Mesonic Second-Class Currents in Nuclear β Decay*

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Mesonic second-class currents have been studied in semileptonic $\Delta S=0$ transitions. The closed-loop diagram is finite for the $\pi\omega$ current, and its energy-dependent lepton part is dominant over the contributions from the two-body exchange diagram. The two-body exchange diagram has energy-dependent and energy-independent lepton parts, which are of comparable magnitude for the $A=12$ triplet. A general form of the mesonic current has also been considered. A general theory of second-class effects, including both mesonic currents and the induced-tensor term for ft values and K/β^+ ratios, has been discussed.

I. INTRODUCTION

After series of papers had connected the asymmetry observed in mirror β decays with the induced-tensor interaction,[1] thus initiating renewed interest in theoretical investigations,[2-17] some evidence to the contrary started to emerge.[18,19] It was pointed out that the observed asymmetries are explainable as binding-energy effects arising from electromagnetic sources.[20-24] The phenomenologically introduced induced-tensor interaction[2-4] predicts for the K/β^+ ratio a dependence on the charge Z and the atomic number A which is completely contradictory to experiments.[18,25] Furthermore, the induced tensor leads to an effect proportional to the maximal lepton energy of the compared mirror transition. An important experiment performed for the $A=8$ triplet[19] did not show any energy dependence of the mirror asymmetry $\delta=(ft)^+/(ft)^- -1$. The result obtained by Wilkinson and Alburger[19] prompted Lipkin[12] to argue that mesonic second-class currents (SCC) would lead to both $\delta\neq 0$ and SCC effects which are energy-independent. Lipkin considered a diagram where the mesonic line is exchanged between two nucleons (see Fig. 2). In Sec. III we point out that even this diagram leads to an energy-dependent effect, which was also discussed in Ref. 17. In Sec. IV we show that for the case of the $A=12$ triplet, energy-dependent and energy-independent contributions are comparable.

Besides exchange diagrams there are diagrams where a mesonic loop closes on the same nucleon line[26,27] (see Fig. 1). Generally, such a diagram is highly divergent and should be cutoff-dependent because of nonrenormalizability. For a particular case of $\pi\omega$ mesonic SCC, divergent parts cancel out due to internal symmetry properties. Final results are dominated by an energy-dependent lepton contribution, which is much larger than the one coming from the exchange diagram. The numerical results given in Sec. IV do not support the suggestion that energy-dependent contributions coming from various diagrams might cancel out. However, a general mixture of mesonic SCC alone, or with the phenomenologically induced tensor included, might be sufficiently flexible to accommodate both the K/β^+ ratio[18] and the result for the $A=8$ triplet.[19]

A detailed discussion presented in Sec. V leads to the conclusion that such a theory is reither a very attractive nor convincing one. Further progress in the investigation of SCC will probably follow through the theoretical study of electromagnetic nuclear-structure effects[21-24] and through the experimental investigation of various possible β-decay correlations.[10,11,15,28]

II. CLOSED-LOOP DIAGRAM

The closed-loop diagram shown in Fig. 1 has already been investigated for strangeness-changing semileptonic decays.[26] In this case the contribution turned out to be divergent. In nuclear β decay the $\pi\omega$ current, originally suggested by Lipkin,[12] leads to a result which is cutoff-independent. In the calculations of the decay amplitudes we use the Lagrangians

$$L_W=iG_\omega(\omega_\mu\pi^-\bar{j}_\mu-\omega_\mu\pi^+j_\mu)\,, \tag{1}$$

$$j_\mu=iL_\mu=i\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu\,, \tag{2}$$

$$L_\omega=if_\omega\bar{\psi}\gamma_\mu\psi\omega_\mu\,, \tag{3}$$

$$L_\pi=ig\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\pi}\,. \tag{4}$$

Here G_ω characterizes the strength of mesonic SCC, while f_ω and g are determinable from other experiments. All other notation has the usual meaning. The two diagrams in Fig. 1 have been calculated, leading in the $l^2\to 0$ limit to the result

$$d_L=d_L^1+d_L^2\,, \tag{5}$$

$$d_L^1=-i(1\sqrt{2})G_\omega g f_\omega\pi^2 \times[4I_I^S-2(M/m_\omega)^2I^P+(1/m_\omega{}^2)(I^\lambda+\tfrac{1}{3})] \times\bar{u}i(p_\mu+p'_\mu)\gamma_5\tau^\mp u\bar{L}_\mu\,, \tag{6}$$

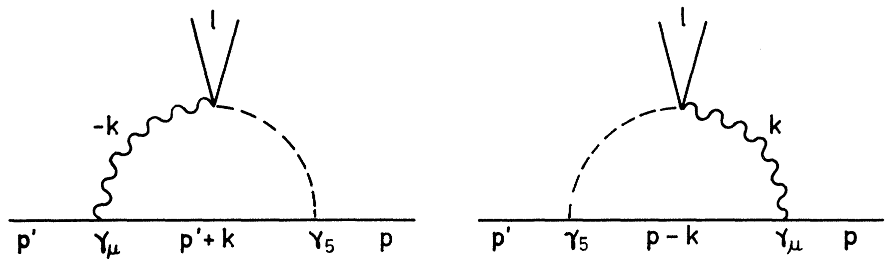

FIG. 1. The full lines are fermions, the wavy line is the vector meson, and the dashed line is the pion. $\vec{p}$ and $\vec{p}'$ are the nucleon moments, and the closed-loop integration goes over $\vec{k}$. $\vec{l}=\vec{p}-\vec{p}'$ is the total momentum carried by leptons.

$$d_L^2 = -i\sqrt{2}G_\omega g f_\omega \pi^2 (M_n - M_p) I_{II}^S \bar{u}\gamma_\mu\gamma_5\tau^\mp u \bar{L}_\mu . \tag{7}$$

In formulas (6) and (7), M is an average nucleon mass, while M_p and M_n are the proton and neutron mass, respectively. The integrals appearing above are given by

$$I_I^S = \int_0^1 d\alpha_1 \alpha_1 \int_0^1 d\alpha_2 \frac{\alpha_1\alpha_2}{D(\alpha_1, \alpha_2)}, \tag{8}$$

$$I_{II}^S = \int_0^1 d\alpha_1 \alpha_1 \int_0^1 d\alpha_2 \frac{1+\alpha_1 - 2\alpha_1\alpha_2}{D(\alpha_1, \alpha_2)}, \tag{9}$$

$$I^P = \int_0^1 d\alpha_1 \alpha_1 \int_0^1 d\alpha_2 \frac{(1+\alpha_1)(1-\alpha_1)^2}{D(\alpha_1, \alpha_2)}, \tag{10}$$

$$I^\lambda = \int_0^1 d\alpha_1 \alpha_1 \int_0^1 d\alpha_2 (6\alpha_1 - 4) \ln \frac{\lambda^2}{D(\alpha_1, \alpha_2)}, \tag{11}$$

where

$$D(\alpha_1, \alpha_2) = m_\omega^2 \alpha_1\alpha_2 + m_\pi^2(\alpha_1 - \alpha_1\alpha_2) + M^2(1-\alpha_1)^2 . \tag{12}$$

The integral I^λ is actually independent of the cutoff λ, thus making our result finite. Other possible mesonic currents, similar to those listed in Ref. 27, do not always lead to finite contributions. The contribution to the axial-vector part [i.e., to d_L^2 in Eq. (7)] coming from the $k_\mu k_\nu$ part in the vector-meson propagator is of the form

$$d_\lambda^2(V) = i(1/m_V^2)\tfrac{1}{2}\pi^2 C_V (M_n - M_p) \ln(\lambda^2/M_{\text{eff}}^2) \bar{u}\gamma_\mu\gamma_5\tau^\mp u L_\mu . \tag{13}$$

Here the index V refers to the particular vector meson, while C_V is given in Table I.

When the cutoff is fixed, from Pietschmann and Rupertsberger's results[27] one finds

$$|d_\lambda^2| \sim |d_L^2| \quad \text{if } G_V \approx G_\omega . \tag{14}$$

It can be shown that the ρ-meson anomalous magnetic moment gives a cutoff-independent contribution to formula (13). The induced-tensor term $d_\lambda^1(V)$ is also cutoff-independent for β decay. Its magnitude is comparable with d_L^1 in Eq. (6). It seems very likely that divergent terms are comparable with $\pi\omega$ terms. However, one may question the comparison between nucleon β decay and $\Sigma\Lambda\beta$ decay, since the β-decay result is proportional to the small nucleonic-mass difference, while in the case of $\Sigma\Lambda\beta$-decay baryonic mass differences appearing as a common multiplicator are much larger. The alternative choice would be to leave the cutoff as an additional parameter, thus making it impossible to compare the closed-loop diagram with the contributions of the exchange diagram. Therefore, we have decided to make an analysis only for the $\pi\omega$ current, thus staying with Lipkin's suggestion.[12]

The effective contribution of $\pi\omega$ SCC to the nuclear β-decay interaction is as follows:

$$D_L^1 = \mp(1/2MG)G_\omega g f_\omega 0.453\times 10^{-2}\,\text{fm} \times \bar{u} i(p_\mu + p'_\mu)\gamma_5\tau^\mp u \bar{L}_\mu , \tag{15}$$

$$D_L^2 = \pm(1/2MG)G_\omega g f_\omega 0.818\times 10^{-4} \bar{u}\gamma_\mu\gamma_5\tau^\mp u \bar{L}_\mu . \tag{16}$$

The system of units $\hbar = 1$, $c = 1$, and fm $= 10^{-13}$ cm was used.

We should keep in mind that the nuclear-structure independence of the above results follows from the nature of our approximation. A more exact approach would be to work in the bound-state-interaction representation[29] with the shell-model potential, for example, in the role of the external field. As the integration over $\vec{k}$ (see Fig. 1) is dominated by the "high-energy" part of the propagator, our approximation, in which we use plane-

TABLE I. Values of C_V expressed through the coupling constants defined in Ref. 26.

Mesonic current constituents	C_V
$\pi^\pm\phi^0$	0
$\eta\rho^\pm$	$G_a f_V(f_p - \frac{1}{3}d_p)$
$K^\pm K^{*0} + K^0 K^{*\pm}$	$G_a f_V(f_p + d_p)$
$\eta'\rho^\pm$	$G_{\eta'}\sqrt{2}\, g f_V$

wave nucleon propagators, is most probably satisfactory.

III. EXCHANGE DIAGRAM

It has been argued by Lipkin[12] that the contribution coming from the exchange diagram shown in Fig. 2, should have negligible energy dependence, thus avoiding the contradiction to measurements of the β decay of ^{8}Li and ^{8}B.[19] However, there is a sizable contribution coming from the fourth component of the vector-meson vertex (x_1 in Fig. 2). The product of mesonic vertices for this term behaves as a pseudoscalar and, therefore, in the case of the Gamow-Teller transition the necessary vector ingredient can come only from the multipole expansion of the lepton covariant:

$$L_4(r)=\cdots - i\,\vec{r}\cdot\vec{l}\,L_4(0)+\cdots\,. \tag{17}$$

This makes the result proportional to the total lepton momenta $\vec{l}$ and, therefore, energy-dependent. Contributions from the spatial components of the vector-meson vertex are energy-independent.

The following expression corresponds to the diagram in Fig. 2:

$$D_E=\kappa i\int d^4x_1 d^4x_2 d^4x_3\, i\bar{\psi}(x_1)\gamma_\mu\psi(x_1) \times iD_{\mu\nu}(x_1-x_3)\,j_\nu(x_3)\,i\Delta(x_3-x_2)\,i\bar{\psi}(x_2)\gamma_5\tau_2^{\pm}\psi(x_2)\,, \tag{18}$$

$$\kappa=2gf_\omega G_\omega/G\,. \tag{19}$$

Here $D_{\mu\nu}$ is the ω-meson propagator and Δ is the pionic propagator. The functions $\psi(x_i)$ appearing in (18) are bound-state nucleon wave functions.[29]

We first calculate the part corresponding to the exchanged line in Fig. 2:

$$\begin{aligned}P(x_1;x_2)&=\int d^4x_3 D_{\mu\nu}(x_1-x_3)e^{-ilx_3}L_\nu(0)\Delta(x_3-x_1)\\ &=(2\pi)^{-4}e^{-ilx_1}L_\nu(0)\\ &\quad\times\int d^4k\,e^{ik(x_1-x_2)}D_{\mu\nu}(k-l)\Delta(k)\,.\end{aligned} \tag{20}$$

The integrations in Eq. (19) over t_1 and t_2 fix k_0 and k_0-l_0 as relatively small constants in the nonrelativistic approximation (NRA) when compared with the integration variable $\vec{k}$ or with the mesonic mass squared.

In the following we also neglect the term $(k-l)_\mu(k-l)_\nu/m_\omega^2$ appearing in the vector-meson propagator. As $m_\omega^2 \gg (k_0-l_0)^2$, it can obviously be neglected for $\mu=\nu=4$. For other combinations of indices the analysis becomes somewhat more involved. The combination $\mu=4$, $\nu=j$, for example, leads to a contribution proportional to E but 20 times smaller than E [E is defined in Eq. (32)].

In order to proceed, one has to deal with the integral

$$J=\int d^3k\frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{[(\vec{k}-\vec{l})^2+m_\omega^2][\vec{k}^2+m_\pi^2]}\,. \tag{21}$$

A similar integral has already been considered in Ref. 30. Using the Feynman parametrization, the integral (21) can be transformed into

$$J=\int_0^1 d\alpha\, e^{i\alpha\vec{l}\cdot(\vec{r}_1-\vec{r}_2)} \times\frac{d^3k\,e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{[\vec{k}^2+m^2+\vec{l}^2\alpha(1-\alpha)+(m_\omega^2-m_\pi^2)]^2}\,. \tag{22}$$

The leptonic momenta $\vec{l}^2$ appearing in the denominator of J can be neglected.

Using the NRA for the nucleon wave functions, we arrive at the following two-body operator corresponding to the combination of indices $\mu=\nu=4$:

$$D_E(4)=-\kappa(A+B_1+B_2)\,, \tag{23}$$

where

$$A=\pm\frac{1}{2M}[(\vec{r}_1\cdot\vec{l})(\vec{\sigma}_2\cdot\vec{\nabla}_2)Q_A\tau_2^{\mp} +(\vec{r}_2\cdot\vec{l})(\vec{\sigma}_1\cdot\vec{\nabla}_1)Q_A\tau_1^{\mp}]L_4(0)\,, \tag{24}$$

$$Q_A=\frac{1}{(2\pi)^3}\int d^3k\frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{(\vec{k}^2+m_\pi^2)(\vec{k}^2+m_\omega^2)}\,, \tag{25}$$

$$B_1=\mp\frac{1}{2M}(\vec{r}_1-\vec{r}_2)\cdot\vec{l} \times[(\vec{\sigma}_2\cdot\vec{\nabla}_2)Q_B\tau_2^{\mp}-(\vec{\sigma}_1\cdot\vec{\nabla}_1)Q_B\tau_1^{\mp}]L_4(0)\,, \tag{26}$$

$$B_2=\pm\frac{1}{2M}[(\vec{\sigma}_2\cdot\vec{l})Q_B\tau_2^{\mp}+(\vec{\sigma}_1\cdot\vec{l})Q_B\tau_1^{\mp}]L_4(0)\,, \tag{27}$$

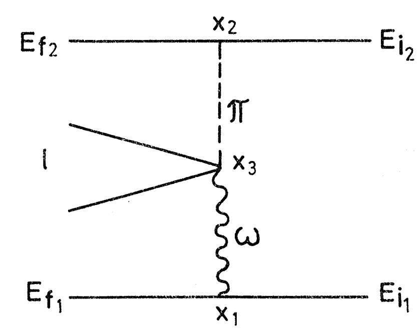

FIG. 2. The exchange diagram. The lines correspond to fermions, ω meson, and the pion, respectively. The coordinate dependence of the vertices and the energies of the nucleon lines are indicated.

$$Q_B = \frac{1}{(m_\omega^2 - m_\pi^2)^2 (2\pi)^3} \int d^3k\, e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} \times \left[\ln\left(\frac{\vec{k}^2 + m_\omega^2}{\vec{k}^2 + m_\pi^2}\right) - \frac{m_\omega^2 - m_\pi^2}{k^2 + m_\omega^2}\right]. \tag{28}$$

When $\mu = \nu = j$, we obtain the two-body operator

$$D_E(j) = -i\kappa(C + D + E), \tag{29}$$

where

$$C = \pm(1/2M^2)[\vec{p}_1(\vec{\sigma}_2\cdot\vec{\nabla}_2)Q_A\tau_2^\mp + \vec{p}_2(\vec{\sigma}_1\cdot\vec{\nabla}_1)Q_A\tau_1^\mp]\cdot\vec{L}_4(0), \tag{30}$$

$$D = \pm(1/4M^2)[(\vec{\sigma}_1\times\vec{\nabla}_1)(\vec{\sigma}_2\cdot\vec{\nabla}_2)Q_A\tau_2^\mp + (\vec{\sigma}_2\times\vec{\nabla}_2)(\vec{\sigma}_1\cdot\vec{\nabla}_1)Q_A\tau_1^\mp]\cdot\vec{L}_4(0), \tag{31}$$

$$E = \mp i(1/4M^2)[\vec{\nabla}_1(\vec{\sigma}_2\cdot\vec{\nabla}_2)Q_A\tau_2^\mp + \vec{\nabla}_2(\vec{\sigma}_1\cdot\vec{\nabla}_1)Q_A\tau_1^\mp]\cdot\vec{L}_4(0). \tag{32}$$

Here $L_4 = \psi_e^\dagger\gamma_5\psi_\nu$ and $\vec{L}_4 = \psi_e^\dagger\vec{\sigma}\psi_\nu$. The effective nuclear matrix elements transforming like axial vectors are found by omitting $\vec{1}L_4$ in the two-body operator $D_E(4)$ in Eq. (23) or L_4 in the two-body operator $D_E(j)$ in Eq. (29). In the following we write $A = \vec{A}\cdot\vec{1}L_4$, $C = \vec{C}\cdot\vec{L}_4$, etc.

In order to establish the relative importance of the various contributions listed above and thus test for the possibility of energy dependence of the theory, we described the nuclear-physics side by a simple model. Our numerical calculations were performed for the $A = 12$ triplet using the shell-model wave functions. The formalism is very similar to the one encountered in the study of parity-violating nuclear forces,[31] which means that all 12 nucleons have been considered.

Introducing the notation

$$-\kappa\langle f|A|i\rangle = av, \quad \text{etc.}, \tag{33}$$

where $\vec{v}$ is a unit axial vector, we can write the results as follows

$$\begin{aligned} a &= \mp Y_E 2.908\times10^{-4}\ \text{fm}, \\ b_1 &= \mp Y_E 3.953\times10^{-4}\ \text{fm}, \\ b_2 &= \mp Y_E 4.641\times10^{-4}\ \text{fm}, \\ c &= \pm Y_E 1.076\times10^{-6}, \\ d &= \pm Y_E 2.565\times10^{-5}, \\ e &= \pm Y_E 5.522\times10^{-5}. \end{aligned} \tag{34}$$

The upper and lower sign refer to β^+ and β^- decay, respectively. The dimensionless constant Y_E is defined by

$$Y_E = gf_\omega G_\omega/M. \tag{35}$$

The approximation used by Kubodera, Delorme, and Rho[17] corresponds to

$$Q_A \to \frac{1}{4\pi m_\omega^2}\frac{e^{-m_\pi r}}{r}, \quad Q_B \to 0, \tag{36}$$

thus omitting the large terms b_1 and b_2.

The situation is particularly simplified for $0^- \to 0^+$ first-forbidden β-decay transitions. The effective nuclear operator in this case has to behave as a pseudoscalar, which means that the exponentials in Eqs. (20) and (25) can be replaced by unity. The only important contribution is then

$$D_E(4, 0^-\to 0^+) = \pm\kappa(1/2M)[(\vec{\sigma}_2\cdot\vec{\nabla}_2)Q_A\tau_2^\mp + (\vec{\sigma}_1\cdot\vec{\nabla}_1)Q_A\tau_1^\mp]L_4(0). \tag{37}$$

Contributions of the $D_E(j)$ type require multipole expansion, in contrast to Gamow-Teller transitions, thus becoming negligible. This, in principle, makes $0^- \to 0^+$ transitions very suitable for investigation. (In the case of unique forbidden transitions all types of contributions would be important.) However, the nuclear-physics side of the problem is extremely difficult, as one has to deal with heavy and very complex nuclei.

IV. EFFECTIVE β-DECAY INTERACTION

In this section we present expressions for the correction factor for Gamow-Teller transitions, including both closed-loop and exchange-diagram contributions. For the sake of comparison, we also write the phenomenologically introduced induced tensor (IT)[2-4] which enters the interaction Hamiltonian (for $\beta^\mp$ transitions) as

$$H_{\text{int}} = g_A\vec{\sigma}\cdot\vec{L}_4 \pm \frac{Y}{2M}\vec{\sigma}\cdot(-i\vec{\nabla})L_4 + \cdots. \tag{38}$$

All notation has the same meaning as in Ref. 15. Equation (38) corresponds to case A in this reference. The exchange-diagram contributions to the interaction Hamiltonian (for $\beta^\mp$ transitions) are as follows:

$$E(\text{IT}) = \pm Y_E V\vec{\sigma}\cdot\vec{1}L_4', \quad V = -3.33\times10^{-4}\ \text{fm} \tag{39}$$

and

$$E(\Sigma) = \mp Y_E W\vec{\sigma}\cdot\vec{L}_4, \quad W = -1.23\times10^{-4}. \tag{40}$$

Here $\vec{1}L_4'$ indicates the particular multipole in the expansion of the lepton covariant. It is important to note that $\vec{1}L_4'$ is not equal to the derivative $i\vec{\nabla}L_4$ when electron Coulomb wave functions are introduced.[32] The numerical values correspond to the $A = 12$ triplet. The effective single-particle interaction has a formal meaning. It is obtained by writing, for example, $a\vec{v} = a\vec{\sigma}/\langle\sigma\rangle$ and using $\langle\sigma\rangle = -\frac{2}{3}$.

TABLE II. Partial contributions δ_i to the $\delta = C^{\beta^+}/C^{\beta^-} - 1 = \sum_i \delta_i$ asymmetry.

δ_i	δ_i analytic	δ_i numeric	Remark
δ_1	$-\frac{2}{3}\hat{x}(E_0^+ + E_0^-)$ fm	$-\hat{x}\,0.1003$	phenomenological IT
δ_2	$x_E V \frac{2}{3}(E_0^+ + E_0^-)$	$-x_E\,0.33\times 10^{-4}$	energy-dependent exchange contribution
δ_3	$2x_E W$	$x_E\,0.25\times 10^{-4}$	energy-independent exchange contribution
δ_4	$-x_E(0.23\times 10^{-2}\text{ fm})\frac{2}{3}(E_0^+ + E_0^-)$	$-x_E\,0.23\times 10^{-3}$	energy-dependent closed-loop contribution
δ_5	$2x_E\,0.41\times 10^{-4}$	$x_E\,0.81\times 10^{-4}$	energy-independent closed-loop contribution

The closed-loop diagrams give the following contributions to Eq. (38):

$$C(\text{IT}) = \pm Y_E(0.227\times 10^{-2}\text{ fm})\vec{\sigma}\cdot(-i\vec{\nabla})L_4 \qquad (41)$$

and

$$C(\Sigma) = \mp Y_E 0.490\times 10^{-4}\vec{\sigma}\cdot\vec{L}_4 . \qquad (42)$$

The spectrum-shape correction factor for Gamow-Teller transitions is of the form

$$\begin{aligned} C^\beta = {} & C_{1\text{GT}} \pm \hat{x}(C_2 + \beta_3 C_3)\text{ fm} \mp x_E VMC_{9\text{GT}} \\ & \mp x_E W C_{1\text{GT}} \pm x_E 0.227\times 10^{-2}(C_2 + \beta_3 C_3)\text{ fm} \\ & \mp x_E 0.409\times 10^{-4} C_{1\text{GT}} . \end{aligned} \qquad (43)$$

The parameters appearing here are defined as follows:

$$\hat{x} = b/2M\text{ fm}, \quad b = Y/g_A, \quad x_E = Y_E/g_A, \qquad (44)$$

while all other quantities are defined as in Ref. 15. Equation (43) completes expression ($B2$) reported previously.[15]

In order to be able to discuss the importance of various mesonic-current contributions to the correction factor C^β, we have to calculate the mirror asymmetry δ for $A = 12$ from Eq. (43). To simplify the estimate of δ, we use the low-Z approximation and neglect β_3. The results obtained are given in Table II. It is evident from the table that the energy-dependent term C(IT) in Eq. (41), coming from the closed-loop diagram, is a dominant one.

As maximal β-decay energies have to be expressed in fm^{-1} units ($E_0^+ + E_0^-$ is 0.1376 and 0.1505 for $A = 8$ and $A = 12$, respectively), the energy-dependent term δ_2 coming from the exchange diagram is comparable with the energy-independent term δ_3. A similar conclusion can be inferred from Ref. 17, where an entirely different nuclear model was exploited. The approximation (36) introduced into our calculational scheme leads to $V = -5.28\times 10^{-4}$ fm, which is appreciably different from the value $V = -3.33\times 10^{-4}$ fm in Eq. (39) obtained using the full expressions (23) and (29). This indicates that the two-body operator should not be approximated too severely. As the relative magnitudes of our parameters V and W are qualitatively comparable with the relative magnitudes of the corresponding quantities L and I in Ref. 17, we estimate that changes in nuclear-model wave functions do not change this ratio by more than 50%.

V. K/β^+ RATIO AND CONCLUSION

It was suggested[17] that experimental results might follow from a suitable mixture of the "normal" induced tensor IT[2-11, 13-16] and of the exchange term coming from mesonic SCC. Our calculations do not support the expectations[17] that the energy-dependent closed-loop contribution cancels the energy-dependent exchange-diagram contribution. Therefore, mesonic SCC do not seem to be able to explain the result for the $A = 8$ triplet. As we are dealing with an essentially nonrenormalizable theory, there is some arbitrariness in the theoretical description. It seems, therefore, interesting to make some additional considerations of the "mixed-model" theory. Let us show how the assumption necessary to explain the energy independence of the result for the $A = 8$ triplet[19] leads to somewhat better agreement with Vatai's results for the K/β^+ ratio than in the standard approach.[18]

When the Coulomb charge Z is low, the spectrum-shape correction factors can be written as

$$C^{\beta^\pm} = 1 \pm \beta\tfrac{2}{3}(E_0^\pm \mp 2\xi - 1/W) \pm \gamma\tfrac{2}{3}(E_0^\pm \mp 3\xi - 1/W) \pm \eta , \qquad (45)$$

$$C^K = 1 + \beta\tfrac{2}{3}(-E_b + 2\xi) + \gamma\tfrac{2}{3}(-E_b + 3\xi) + \eta . \qquad (46)$$

Here E_0 are the maximal kinetic energies and E_b is the electron binding energy. All other notation has the usual meaning.[3, 15] The arbitrary constants β, γ, and η parametrize the contributions from both the phenomenologically induced tensor and mesonic currents. The most general mesonic cur-

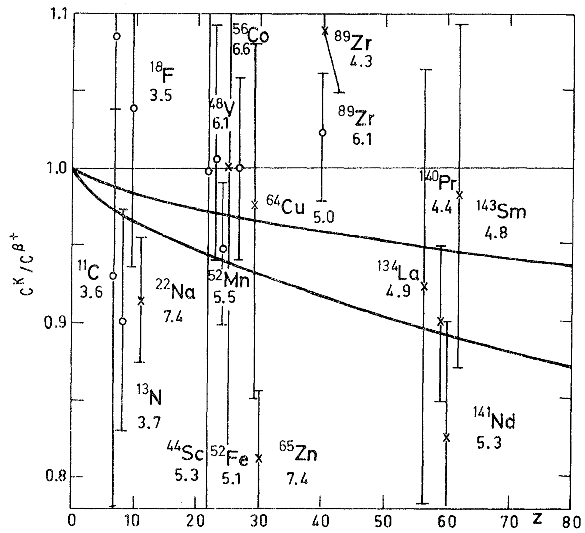

FIG. 3 C^K/C^{β^+} ratios for $\beta=3\times10^{-3}$ and $\beta=6\times10^{-3}$. The experimental points are those compiled in Ref. 18.

rents[27] may be assumed as being included.[33]

For small β, γ, and η we find the ratios

$$C^{\beta^+}/C^{\beta^-} \cong 1 + \tfrac{2}{3}(\beta+\gamma)(E_0^+ + E_0^-) + 2\eta\,, \tag{47}$$

$$C^K/C^{\beta^+} = 1 + \tfrac{2}{3}(4\beta + 6\gamma)\xi + \tfrac{2}{3}(\beta+\gamma)(E_0^+ - E_b)\,. \tag{48}$$

In order to make the ratio (47) energy-independent, we have to select

$$\beta+\gamma=0\,, \tag{49a}$$

as suggested before,[17] or

$$\beta=0, \quad \gamma=0\,. \tag{49b}$$

The first selection leads to

$$C^{\beta^+}/C^{\beta^-} = 1 + 2\eta > 1 \tag{50}$$

and

$$C^K/C^{\beta^+} \cong 1 - \tfrac{4}{3}\beta\xi < 1\,. \tag{51}$$

Figure 3 shows the C^K/C^{β^+} ratio for $\beta=3\times10^{-3}$ and $\beta=6\times10^{-3}$. The alternative choice (49b) can also be easily explained in the mixed model. It should be considered in the case when Vatai's deviations are explainable as pure nuclear-structure effects. As the mirror asymmetry δ is actually ascribable to the same causes,[20-24] we may end with the result $\beta=\gamma=\eta$. We might even insist that second-class currents are present, but "hidden" by interference.

Thus, the mixed theory is aesthetically unpleasing. On the other hand, the $\pi\omega$ mesonic current alone does not seem to be sufficient to explain experimental results. The energy-dependent closed-loop contribution δ_4 in Table I is by far the largest, and the energy-dependent exchange-diagram contribution δ_2 in Table II does not have even the correct sign to compensate it.

However, we should keep in mind that we are dealing with an essentially nonrenormalizable theory, which makes all closed-loop results somewhat suspect. It is difficult, however, to imagine why they should be any smaller than as calculated.

Thus, on balance, neither the phenomenologically introduced induced tensor[34] nor mesonic currents lead to an easy and natural fit of experimental results.

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[33]It is possible that a suitable combination of cutoff-dependent contributions might enable us to construct expressions of the form (15) and (16) even with no phenomenologically induced tensor included.
[34]This conclusion is supported by the analysis of forbidden transitions.