PHYSICAL REVIEW C VOLUME 8, NUMBER 3 SEPTEMBER 1973

New Expression for the Moment-of-Inertia Parameter

Nazakat Ullah and K. R. Sandhya Devi
Tata Institute of Fundamental Research, Bombay-5, India
(Received 9 May 1973)

A new expression for the moment-of-inertia parameter has been derived using the method of moments. The relation between the present expression and the Skyrme expression is given and its implications are discussed.

A fairly large number of calculations[1] have been performed to calculate the moment of inertia of the nucleus using Skyrme's expression.[2] This expression is given by

$$\frac{\hbar^2}{2\mathscr{I}_S}=A_S=\frac{\langle|HJ^2|\rangle-\langle|H|\rangle\langle|J^2|\rangle}{\langle|J^4|\rangle-\langle|J^2|\rangle^2}, \tag{1}$$

where $\mathscr{I}_S$ is the moment of inertia and $|\rangle$ is the intrinsic wave function, which is generated by a variational calculation.

The energies E_J of the various states in the band having a good total angular momentum J are given by

$$E_J=E_{0S}+A_SJ(J+1), \tag{2}$$

where the constant E_{0S} is given by

$$E_{0S}=\langle|H|\rangle-A_S\langle|J^2|\rangle. \tag{3}$$

The derivation of expression (1) is based on a variational problem[2] in which one minimizes the quantity $\langle|(H-E_0-AJ^2)^2|\rangle$ with respect to E_0 and A.

The purpose of the present note is to derive a new expression for the moment of inertia using the method of moments.[3,4] We shall also show the relation between the new expression and the Skyrme expression. A few remarks about the advantages of the present expression will be passed at the end of the manuscript.

Let us consider a normalized intrinsic wave function $|\rangle$, which is made up of several angular momentum states each occurring with a probability $a_J{}^2$. Then the first and second moments of the Hamiltonian H and the square of total angular momentum J^2 are given by

$$\langle|H|\rangle=\sum E_Ja_J{}^2, \tag{4a}$$

$$\langle|H^2|\rangle=\sum E_J{}^2a_J{}^2, \tag{4b}$$

$$\langle|J^2|\rangle=\sum J(J+1)a_J{}^2, \tag{5a}$$

$$\langle|J^4|\rangle=\sum[J(J+1)]^2a_J{}^2, \tag{5b}$$

where E_J's are the energies of the good J states.

One could also write the higher moments of H and J^2 in the same way, but since it is known[3] that the lower moments contain most of the useful information, we shall work with the set of Eqs. (4) and (5).

We next suppose that the energies E_J are given by

$$E_J=E_{0M}+A_MJ(J+1). \tag{6}$$

This expression is of the same form as expression (2), except that the parameters E_{0M}, A_M will be determined from Eqs. (4) and (5).

From expressions (4) and (5), we find that the inverse moment-of-inertia parameter A_M is given by

$$A_M=\left(\frac{\langle|H^2|\rangle-\langle|H|\rangle^2}{\langle|J^4|\rangle-\langle|J^2|\rangle^2}\right)^{1/2}, \tag{7}$$

and E_{0M} is given by expression (3) with A_S replaced by A_M.

Expression (7) provides a new expression for the moment-of-inertia parameter. We would first like to show the relation between A_M and A_S and discuss later if Eq. (7) yields better values of the excited energies E_J, or has any advantages over the Skyrme's expression.

Let us consider the correlation coefficient ρ between two quantities P and Q. It is given by

$$\rho=\frac{\langle PQ\rangle-\langle P\rangle\langle Q\rangle}{[(\langle P^2\rangle-\langle P\rangle^2)(\langle Q^2\rangle-\langle Q\rangle^2)]^{1/2}}. \tag{8}$$

If we now let P be the Hamiltonian and Q the square of total angular momentum J^2, then expressions (1), (7), and (8) give us

$$A_S=\rho A_M. \tag{9}$$

Since the correlation coefficient ρ is always less than or equal to 1, we find that the inverse moment-of-inertia parameter A_M using the method of moments will always be bigger or equal to the one given by the Skyrme expression.

We would now like to see if expression (7) yields better values of E_J than those one obtains from Skyrme's expression. Using expressions (2), (3), (6), and (9), we can show that the lowest levels calculated using the new expression will always

TABLE I. Energies E_J for the nuclei ^{12}C, ^{20}Ne, ^{28}Si, and ^{36}Ar using (1) exact projection, (2) the Skyrme expression for the moment of inertia, and (3) the present expression based on the method of moments. The zero of the energy is taken to be the total Hartree-Fock energy in each case.

	Energies E_J (MeV)		
J	Exact projection	Skyrme	Moments
	^{12}C		
0	−3.97	−4.02	−4.02
2	−0.73	−0.69	−0.69
4	7.13	7.08	−7.09
	^{20}Ne		
0	−3.06	−2.80	−2.81
2	−1.84	−1.77	−1.78
4	0.80	0.63	0.63
6	4.64	4.39	4.42
8	8.02	9.53	9.58
	^{28}Si		
0	−2.65	−2.17	−2.39
2	−1.95	−1.57	−1.72
4	−0.35	−0.17	−0.18
6	2.11	2.04	2.24
8	5.30	5.04	5.54
	^{36}Ar		
0	−2.37	−2.27	−2.28
2	−1.24	−1.30	−1.30
4	0.84	0.97	0.97
6	4.84	4.53	4.54
8	8.60	9.39	9.40

be closer to their exact values than the ones calculated using the Skyrme expression. As a matter of fact, it could be shown that the quantity $[\langle J^2\rangle - J(J+1)]$ determines up to what level the method of moments will give better values of E_J. To check this point further, we have used the deformed Hartree-Fock wave functions of Ripka[5] and calculated the values of E_J using both the Skyrme expression and the present expression. The results for nuclei ^{12}C, ^{20}Ne, ^{28}Si, and ^{36}Ar are shown in Table I. The first column of the table gives the values of J. In the second column we have given the exact projected energies.[5] The last two columns give the values of E_J which are found using Skyrme and the method of moments, respectively. These should be compared with the exact projected energies. All the energies are measured from E_{HF}, the total Hartree-Fock energy.[5] From Table I, we find that for all the nuclei and for all values of J, except one or two highest ones, the values of E_J calculated using the present expression are slightly closer to their exact values than the ones obtained using the Skyrme expression.

Next, we would like to add what the correlation coefficient tells us. It is obvious that if ρ is equal to 1, then both the expressions for the moment of inertia become the same. In this case the theory of probability[3] tells us that $H - \langle H\rangle$ must be proportional to $J^2 - \langle J^2\rangle$. When $\rho \neq 1$, then, in addition to $J^2 - \langle J^2\rangle$, one has other correction terms in $H - \langle H\rangle$ also. The magnitude of these correction terms will depend on the deviation of ρ from unity. For large deviations, one could employ one of the variable-moment-of-inertia models,[6] and calculate its parameters using the relations (4) and (9).

In the end we would like to remark that in some situations it will be easier to work with the new expression (7). This will happen when one uses the elegant quasispin formalism to handle such Hamiltonians as the pairing Hamiltonian or the Lipkin-Meshkov-Glick Hamiltonian.[7] In the quasispin formalism the Hamiltonian becomes a function of the quasi-angular-momentum operators and the wave functions belong to the usual $|jm\rangle$ representation. This makes it easier to evaluate matrices in the numerator of expression (7) rather than expression (1).

[1]S. K. M. Wong, M. I. Tienhaara, and L. E. H. Trainor, Phys. Rev. Lett. 21, 1819 (1968); I. Kelson and C.A. Levinson, Phys. Rev. 134, B269 (1964).

[2]T. H. R. Skyrme, Proc. Phys. Soc. Lond. A70, 433 (1957).

[3]M. G. Kendall, *The Advanced Theory of Statistics* (Charles Griffin, London, 1945), Vol. I.

[4]J. B. French, in *Nuclear Structure,* edited by A. Hossain, Harun-ar-Rashid, and M. Islam (North-Holland, Amsterdam, 1967).

[5]G. Ripka, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1968), Vol. 1.

[6]P. Holmberg and P. O. Lipas, Nucl. Phys. A117, 552 (1968); M. A. J. Mariscotti, G. Scharff-Goldhaber, and B. Buck, Phys. Rev. 178, 1864 (1969).

[7]N. Ullah, Nucl. Phys. A206, 332 (1973).