation energy (unmarked), other peaks are attributed to contaminants such as oxygen and silicon. Angular distributions were obtained between 8 and 50° in 2–5° intervals, depending on the target.

A monitor counter was generally not available for these experiments. However, the data for some angles were occasionally repeated, or were obtained for angles intermediate to those of previous runs. Good consistency of results was obtained.

B. Absolute Cross Sections

The absolute cross-section scales were obtained from a measurement of deuteron elastic scattering yields at an energy of 28 MeV and a comparison with optical-model calculations.

Deuteron optical-model parameter sets which fit the shape of the elastic scattering angular distributions should provide a normalization that is accurate to within 10%, and should be even more accurate at the forward angles where the Coulomb contributions are dominant. The appropriate optical-model parameters are discussed in Sec. III. We note here that the yield, at angles less than

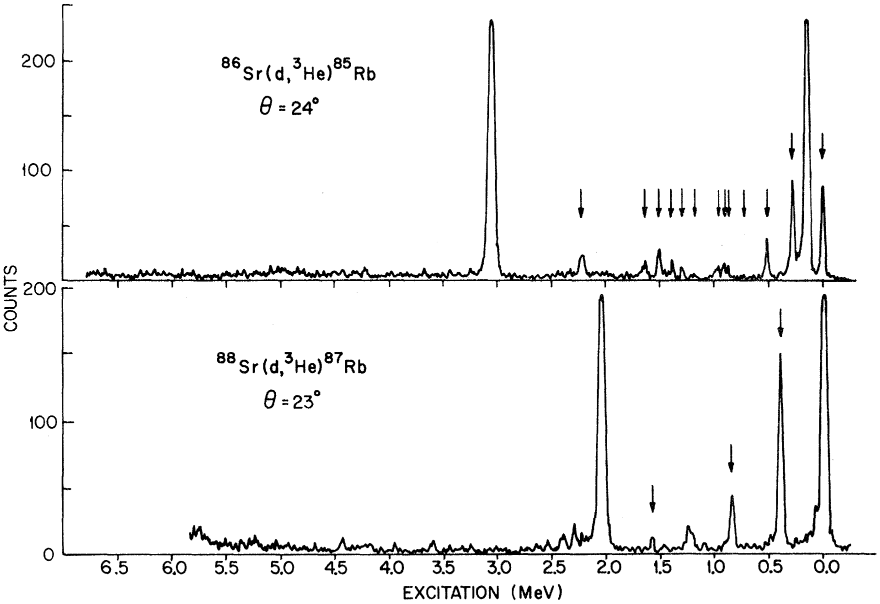

FIG. 1. Spectra for the reactions ^{86,88}Sr(d,^{3}He)^{85,87}Rb for a deuteron energy of 27.8 MeV. With the exception of strong $p_{3/2}$ transitions to states near zero excitation energy, arrows indicate the positions of states identified in the residual nuclei.

18°, is virtually insensitive to sizable changes in the optical parameters.

The elastic scattering of deuterons from ^{88}Sr was measured in 2–4° intervals between 12 and 88°. The angular distributions and the optical-model fits are shown in Fig. 2. Most of the data for the ^{88}Sr(d, ^{3}He) reaction were obtained from a separate target. These were normalized to the target used for the elastic scattering measurements at 25°.

Elastic deuteron scattering was not measured for the ^{86}Sr target. Instead, the yields from the strongest (d, ^{3}He) transitions from the ^{86}Sr and ^{88}Sr isotopes were observed at 10 and 23° for a natural Sr target. The known natural abundances of these isotopes and the previously determined cross sections from ^{88}Sr were used to determine the cross-section scale for the ^{86}Sr(d, ^{3}He) data.

The absolute cross sections are believed to be reliable to within 15% for the ^{86}Sr target, and to within 10% for the ^{88}Sr target.

III. DISTORTED-WAVE CALCULATIONS

The reaction mechanism is treated here in the distorted-wave Born approximation (DWBA) as a direct, one-step pickup of a single proton from the target nucleus. All calculations were made with the program DWUCK2.[15] The absolute cross sections are given by

$$\frac{d\sigma}{d\Omega}=NS\,\frac{\sigma_{\rm DW}}{2j+1},$$

where j is the total angular momentum transfer, S is the spectroscopic factor, and the normalization constant N is taken to be 2.95.[16]

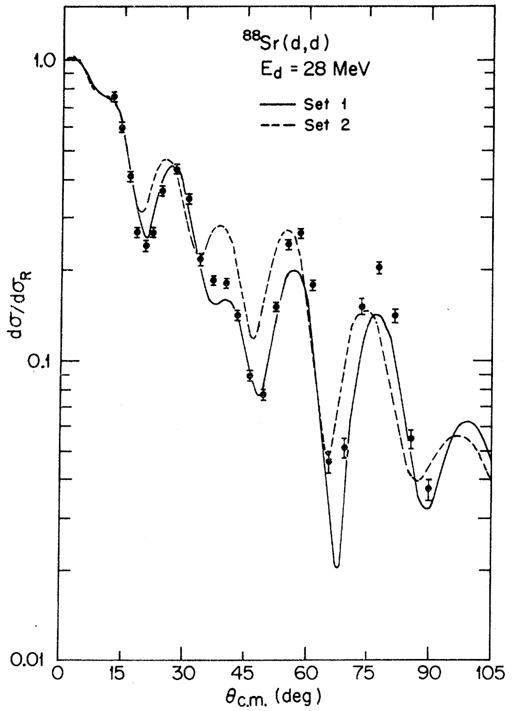

FIG. 2. The angular distribution for deuteron elastic scattering from ^{88}Sr, in ratio to the Rutherford cross section, at an energy of 27.8 MeV. The curves are the results for optical-model calculations with two separate parameter sets. The set corresponding to the solid curve was used for the (d, ^{3}He) calculations.

The optical potentials consisted of the usual complex Woods-Saxon potentials with a surface-derivative absorption term and spin-orbit term for the deuteron channel. The effects of various parameter sets were investigated. These sets included, among others, those used in the analysis of the ^{86}Sr(^{3}He, d)^{87}Y reaction in Ref. 11. The parameter set $D1$ of Ref. 11 did not give a satisfactory fit to the measured deuteron elastic scattering from ^{88}Sr at 28 MeV, as shown by the dashed line in Fig. 2. However, the parameter set determined for the 30-MeV elastic scattering from ^{89}Y in Ref. 9 gave a very good fit and was used, without modification, in the final calculations. The ^{3}He parameter set is the same as that used in Ref. 11, except for a slight decrease in the volume imaginary potential. The final parameter sets are given in Table II.

The parameters for the bound-state wave function are more difficult to determine. Usually, the geometrical parameters can be varied considerably without changing the shapes of angular distributions, but producing very large changes in the cross sections. It was found that values no less than $r_0 = 1.25$ and $a = 0.70$ fm were required to keep the spectroscopic factors from greatly exceeding sum rules. The potential strength was varied in order to fit the separation energies of the residual states, corresponding to the (γ, p) Q values from the target nuclei. Since these strengths were typically about 60 MeV, the spin-orbit parameter was taken to be $\lambda = 18$ so as to correspond to a spin-orbit strength of about 6 MeV.

The usual nonlocality corrections were included for the scattering channels, but not for the form factors. A finite-range parameter $R = 0.77$ fm (DWUCK2's convention) was also used in the final calculations. These factors[16] assisted in reducing the spectroscopic factors to reasonable values.

In view of the importance attached to the spectroscopic factors in this study, an investigation was made of the sensitivity of relative values to optical parameters. It was found that, whereas the potential sets in Table II provided excellent fits to the data and also gave good absolute spectroscopic factors, relative values for states with different l or Q values were, in addition, quite

TABLE II. The optical-model parameter sets used in the DWBA calculations. The potentials have units MeV and the geometrical parameters have units fm. All magnitudes are given as used in the program DWUCK2 (see Ref. 15).

Particle	V	r_0	a	W	W'	r'	a'	V_{so}	r_c	β[a]
d	100.	1.10	0.80	0	49.9	1.27	0.844	14.4	1.30	0.54
^{3}He	170.	1.14	0.75	18	0	1.60	0.80	0	1.40	0.25
p	~60.[b]	1.25	0.70					$\lambda = 18$	1.25	0

[a] Nonlocality parameter.

[b] The potential strength was adjusted to reproduce the separation energy for the residual state.

insensitive to changes in the parameters. This was true even when the absolute magnitudes and shapes of the angular distributions were in poor agreement with the data. The principal uncertainty in relative values comes from the weak population of many states.

IV. STATES OF THE RUBIDIUM ISOTOPES

The experimental angular distributions and DWBA curves are shown in Figs. 3 and 4. For the sum-rule analysis, it is important that the correct l and j values transferred in the (d, ^{3}He) reactions be identified. The identification of the l values in ^{85,87}Rb was usually unambiguous. The j values were more uncertain, particularly since the small cross sections prevented detection of possible j-dependent effects, such as were noticed at higher energies.[9]

The single-hole $2p_{3/2}$ and $1f_{5/2}$ states of the $Z = 38$ core were readily identified in ^{85,87}Rb. However, no clear evidence was found for any large fragments of the $1f_{7/2}$ pickup strength even though spectra were obtained up to 9 MeV excitation. Estimates of the single-particle orbitals place the $f_{7/2}$ centroid near 5 MeV. Large contaminant groups are present above 6 MeV, but their kinematic behavior does not cause them to obscure totally the main regions of interest. Distorted-wave calculations for a pure $f_{7/2}$ state at 5 MeV excitation indicate that the maximum cross sections near 20–25° would be about 200 μb/sr. The suppression from the maximum $f_{5/2}$ cross sections of about 500 μb/sr is due largely to Q dependence. States with maximum cross sections less than about 40 μb/sr probably could not be discovered in these experiments. It is very probable that the $f_{7/2}$ strength is fragmented over many states.

A. Levels in ^{87}Rb

Angular distributions have been obtained previously for the ^{88}Sr(d, ^{3}He)^{87}Rb reaction at 21 MeV for three states at 0.00, 0.403, and 0.846 MeV.[17] A fourth state, at 1.58 MeV, was also observed[9] in the reaction initiated with 35-MeV deuterons. All four states were also observed in the present experiment.

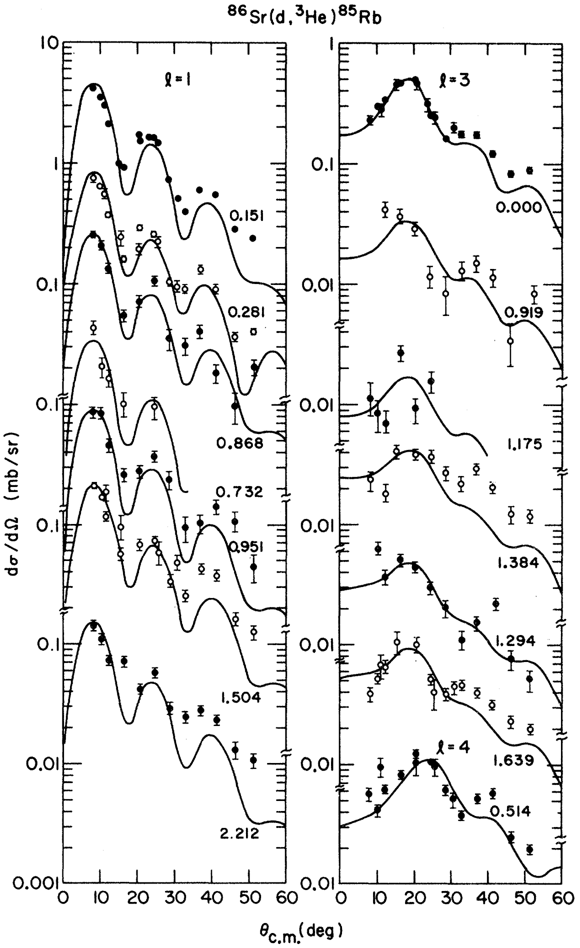

FIG. 3. Measured angular distributions for the reaction ^{86}Sr(d,^{3}He)^{85}Rb. They are grouped according to the l values indicated above them. The numbers associated with each angular distribution give the excitation energies in ^{85}Rb in MeV. The curves were obtained from DWBA calculations.

Other levels in ^{87}Rb have been identified[18] from the decay of ^{87}Kr and inelastic α-particle scattering.[19] The 0.00- and 0.403-MeV states are known[20] to have J^{π} assignments of $\frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively. Assignments of $\frac{1}{2}^-$, $\frac{9}{2}^+$, and $\frac{7}{2}^-$, respectively, are preferred[18] for states at 0.846, 1.349, and 1.578 MeV. In the $(d, {}^3\mathrm{He})$ experiment of Ref. 9, the 1.58-MeV state was observed to have an $l = 1$ angular distribution, which is inconsistent with a $\frac{7}{2}^-$ assignment. The state was very weakly populated in our data and the crucial forward angles were obscured by the ^{16}O contamination. The discrepancy cannot be resolved in the present data.

A careful search was made for the 1.349-MeV state since its possible $\frac{9}{2}^+$ assignment makes it especially attractive for evaluating the proton distribution in ^{88}Sr and since its probable counterpart in ^{85}Rb was clearly seen (see Sec. IV B). No evidence of its presence was found. The upper limit on its maximum cross section is 20 μb/sr, in contrast with the observed 100-μb/sr cross section observed for the 0.514-MeV state in ^{85}Rb. Also, no states above 1.6 MeV were identified in ^{87}Rb, except possibly for a level near 2.40 MeV (maximum cross section ≤ 40 μb/sr). This possibly has a $\frac{7}{2}^+$ assignment[18] and is of less direct interest for the present study.

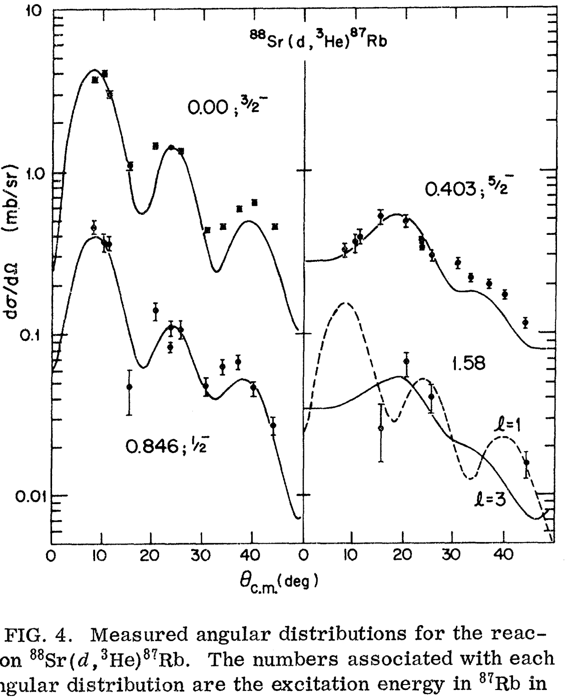

FIG. 4. Measured angular distributions for the reaction ^{88}Sr$(d, {}^3\mathrm{He})^{87}$Rb. The numbers associated with each angular distribution are the excitation energy in ^{87}Rb in MeV and the known J^{π}. The curves were obtained from DWBA calculations.

B. Levels in ^{85}Rb

Early Coulomb-excitation experiments had established the presence of states in ^{85}Rb at 0.00, 0.151, 0.514, and 0.870 MeV with assignments of $\frac{5}{2}^-$, $\frac{3}{2}^-$, $(\frac{9}{2})^+$ and $(\frac{5}{2}, \frac{7}{2}, \frac{9}{2})^-$, respectively.[21] Additional levels have been inferred from studies of the $(n, n'\gamma)$ reaction[22] and the decay[23] of ^{85}Sr and ^{85}Kr.

The decay studies[23] revealed possible new states at 0.281, 0.732, 0.878, and 0.919 MeV. The 0.281-MeV state is confirmed by the $(d, {}^3\mathrm{He})$ reaction. The levels at 0.732 and 0.919 also appear to be weakly populated, with cross sections about 10–40 μb/sr. Both levels are needed to account for some of the γ rays from the $(n, n'\gamma)$ reaction.[22] The 0.878-MeV level is more doubtful. No evidence for it is available from the $(n, n'\gamma)$ reaction.[22] The $(d, {}^3\mathrm{He})$ data require a level near 0.868 [22] or 0.878 [23] MeV, but a distinction is not possible. The observed $l = 1$ angular distribution in the $(d, {}^3\mathrm{He})$ reaction to a state at this energy is not consistent with the suggested $\frac{5}{2}^-$ or $\frac{7}{2}^-$ assignment[23] for the 0.868-MeV state.

The $(d, {}^3\mathrm{He})$ reaction populates seven additional states above 0.92 MeV. Five of them, at 0.951, 1.175, 1.294, 1.384, and 1.639 MeV, agree very well (within their small errors) with levels inferred from the $(n, n'\gamma)$ data.[22] Two other levels, near 1.504 and 2.212 MeV, were identified, but do not account for additional γ rays. It is important to point out that the Q value for the 0.951-MeV state, $Q = -5.096$ MeV, is within 15 keV of the Q value, $Q = -5.110$ MeV, for the ground-state transition from ^{88}Sr as determined from the 1971 Mass Tables.[24] This isotope had less than a 2% abundance in the target, but could account for about half of the observed yield.

V. SUM-RULE ANALYSIS

A. Orbit Assignments

Table III summarizes the spectroscopic information of the present data for the rubidium isotopes. A discussion of the known J^{π} assignments was given in Sec. IV. The sum-rule analysis requires assignments for all of the states observed in the proton pickup and stripping reactions.

In the ^{88}Sr$(d, {}^3\mathrm{He})^{87}$Rb reaction, only the weak state at 1.58 MeV has a disputed assignment.[9, 18] We assume here that the $\frac{7}{2}^-$ assignment from β-decay measurements[18] is correct. A choice of $\frac{3}{2}^-$ or $\frac{1}{2}^-$ will not noticeably affect the final results.

For the ^{86}Sr$(d, {}^3\mathrm{He})^{85}$Rb reaction, the known or probable assignments for states below 0.6 MeV are consistent with the present data and are assumed to be correct. Above this energy, many levels with $l=1$ and $l=3$ angular distributions are observed. There is no reason to ascribe them all to $p_{3/2}$ or $f_{5/2}$ transfers. The analysis is therefore made for three different assumptions concerning the distribution of strength for states above 0.6 MeV: (a) All transfers correspond to $p_{3/2}$ and $f_{5/2}$ orbits; (b) all transfers correspond to $p_{1/2}$ and $f_{7/2}$ orbits; and (c) the strengths are divided among all four orbits with favor given to the $p_{3/2}$ and $f_{5/2}$ orbits.

The proton-stripping strengths for the ^{88}Sr target are taken directly from Ref. 8. Those for the ^{86}Sr target can be obtained from the ^{86}Sr(^{3}He, d)^{87}Y data of Ref. 11. In that work, the $l=3$ strengths were all assumed to be $f_{5/2}$ transfers, but the $l=1$ strengths could not be uniquely assigned. We have investigated the ^{89}Y(p, t)^{87}Y reaction in a companion experiment[25] in order to extract the j assignments. The $p_{1/2}$ and $p_{3/2}$ states of ^{87}Y should be populated by $L=0$ and 2 transfers in this reaction, respectively. From these data, the states at 0.000 and 1.848 are assigned $\frac{1}{2}^-$ and those at 0.982 and 2.085 are assigned $\frac{3}{2}^-$.

TABLE III. Spectroscopic results of the DWBA analysis for the ^{86,88}Sr(d, ^{3}He)^{85,87}Rb reactions. For each state of excitation energy E_x (MeV), the l values are determined from the present data except where presumed J^π values are given. The spectroscopic factors S were determined from the distorted-wave calculations by using the parameters of Table II. They have not been renormalized.

			Choice 1		Choice 2	
E_x	J^π	l	j	S	j	S
			^{85}Rb			
0.000	$\frac{5}{2}^-$	3	$\frac{5}{2}$	3.1		
0.151	$\frac{3}{2}^-$	1	$\frac{3}{2}$	2.3		
0.281	$(\frac{1}{2})^-$	1	$\frac{1}{2}$	0.51		
0.514	$(\frac{9}{2})^+$	4	$\frac{9}{2}$	0.91		
0.732		1	$\frac{3}{2}$	0.023	$\frac{1}{2}$	0.027
0.868		1	$\frac{3}{2}$	0.088	$\frac{1}{2}$	0.10
0.919		3	$\frac{5}{2}$	0.28	$\frac{7}{2}$	0.19
0.951		1	$\frac{3}{2}$	0.062	$\frac{1}{2}$	0.075
1.175		(3)	($\frac{5}{2}$	0.14	$\frac{7}{2}$	0.095)
1.294		3	$\frac{5}{2}$	0.71	$\frac{7}{2}$	0.46
1.384		3	$\frac{5}{2}$	0.45	$\frac{7}{2}$	0.30
1.504		1	$\frac{3}{2}$	0.19	$\frac{1}{2}$	0.22
1.639		3	$\frac{5}{2}$	0.98	$\frac{7}{2}$	0.68
2.212		1	$\frac{3}{2}$	0.14	$\frac{1}{2}$	0.16
			^{87}Rb			
0.000	$\frac{3}{2}^-$	1	$\frac{3}{2}$	3.1		
0.403	$\frac{5}{2}^-$	3	$\frac{5}{2}$	5.3		
0.846	$(\frac{1}{2})^-$	1	$\frac{1}{2}$	0.48		
1.578	$(\frac{7}{2})^-$	3	$\frac{7}{2}$	$\leq$0.54		
		(1)	($\frac{3}{2}$	$\leq$0.25	$\frac{1}{2}$	$\leq$0.30)

In order to aid in the comparison with ^{90}Zr, the spectroscopic strengths for proton-transfer reactions on ^{90}Zr have been obtained from the literature. A study of the (d, ^{3}He) reaction by Preedom *et al.*,[26] at 34.4 MeV revealed a population of only the four lowest states of ^{89}Y, whose spins are known. Strengths for the (^{3}He, d) reaction may be taken from Ref. 8. Most of the ambiguity in the j values for the $l=1$ transitions may be removed by a comparison with the results of the ^{92}Mo(d, ^{3}He)^{91}Nb reaction.[27] The states near 1.31 and 1.62 are populated quite strongly in the pickup reaction and may thus be assigned $J^\pi=\frac{3}{2}^-$. This same assignment is also given to the states near 1.97 and 2.36 MeV, which are only weakly populated by the (^{3}He, d) reaction.

Table IV lists the spectroscopic sums $\sum(2J_f+1)C^2S$ for the stripping[8, 11] reactions and $\sum C^2S'$ for the pickup reactions on ^{86,88}Sr and ^{90}Zr.

B. Computations

The orbits of interest are the $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$ orbits which can hold a total of 22 protons. The proton-transfer spectroscopic factors satisfy sum rules which are[28]:

$$\sum\frac{2J_f+1}{2J_i+1}(C^2S)_< + \sum\frac{2J_f+1}{2J_i+1}(C^2S)_> = \text{proton holes in target}\,, \tag{1}$$

$$\sum(C^2S')_> = \text{proton particles in target} \tag{2}$$

for stripping and pickup reactions, respectively. The notation $(C^2S)_>$ and $(C^2S)_<$ refer to final states with isospin $T_f=T_i\pm\frac{1}{2}$. For each shell-model orbital j, the right-hand sides of Eqs. (1) and (2) must sum to the value $2j+1$. The (^{3}He, d) experiments only measured the first term on the left side in Eq. (1). The second term is related to neutron-stripping strengths for the orbital j. Assuming the conventional shell model for neutrons, this term is zero except only for $g_{9/2}$ stripping on ^{86}Sr, for which the value is $\frac{2}{11}$.

Since $J_i=0$, we can set $x_j=\sum(2J_f+1)(C^2S)_<$, $y_j=\sum(C^2S')_>$, and $n_j=(2j+1)-\sum(2J_f+1)(C^2S)_>$. The experimental quantities x_j and y_j may need to be renormalized by factors α and β, respectively. Thus we can form the equation

$$\alpha x_j+\beta y_j=n_j \tag{3}$$

for each of the four orbits under consideration. It is most convenient to obtain the "best" values of α and β by applying a least-squares method to the four equations. This method need not preserve the sum $\sum n_j \simeq 22$, but the discrepancy, in practice, is small. We thus obtain the simultaneous equations

$$\alpha \sum x_j^2 + \beta \sum x_j y_j = \sum n_j x_j \quad \text{and} \quad \alpha \sum x_j y_j + \beta \sum y_j^2 = \sum n_j y_j . \tag{4}$$

These are readily solved.

The results of the least-squares analysis are presented in Table V. The renormalization coefficients α and β are applied to the experimental sums of Table IV and can be used to renormalize the original data for the stripping reactions on Refs. 8 and 11, and the pickup reactions of the present work (see Table III). The number of transferred particles in each reaction is summed for each orbital for comparison with the sum-rule limits. The fullness probabilities V_j^2 for the target nuclei were computed by dividing the number for the $(d, {}^3\text{He})$ reaction by the actual sum rather than by the sum limit.

VI. DISCUSSION

A. Spectroscopic Strengths

It is encouraging that the renormalization factors α and β in Table V are not much different from unity, especially in case C for the ^{86}Sr target where the assigned $l=1$ and 3 strengths were divided among several j orbitals. This supports the assumption that the one-step DWBA calculations may be a fairly accurate description of the reaction mechanism for these reactions and that the parameters were chosen well.

TABLE IV. The experimental sums $\sum(2J_f+1)C^2S$ and $\sum C^2S'$ and $({}^3\text{He}, d)$ and $(d, {}^3\text{He})$ reactions, respectively, for ^{86,88}Sr. Three options are given for the pickup reaction on ^{86}Sr.

Orbit	Stripping	Pickup		
^{86}Sr target		(A)	(B)	(C)
$f_{5/2}$	1.4	5.7	3.1	4.7
$p_{3/2}$	0.63	2.8	2.3	2.6
$p_{1/2}$	1.2	0.51	1.1	0.71
$g_{9/2}$	8.5	0.91	0.91	0.91
^{88}Sr target				
$f_{5/2}$	0.55	5.3		
$p_{3/2}$	0.51	3.1		
$p_{1/2}$	1.8	0.48		
$g_{9/2}$	8.8	0.0		
^{90}Zr target				
$f_{5/2}$	0	7.80		
$p_{3/2}$	0.44	4.25		
$p_{1/2}$	0.60	1.91		
$g_{9/2}$	9.0	1.10		

In general, the experimental sums of the spectroscopic strengths are in good agreement with the sum-rule limits. There is some fluctuation due to the weak population of some states and possible irregularities in the analysis.

Nevertheless, there are some puzzles in the results of Table V that need to be discussed. The sum for the $p_{1/2}$ orbit for the ^{88}Sr target is 2.5 and exceeds the sum-rule limit of 2. This apparently does not occur for the ^{86}Sr target except

TABLE V. The number of protons of each orbit j transferred in $({}^3\text{He}, d)$ and $(d, {}^3\text{He})$ reactions on ^{86,88}Sr, as renormalized by the sum-rule constraints. The renormalization factors α and β for the $({}^3\text{He}, d)$ and $(d, {}^3\text{He})$ reactions, respectively, are given for each case. Also listed are the fullness probabilities $V_j{}^2$ for each orbit in the target nucleus.

	Transferred particles				
Orbit	$({}^3\text{He}, d)$	$(d, {}^3\text{He})$	Sum	Limit	$V_j{}^2$
	^{86}Sr (case A) $\alpha=1.06$, $\beta=0.87$				
$f_{5/2}$	1.5	5.0	6.5	6	0.77
$p_{3/2}$	0.67	2.4	3.1	4	0.78
$p_{1/2}$	1.3	0.44	1.7	2	0.26
$g_{9/2}$	9.1	0.79	9.9	9.8	0.08
Sum	12.5	8.6	21.2		
	^{86}Sr (case B) $\alpha=1.00$, $\beta=1.43$				
$f_{5/2}$	1.4	4.4	5.8	6	0.76
$p_{3/2}$	0.63	3.3	3.9	4	0.85
$p_{1/2}$	1.2	1.6	2.8	2	0.57
$g_{9/2}$	8.5	1.3	9.8	9.8	0.13
Sum	11.7	10.6	22.3		
	^{86}Sr (case C) $\alpha=1.04$, $\beta=1.0$				
$f_{5/2}$	1.5	4.7	6.2	6	0.76
$p_{3/2}$	0.65	2.6	3.2	4	0.81
$p_{1/2}$	1.2	0.71	1.9	2	0.37
$g_{9/2}$	8.8	0.91	9.7	9.8	0.09
Sum	12.1	8.9	21.0		
	^{88}Sr $\alpha=1.12$, $\beta=1.03$				
$f_{5/2}$	0.62	5.5	6.1	6	0.90
$p_{3/2}$	0.57	3.2	3.8	4	0.84
$p_{1/2}$	2.0	0.50	2.5	2	0.20
$g_{9/2}$	9.9	0	9.9	10	0
Sum	13.1	9.2	22.3		
	^{90}Zr $\alpha=1.02$, $\beta=0.78$				
$f_{5/2}$	0	6.1	6.1	6	1.0
$p_{3/2}$	0.45	3.3	3.8	4	0.87
$p_{1/2}$	0.61	1.5	2.1	2	0.71
$g_{9/2}$	9.2	0.86	10.1	10	0.09
Sum	10.3	11.8	22.1		

for the extreme case B where all the $l=1$ strength to states of unknown J^π in ^{85}Rb was attributed to $p_{1/2}$ pickup. We also note that the ratio of the sums for the $p_{3/2}$ and $p_{1/2}$ orbits is less than the expected value of 2 in every case, including ^{90}Zr and the extreme case A for ^{86}Sr in which $p_{3/2}$ transfer is strongly favored. Typically the ratio is about 1.7.

An explanation for this problem is not immediately forthcoming. Quite possibly some $p_{3/2}$ strength is fragmented into very weakly populated states that were not observed. Multistep processes in the reaction mechanism could also disturb the assignment of spectroscopic strength. For example, inelastic excitation of a collective 2^+ state in the target nucleus would result in $p_{1/2}$ transfers to $J^\pi = \frac{3}{2}^-$ states, and vice versa. The $B(E2)$ to the lowest 2^+ state in ^{88}Sr is about 10 single-particle units.[29]

B. Stronium Ground States

In considering the fullness probabilities V_j^2 in Table V for ^{86,88}Sr it is clear that one cannot fairly consider the $Z=38$ number to be magic. In both nuclei there appears to be some substantial excitation of protons from the $p_{3/2}$ and $f_{5/2}$ orbitals to the higher $p_{1/2}$ and/or $g_{9/2}$ orbitals. One should note that if $V_j^2 = 0.20$ for the $p_{1/2}$ orbitals in ^{88}Sr, $|V_j| = 0.45$ and amplitudes of this size can have very significant effects on nuclear structure and reaction processes.

In the case of ^{88}Sr, the configuration-mixed theoretical wave function for the ground state calculated by Hughes[10] is in very good agreement with the present data. The Hughes wave function produces V_j^2 of 0.92, 0.79, and 0.30 for the $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ orbitals, respectively.

A more dramatic effect is seen in ^{86}Sr. In comparison with ^{88}Sr, the two $g_{9/2}$ *neutron* holes in ^{86}Sr produces a substantial redistribution of *proton* orbital populations, even across a major shell. Particles appear to be removed from the $f_{5/2}$ orbital and scattered among the $p_{1/2}$ and $g_{9/2}$ orbitals. This effect has been previously noted[11] and is substantiated here in more detail. It is understood that two $g_{9/2}$ neutron holes will result in more complicated spectra for ^{86}Sr or ^{85}Rb. The data suggest, however, that the additional states are mixed by strong residual interactions with the ground state. In contrast with nuclei near mass 90,[5] seniority may not be a reasonably good quantum number near the strontium isotopes.

The analysis of the ^{90}Zr ground state in this paper reveals that configuration mixing with the $p_{3/2}$ orbitals should also be taken in serious consideration. The mixing between the $p_{1/2}$ and $g_{9/2}$ orbitals that is normally assigned[1, 2] leads to V_j^2 values of 1.0, 1.0, 0.64, and 0.07 for the $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$ orbitals, respectively. The data indicate that some of the $p_{3/2}$ particles have been redistributed among the $p_{1/2}$ and $g_{9/2}$ orbitals.

Although ^{88}Sr does not appear to be as configuration mixed in its ground state as ^{90}Zr, it also does not appear to be very good doubly magic nucleus. In any case, the possible closed-shell properties of $Z=38$ are not preserved throughout the strontium isotopes. It may be necessary for the shell-model calculations of nuclei in this mass region to consider the $f_{5/2}$ and $p_{3/2}$ protons as active along with $p_{1/2}$ and $g_{9/2}$ protons. The coupling of active neutrons with "closed-shell" protons is a phenomenon that certainly needs further study.

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