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Charge Asymmetry Effects and the Trinucleon Binding-Energy Difference*

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We demonstrate, using a separable-potential model, that the discrepancy between the observed ^{3}H-^{3}He binding-enery difference and the calculated Coulomb energy does not necessarily imply that $|a_{nn}| > |a_{pp}|$. This possibility arises from the relative insensitivity of the trinucleon binding energy to the scattering length as compared to the sensitivity to the effective range.

I. INTRODUCTION

Because the singlet nucleon-nucleon (N-N) scattering lengths are large and therefore sensitive to small differences in the corresponding N-N interactions, their measurement provides a good test of charge asymmetry—differences between the nuclear part of the p-p and the n-n interactions. On the other hand, small deviations from charge symmetry resulting in $|a_{nn} - a_{pp}| < 2$ fm will probably not be sufficient to account for the apparent discrepancy in the measured binding-energy difference in the trinucleon isodoublet. We wish to emphasize the sensitivity of the three-body binding energy to the N-N effective range r as opposed to the N-N scattering length a by means of a simple, separable-potential model calculation. More importantly, we wish to point out that, because of the relative insensitivity of the binding energy to the scattering length, the discrepancy between the binding-energy difference

$$\Delta = |E_B(^3\mathrm{H}) - E_B(^3\mathrm{He})| \tag{1}$$

and the best theoretical estimates of the Coulomb energy difference E_C (assuming charge symmetry) is compatible with

$$|a_{nn}| < |a_{pp}| .$$

The measured, low-energy proton-proton scattering parameters are[1-4]

$$a_{pp}^C = -7.823 \pm 0.01 \text{ fm},$$
$$r_{pp}^C = 2.794 \pm 0.015 \text{ fm},$$

where the superscript C indicates that the Coulomb effects are included. The removal of Coulomb effects adds measurably to the uncertainty in these scattering parameters because of possible variations in the short-range part of the nuclear potential. The current estimate of the Coulomb-corrected values is[1,4-6]

$$a_{pp} = -17.1 \pm 0.3 \text{ fm},$$
$$r_{pp} = 2.84 \pm 0.03 \text{ fm}.$$

For comparison, the neutron-neutron low-energy scattering parameters are estimated to be[1]

$$a_{nn} = -16.4 \pm 0.9 \text{ fm},$$
$$r_{nn} = 2.84 \pm {\sim}0.5 \text{ fm}.$$

(It should be noted that in many experiments equivalence of the neutron-neutron and Coulomb-corrected proton-proton effective ranges was assumed in order to determine a_{nn}.) It is clear that, if there exists any difference between a_{pp} and a_{nn}, the p-p value appears to be slightly larger, indicating that the over-all strength of the nuclear part of the proton-proton interaction is somewhat greater than that of the neutron-neutron interaction. In contrast to this, the experimental binding-energy difference ($\Delta \approx 0.76$ MeV) is larger than the Coulomb-energy difference ($E_C \approx 0.60$ to 0.66 MeV) calculated in perturbation theory for all reasonable models of the triton.[7] If this discrepancy is ascribed to charge asymmetry in the N-N interactions, it requires that the n-n interaction contribute more to the binding of ^{3}H than does the p-p interaction to the binding of ^{3}He. This has been interpreted as indicating $|a_{nn}| > |a_{pp}|$. However, we wish to point out that this conclusion does not necessarily follow. The trinucleon binding energies are much more sensitive to small differences in effective ranges than to small differences in scattering lengths.[8] Thus it is quite possible that one may have $|a_{nn}| < |a_{pp}|$ and still be able to explain the missing energy in the theoretical charge-symmetric estimate of Δ.

II. THEORETICAL MODEL

For simplicity we have used a separable-potential model of the trinucleons in which we assume central N-N interactions but allow for charge dependence and charge asymmetry:

$$V_{np}^t \neq V_{np}^s \neq V_{nn} \neq V_{pp} .$$

The potentials were chosen to be of the simple attractive, rank-one form

$$V_i(k,k') = -(\lambda_i/M)g_i(k)g_i(k'),\quad g_i(k) = (\beta_i^2 + k^2)^{-1}. \tag{2}$$

Because the formulation of the three-body separable-potential model from the Schrödinger equation is well known,[8-11] we quote only the system of coupled integral equations that were solved to obtain the triton (or ^{3}He) binding energy:

$$\begin{aligned}
u^t_{np}(p) &= \pi\lambda^t_{np}[1-\lambda^t_{np}\Lambda^t_{np}(E)]^{-1}\int\{I_{tt}(p,p')u^t_{np}(p') + I_{ts}(p,p')u^s_{np}(p') + 2I_{tn}(p,p')u_{nn}(p')\}p'^2dp',\\
u^s_{np}(p) &= \pi\lambda^s_{np}[1-\lambda^s_{np}\Lambda^s_{np}(E)]^{-1}\int\{3I_{st}(p,p')u^t_{np}(p') - I_{ss}(p,p')u^s_{np}(p') + 2I_{sn}(p,p')u_{nn}(p')\}p'^2dp',\\
u_{nn}(p) &= \pi\lambda_{nn}[1-\lambda_{nn}\Lambda_{nn}(E)]^{-1}\int\{3I_{nt}(p,p')u^t_{np}(p') + I_{ns}(p,p')u^s_{np}(p')\}p'^2dp'.
\end{aligned}\tag{3}$$

The $u^t_{np}(p)$, $u^s_{np}(p)$, and $u_{nn}(p)$ are the spectator functions associated with each of the form factors $g^t_{np}(k)$, $g^s_{np}(k)$, and $g_{nn}(k)$ in constructing the triton wave function, the completely symmetric part of which is

$$\psi(p,k) = \psi^{(1)} + \psi^{(2)} + \psi^{(3)}, \tag{4}$$

$$\psi^{(1)} = N\frac{[g^t_{np}(k)u^t_{np}(p) + \frac{1}{3}g^s_{np}(k)u^s_{np}(p) + \frac{2}{3}g_{nn}(k)u_{nn}(p)]}{(k^2 + \frac{3}{4}p^2 + ME_B)},$$

where $\psi^{(i)}$ indicates the cyclic permutation of the Jacobi variables $\vec{p} = \vec{p}_1 - \frac{1}{2}(\vec{p}_2 + \vec{p}_3)$ and $\vec{k} = \frac{1}{2}(\vec{p}_2 - \vec{p}_3)$. The functions $\Lambda_i(E)$ and $I_{ij}(p,p')$ are the integrals

$$\begin{aligned}
\Lambda_i^{-1}(E) &= \int\frac{g_i^2(k)d^3k}{k^2 + \frac{3}{4}p^2 + ME_B},\\
I_{ij} &= \int_{-1}^{1}\frac{g_i(|\vec{p} + \frac{1}{2}\vec{p}'|)g_j(|\frac{1}{2}\vec{p} + \vec{p}'|)}{ME_B + p^2 + p'^2 + pp'x}dx.
\end{aligned}\tag{5}$$

It should be clear that if we allow $V^s_{np} = V_{nn}$, the equations reduce to those of Ref. 9. A Gaussian-Gegenbauer integration scheme was used in obtaining the iterated solution of the eigenvalue problem, the Gegenbauer abscissas and weights being optimal for the separable potentials used.

We emphasize that we do not consider the simple model described to be an exact representation of either ^{3}H or ^{3}He. However, the model is adequate to investigate small differences in binding as a function of small differences in the N-N interactions. Although our two-parameter potentials are not sufficiently general to permit variation in a and r while holding fixed the high-energy phase shifts, the results of Ref. 8 indicate that reasonable values for a and r essentially determine the binding for potentials of the form $(\beta^2 + k^2)^{-n}$. Recent investigations of the sensitivity of the trinucleon properties to variations in the two-nucleon off-shell scattering amplitude indicate that the properties of the deuteron and anti-bound-state poles dominate the calculation.[12,13] In this model those properties are fixed by a and r.

III. NUMERICAL RESULTS

Let us first consider binding energies for which the n-p singlet and triplet parameters were determined by the low-energy scattering data. The triplet parameters also yield a deuteron binding energy of 2.225 MeV. The n-n scattering length and effective range were allowed to vary as shown in Table I. The effect of charge dependence[11,14,15] on the binding energy of the triton is evident.

Because the combination of central potentials described above is known to overbind the triton,[8-10] we have also performed the calculations with an effective triplet interaction whose strength λ^t_{np} was adjusted slightly to obtain the correct triton binding energy [$E_B(^3\mathrm{H}) = 8.48$ MeV] when the Coulomb-corrected p-p parameters were used for the n-n interaction (i.e., parameters yielding $a_{nn} = -17$ fm, $r_{nn} = 2.84$ fm). This adjustment accounts in part for the fact that the long-range tensor part of the triplet interaction is slightly less effective in the relatively tightly bound triton as compared to the loosely bound deuteron. Variation of the ^{3}H binding with a_{nn} and r_{nn} is shown in Table II.

By examining both Tables I and II it can easily be seen that small differences in the scattering length alone result in only minor differences in the three-body binding energy ($\Delta a_{nn} = 1$ fm $\rightarrow \Delta E_B \approx 0.03$ MeV). A comparable percentage difference

TABLE I. Triton binding energy (MeV) for different values of a_{nn} and r_{nn} assuming $\lambda^t_{np} = 0.3693$, $\beta^t_{np} = 1.389$ ($a^t_{np} = 5.425$ fm, $r^t_{np} = 1.777$ fm), and $\lambda^s_{np} = 0.1430$, $\beta^s_{np} = 1.150$ ($a^s_{np} = -23.71$ fm, $r^s_{np} = 2.74$ fm).

a_{nn} (fm) \ r_{nn} (fm)	2.74	2.84	2.94
−23.71	10.64		
−18.0	10.51	10.33	10.14
−17.0	10.48	10.29	10.10
−16.0	10.43	10.25	10.07

TABLE II. Triton binding energy (MeV) for different values of a_{nn} and r_{nn} assuming $\lambda_{np}^{t}=0.3495$, $\beta_{np}^{t}=1.389$, and $\lambda_{np}^{s}=0.1430$, $\beta_{np}^{s}=1.150$.

a_{nn} (fm) \ r_{nn} (fm)	2.74	2.84	2.94
−23.71	8.82		
−18.0	8.68	8.52	8.37
−17.0	8.65	8.48	8.33
−16.0	8.61	8.45	8.29

in the less well-determined effective range leads to much larger differences in the binding energy ($\Delta r_{nn}=0.1$ fm $\rightarrow \Delta E_B \approx 0.19$ MeV), demonstrating that the triton binding energy is more sensitive to the effective range than to the scattering length.[8,16]

To illustrate our point about charge asymmetry clearly, we have adjusted the triplet strength further to produce the correct ^{3}He binding energy when the p-p interaction is assumed to be determined by the low-energy (Coulomb-uncorrected) p-p scattering data. Clearly we introduce an error when we assume that the Coulomb part of the p-p interaction can be treated as merely a modification to the parameters of the strong interaction.[17] However, our results indicate that the error is not very large. In Table III we have compiled a comparison of the triton binding energy as a function of a_{nn} and r_{nn}, where the n-p triplet parameters were chosen as described above. It is clear that the weaker dependence of the binding energy upon variation in scattering length as compared to effective range allows for the possibility of having $|a_{nn}|<|a_{pp}|$, while at the same time obtaining the experimental value of Δ if $r_{nn}<r_{pp}$. While this would contradict certain theoretical models of charge asymmetry,[18,19] an experimental determination of $|a_{nn}|<|a_{pp}|$ need not be in contradiction to the apparent requirement that the n-n force in ^{3}H be stronger than the p-p force in ^{3}He if Δ is to be understood. The scattering length is a measure of the over-all strength of the interaction whereas its contribution to the binding of a nucleus is a measure of its strength over the limited extent of the nucleus.

This general feature has been well known for a long time, being reviewed, in fact, by Bethe and Bacher.[20] While decreasing the effective range and keeping the scattering length constant would decrease the two-body binding energy slightly, the effect in the three-body binding energy is probably best understood in the model which Thomas used to demonstrate the finite range of nuclear forces.[21]

TABLE III. Triton binding energy (MeV) for different values of a_{nn} and r_{nn} assuming $\lambda_{np}^{t}=0.3476$, $\beta_{np}^{t}=1.389$, and $\lambda_{np}^{s}=0.1430$, $\beta_{np}^{s}=1.150$. The ^{3}He binding energy is 7.71 MeV assuming p-p parameters $\lambda_{pp}=0.1534$, $\beta_{pp}=1.223$ ($a_{pp}^{c}=-7.823$ fm, $r_{pp}^{c}=2.79$ fm); the corresponding charge symmetric $a_{nn}\approx-17$ fm, $r_{nn}\approx 2.84$ fm yield a ^{3}H binding of 8.32 MeV.

a_{nn} (fm) \ r_{nn} (fm)	2.74	2.84	2.94
−23.71	8.65		
−18.0	8.52	8.36	8.20
−17.0	8.49	8.32	8.16
−16.0	8.44	8.29	8.13

There the binding energy of the deuteron is kept fixed by increasing the depth of the potential as the range is decreased, the n-n interaction is taken to be zero, and Thomas demonstrated that, as the range of the two-body force goes to zero, the three-body binding energy becomes infinite. This is simply understood from Wigner's argument[22] that in the deuteron one has $V+2T\approx 0$, while in the ^{3}H one has $2V+3T\approx -T$, which has no lower bound as the size of the system is decreased. While these arguments ignore short-range repulsion and the entire problem of saturation, they do give a qualitative understanding of the response to small changes in the potential parameters studied in the present work.

IV. CONCLUSION

In conclusion we have pointed out that the discrepancy in the measured value of Δ and the best theoretical estimates of the Coulomb energy difference E_C is not in contradiction with $|a_{nn}|<|a_{pp}|$. We emphasize that while it is certainly reasonable that one assume $r_{nn}=r_{pp}$ in searching for differences in a_{nn} and a_{pp}, if charge asymmetry is found, the more difficult determination of r_{nn} must be attempted. In addition we wish to point out that including charge dependence in the singlet N-N interaction ($V_{np}^{s}\neq V_{nn}$) adds some 0.1–0.25 MeV to the binding of the triton compared to the usual assumption of $V_{np}^{s}=V_{nn}=V_{pp}$ in most realistic potential calculations.[23–27] Thus it would seem that including charge dependence and possible charge-asymmetry effects in a realistic potential calculation might add as much as 0.4 MeV to the binding energy. This would move the estimates by Hennell and Delves[27] for the Reid potential even closer to the experimental binding energy and reduce further the amount of binding to be attributed to three-body forces.

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[1]E. M. Henley and D. H. Wilkinson, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Slaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North-Holland, Amsterdam, 1972), pp. 229–33, 242–3. This work references all recent experimental values for a_{nn} and gives an estimate of the best average value.

[2]H. P. Noyes, Annu. Rev. Nucl. Sci. 22, 465 (1972).

[3]H. P. Noyes and H. M. Lipinski, Phys. Rev. C 4, 995 (1972).

[4]E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).

[5]R. V. Reid, Ann. Phys. (N. Y.) 50, 411 (1968).

[6]M. S. Sher, P. Signell, and L. Heller, Ann. Phys. (N. Y.) 58, 1 (1970).

[7]K. Okamoto and C. Pask, Ann. Phys. (N. Y.) 68, 18 (1972).

[8]V. F. Kharchenko, N. M. Petrov, and S. A. Storozhenko, Nucl. Phys. A106, 464 (1968).

[9]A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49, 15 (1963).

[10]A. N. Mitra, G. L. Schrenk, and V. S. Bhasin, Ann. Phys. (N. Y.) 40, 357 (1966).

[11]J. Borysowicz and J. Dabrowski, Phys. Lett. 24B, 125 (1967).

[12]M. I. Haftel, Phys. Rev. C 7, 80 (1973).

[13]J. P. Lavene, S. K. Mukhopedhyay, and G. J. Stephenson, Jr., Phys. Rev. C 7, 968 (1973).

[14]J. Dabrowski, F. Fedorynski, P. Haensel, and M. Y. M. Hassan, Phys. Rev. C 4, 1985 (1971).

[15]A. I. Jaffe and A. S. Reiner, Phys. Lett. 26B, 717 (1968). This reference claims results that differ slightly from those presented in Ref. 11.

[16]R. van Wageningen, J. H. Stuivenberg, J. Bruinsma, and G. Erens, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Slaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North-Holland, Amsterdam, 1972), pp. 453-456.

[17]V. A. Alessandrini, C. A. Garcia, and H. Fanchiotti, Phys. Rev. 170, 939 (1968).

[18]K. Okamoto and C. Pask, Phys. Lett. 36B, 317 (1971).

[19]In the more recent work by E. M. Henley and T. E. Keliher [Nucl. Phys. A189, 632 (1972)] certain errors vitiate the numerical conclusions (private communication, E. M. Henley).

[20]H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. 8, 82 (1936). We wish to thank Professor Bethe for an illuminating conversation.

[21]L. H. Thomas, Phys. Rev. 47, 903 (1935).

[22]E. Wigner, Phys. Rev. 43, 252 (1933).

[23]R. A. Malfliet and J. A. Tjon, Ann. Phys. (N. Y.) 61, 425 (1970).

[24]J. A. Tjon, B. F. Gibson, and J. S. O'Connell, Phys. Rev. Lett. 25, 540 (1970).

[25]E. P. Harper, Y. E. Kim, and A. Tubis, Phys. Rev. Lett. 28, 1533 (1972).

[26]S. C. Bhatt, J. S. Levinger, and E. Harms, Phys. Lett. 40B, 23 (1972).

[27]M. A. Hennell and L. M. Delves, Phys. Lett. 40B, 20 (1972); and Nucl. Phys. A204, 552 (1973).