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# Exact $N$-$^2$H Breakup Calculations*

Mahavir Jain and Gary D. Doolen
*Cyclotron Institute and Physics Department, Texas A & M University, College Station, Texas 77843*
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The fivefold differential cross section computed from the exact solution of the three-particle Faddeev equations for separable spin-dependent $s$-wave nucleon-nucleon interactions is compared with the data from the recent kinematically complete measurement of $^2$H($p$,2$p$)$n$ reaction at 39.5 MeV in $4\pi$ geometry. The calculations agree well with the data, which includes regions of phase space where quasifree scattering and final state interactions are not the predominant reaction mechanisms.

## I. INTRODUCTION

Recent theoretical studies[1, 2] of the three-nucleon problem have provided an understanding of the three-nucleon system in terms of the nucleons interacting via two-body forces. The three-nucleon system is the simplest system which allows the study of the features of the nucleon-nucleon interaction which cannot be obtained from nucleon-nucleon scattering, including the determination of neutron-neutron scattering length. The most important features are the off-shell behavior of the two-nucleon $T$ matrix and the possible existence of explicit three-body forces. These aspects manifest themselves in other systems too, but the three-nucleon system is most amenable to exact calculations and offers the possibility of investigating these effects quantitatively.

In general, two different approaches have been used for the accurate calculations of the three-body bound state, $N$-$^2$H elastic scattering, and breakup amplitudes. The traditional approach has been the variational calculations, some of which incorporate the full complexities of the phenomenological nucleon-nucleon interaction.[3] However, for positive energies only one accurate variational $s$-wave calculation of elastic $N$-$^2$H scattering using a central potential of Yukawa type has been performed by McDonald and Nuttall.[4] This method could be used to compute breakup cross sections and could be extended to more complicated potentials and higher angular momenta.[4] The second method is to start from the Faddeev formulation[1] or one of its variants.[5] The solution of the Faddeev equations with the full nucleon-nucleon potential is a very difficult problem and has only been carried out for the bound state.[6] Only in the last few years have exact solutions in the breakup region been obtained using separable potentials.

One result of many previous calculations[3] is that the general features of the three-body breakup results are not very sensitive to details of the two-nucleon interaction. If the few critical parameters of the two-nucleon interaction agree with the experimental data, the resulting general features of the three-body calculations agree reasonably well with the data although a quantitative fit is not obtained. This is characteristic of elastic scattering as well as kinematically incomplete and complete breakup calculations. However, the latter have primarily been compared with data in the phase-space regions where the simple reaction mechanisms, i.e., quasifree scattering (QFS) or final state interaction (FSI) dominate the cross section. Thus, theory and experiment have not been compared in the large region of the three-body phase space. It is this region, away from dominating QFS and FSI processes, where the breakup cross section should be most sensitive to one of the doublet amplitudes, $M_{D2}$, (two like nucleons coupled to spin zero) and hence to the details of nuclear interaction. Data in this region are now available from the recent kinematically complete measurement of the $^2$H($p$, 2$p$)$n$ reaction at 39.5 MeV in $4\pi$ geometry.[7]

We have developed a computer code[8] to calculate the fivefold differential cross sections, $d^5\sigma/d\Omega_3 d\Omega_4 dE$. These calculations will be discussed in Sec. II. In Sec. III, our calculations are compared with the data, which includes phase space regions where QFS and FSI processes do not predominate. A summary of the results and an outline of current work follows in Sec. IV.

## II. BREAKUP CALCULATIONS

The Faddeev equations for $N$-$^2$H breakup have been solved using separable potentials. One of the first calculations of $d\sigma/d\Omega dE$ for the $^2$H($n,p$)-2$n$ reaction was performed by Aaron and Amado[9] using the contour deformation method.[10] Recently, the same model ($YY$ model) was solved by Cahill and Sloan[11] with improved results.

The notation and equations for the off-shell $T$ matrix used in our calculations are from Cahill and Sloan's paper with corrections[12] and extensions necessary for calculating the fivefold differential cross section, $d^5\sigma/d\Omega_3 d\Omega_4 dE$, appropriate for kinematically complete experiments. The separable $s$-wave spin-dependent nucleon-nucleon potential used is a charge-independent Yamaguchi potential[13] with parameters determined from low-energy two-nucleon data.[11] Coulomb effects are not included. From the Lippman-Schwinger equation, the off-energy-shell two-body scattering amplitude has the separable form,

$$t_n(q, e + i\epsilon, q') = g_n(q) F_n(e + i\epsilon) g_n(q'), \qquad (1)$$

where $g_n$ are the form factors of the interaction, $F_n$ is the propagator corresponding to the two-body c.m. energy, $e$. With the above separation and partial-wave decomposition, the Faddeev equations reduce to one-dimensional coupled singular integral equations of the form:

$$T^{(s)}_{nl}(k',k) = B^{(s)}_{nl l}(k',k) + \sum_{n'} A_{nn'} \int B^{(s)}_{nn' l}(k',k'') F_{n'}(E - 3k''^2/4 + i\epsilon) T^{(s)}_{n' l}(k'',k) k''^2 dk'', \qquad (2)$$

where $s$ is the total spin, $l$ is the angular momentum, $T^{(s)}_{nl}$ is the exact amplitude and $B^{(s)}_{nl}$ is the Born term, and $n = 1$, 2 refers to two like nucleons being in spin 1 or 0 state. The $k$, $k'$, and $q'$ are the c.m. wave numbers for the incident particle, one of the breakup nucleons, and relative wave number of the remaining two nucleons, respectively. The main difficulty in solving Eq. (2) is due to the branch points of the kernel arising from the three-body propagator in $B^{(s)}_{nn'l}$. For the breakup region, these singularities occur on the real $k''$ axis. Rotating the integration contour into the lower-half $k''$ plane avoids these singularities in the solution of Eq. (2) for complex $k'$ ($k' = \kappa' e^{-i\phi}$, $\kappa'$ – real). This requires analytic continuation of $B^{(s)}_{nn'l}(k',k'')$ for complex $k'$ and results in an integral equation for $T^{(s)}_{nl}(\kappa' e^{-i\phi},k)$ which is solved numerically. Using this solution in an integration along the deformed contour[11] the $T^{(s)}_{nl}(k',k)$ on the real $k'$ axis are obtained.

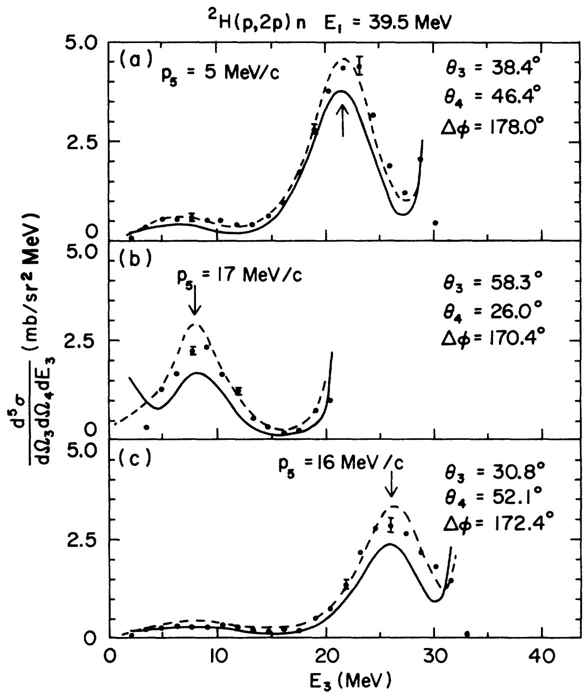


FIG. 1. The experimental cross sections from Ref. 7 for $^2$H($p$,2$p$)$n$ reaction at 39.5 MeV at the angles $\theta_3$, $\theta_4$, and $\Delta\phi = \phi_3 - \phi_4$. The minimum recoil momenta and relative energies are also identified in the figure. Error bars shown are statistical errors. The absolute normalization is accurate to 6%. The solid lines are the results of separable potential calculations discussed in the text. Dashed lines are model predictions from Ref. 7 These angle pairs are selected to emphasize $p$-$p$ QFS scattering.

The numerical integration over $k''$ was performed by a 28-point Gaussian quadrature. The resulting simultaneous equations were solved by Gaussian elimination to calculate $T^{(s)}_{nl}$. The amplitudes, $N^{(s)}_n$, defined by Cahill and Sloan, are obtained by partial wave recomposition of $T^{(s)}_{nl}$ and multiplication by the propagator and the form factor. The three-body transition amplitude, $M$, is expressed in terms of one quartet amplitude, $M_Q$, and doublet spin amplitudes $M_{D1}$ and $M_{D2}$; the latter two correspond to coupling the spins of two like nucleons to spin 1 and spin 0, respectively. $M_Q$, $M_{D1}$, and $M_{D2}$ are linear combinations of the $N^{(s)}_n$'s.

$$|M|^2 = 2|M_Q|^2/3 + |M_{D1}|^2/3 + |M_{D2}|^2/3 . \qquad (3)$$

Taking the nonrelativistic limit of Møller's invariant formula,[14] the fivefold differential cross section in units of mb/sr$^2$ MeV is,

$$d^5\sigma/d\Omega_3 d\Omega_4 dE_3 = \text{K.F.} \times |M|^2 , \qquad (4)$$

where $|M|^2$ is given by Eq. (3) and the kinematic factor (K.F.)

$$\text{K.F.} = \frac{10(2\pi)^4 m^3}{\hbar^7} \frac{p_3 p_4^2}{p_1(2p_4 - p_1\cos\theta_4 + p_3\cos\theta_{34})}, \qquad (5)$$

where the laboratory quantities $p_i$, $E_i$, and $\theta_i$, $i = 1$, 5, refer to the momentum, kinetic energy, polar and azimuthal angles of incident particle, target nucleus, detected particles at ($\theta_3$, $\phi_3$) and ($\theta_4$, $\phi_4$) and the undected particle, respectively. The nucleon masses, $m$, are set equal to 939.2 MeV, $c = 1$, and $\cos\theta_{34} = \hat{p}_3 \cdot \hat{p}_4$.

In our calculations, the exact solution of the integral equation for $T_{nl}^{(s)}$ [Eq. (2)] was used for $l=0$ to 3. We used the second iteration of the integral equation for $l=4$, 5 and first iteration for $l=6$ and 7. We have used the Born term of the integral equation for all $l>7$. This is done by employing $T_{nl}^{(s)} - B_{nl}^{(s)}$ (instead of the usual procedure of using $T_{nl}^{(s)}$) for partial wave recomposition up to $l=7$ and adding the exact Born term to the resulting expression. We observed about 1% difference in the values of cross sections when the exact or iterative solutions up to $l=5$ were used as compared to above. We also compared our calculations of $d^4\sigma/d\Omega_3 d\Omega_4 dE$ for $^2$H$(p, 2p)n$ and $^2$H$(p, pn)p$ reactions at 14.4 MeV ($\theta_3=45°$, $\theta_4=45°$, and $\phi_3-\phi_4=180°$) with the results of Cahill[15] and found excellent agreement in all parts of spectra.

## III. COMPARISON WITH DATA

The calculated cross sections can be compared with the complete set of $^2$H$(p, 2p)n$ data obtained at 39.5 MeV.[7] However, these data consist of about 2000 two-dimensional energy spectra. Therefore, the same representative spectra as in Ref. 7 were selected for comparison with our calculations.

In Fig. 1, the calculated cross sections shown by solid line are compared with the data for three angle pairs in nearly coplanar geometry. These were chosen to emphasize the $pp$ QFS, which peaks at the minimum recoil momentum. Within the experimental errors our calculations generally agree with the data, except at the highest energy of the detected particle. This disagreement is largely due to the finite energy-resolution effects which are not folded in the theoretical calculations. The discrepancy at low energy in Fig. 1(b) is partly due to finite energy-resolution effects and an energy cutoff of about 3 MeV in the data. The theoretical cross section generally lies below the experimental points near the central peak.

Figure 2 shows the data for the angle pairs such that the $np$ FSI dominates. The spectra exhibit peaks for minimum $np$ relative energy. Again the agreement is generally within experimental uncertainty.

In Fig. 3, the data for noncoplanar geometry are compared with the calculations. These angle pairs were selected to minimize the contribution of QFS and FSI. The agreement is again good except for low energies of $E_3$ or $E_4$. At this time, it is not possible to quote the percentage disagreement definitely. The reason for this is that part of this disagreement is due to the experimental difficulty of measuring low-energy protons and to possible contamination from background events arising from detection of uncharged particles at forward angles.

Figure 4 shows the spectra at two angle pairs where $pp$ FSI effects are important. The spectra exhibit a broad peak for low relative $pp$ energy, which is split by the Coulomb repulsion. The depth of the Coulomb minimum increases for lower $pp$ relative energy. The maxima occur for $E_{pp} \sim 0.4$ MeV. The absence of the Coulomb interaction in the theoretical calculations is responsible for dis-

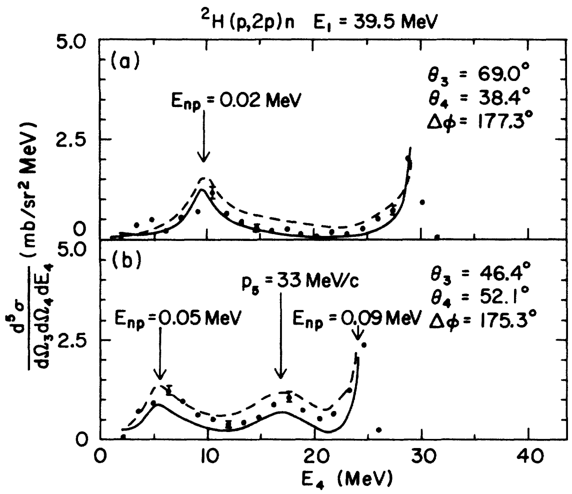


FIG. 2. Same caption as Fig. 1, except that $np$ FSI dominates.

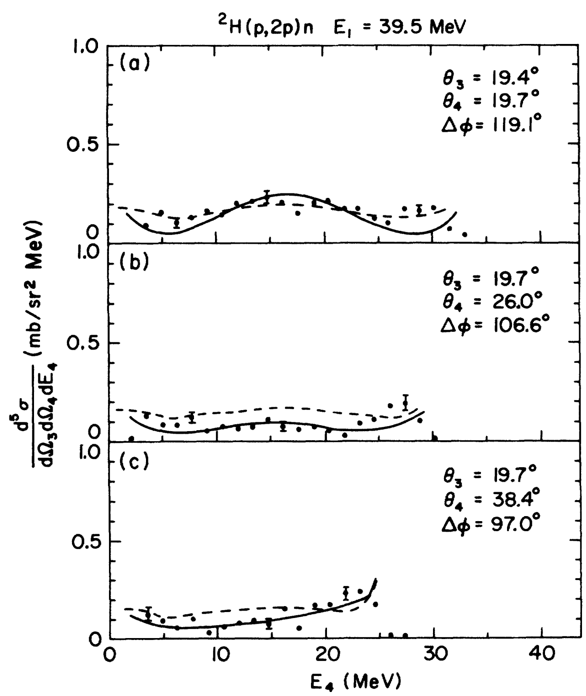


FIG. 3. Same caption as Fig. 1, except that angle pairs are selected to minimize the contributions due to QFS and FSI on the cross sections.

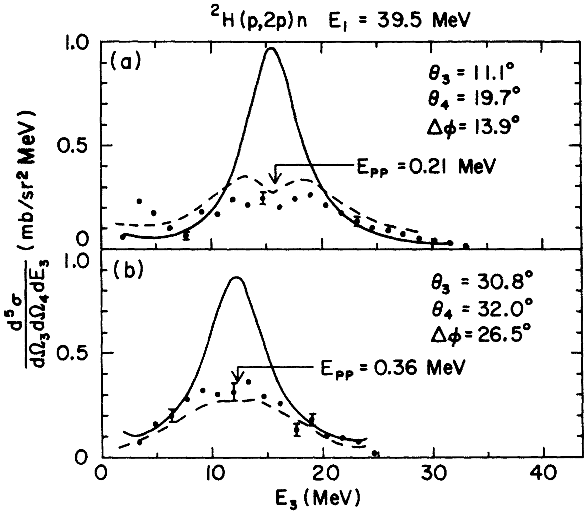


FIG. 4. Same caption as Fig. 1, except that $pp$ FSI is emphasized.

agreement with the data in this region.

For comparison, we have included the dashed lines from Ref. 7, which are the results of a four-parameter model calculation to represent $d\sigma/d\Omega_3 d\Omega_4 dE$ as an incoherent sum of terms representing contributions from phase space (statistical processes), QFS, and FSI. The four adjustable parameters were fitted to a partial Dalitz plot of the data at 43°. This model calculation gives best fits to the data in these spectra. The fits to the data in other regions are also fairly good. However, the model calculation does not reproduce the minimum in the partial Dalitz plot[7] which is interpreted to be due to destructive interference of the Born term with the rest of the multiple-scattering series.[16] This is perhaps an illustrative example of why the simple models have been so successful in fitting the previously available three-body data in the QFS and FSI regions. Their success is possibly due to the fact that these processes are quasi-two-body effects in the first approximation.

Wallace[17] and Ebenhöh have also done similar calculations. Wallace computes the multiple-scattering series and uses Padé approximates to sum the series. He modifies the two-nucleon-interaction propagators to fit two-body cross sections. These calculations at 20, 45, and 100 MeV for the coplanar quasifree scattering consistently overestimate the experimental cross sections at 20 and 45 MeV and give good predictions at 100 MeV. Ebenhöh uses a different numerical technique and concentrates on the FSI region and also gives some results for the QFS region. These calculations are for coplanar cases and are in reasonable agreement with the data.

## IV. CONCLUSION

It is somewhat surprising that such a simple two-body potential can reproduce so well the general features of the three-body breakup data. It is probable that this is due in part to the fact that the deuteron wave function is as so extended that only a small amount of the scattered particles actually contain information about the region where three nucleons interact. It is known that significant variations can be obtained in the $^2S$ phase shifts and inelasticity parameters by varying the two-nucleon potential.[4] The $s$-wave doublet and quartet inelasticity parameters, for nucleon-deuteron scattering at 40 MeV, are ~0.3 and ~0.9, respectively.[19] Also, the $s$-wave quartet phase shift and inelasticity are nearly model-independent. Therefore, variations in the nucleon-nucleon potential would primarily affect the doublet amplitude, $M_{D2}$.

We next plan to study regions in which $M_{D2}$ dominates and hope to separate its effect on the cross section. We hope to determine what corrections are needed to account for the deficiency of the $YY$ model. This correction would be a phenomenological way to include large off-shell effects and possible three-body forces. However, this interpretation is not clear cut due to omission of higher partial waves ($l > 0$) in the nucleon-nucleon interaction, which is not a good approximation at 40 MeV. We are also currently investigating the incorporation of a Coulomb potential and the $YYY$ model (no charge independence) to study the stability of the solutions with respect to variations in the parameters of two-body potentials.


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