Investigation of Off-Shell Effects in Triton Binding-Energy Calculations with Nonlocal-Core Nucleon-Nucleon Potentials

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Using the Faddeev formalism, a study of the off-energy-shell variation of the triton binding energy (E_t) is made. The results are indicative of the insensitivity of E_t to the off-shell differences associated with the various nucleon-nucleon potentials considered, which are equivalent on the energy shell and vary mainly in the shape of their repulsive cores for $r \lesssim 0.5$ fm.

Various nucleon-nucleon (*NN*) potentials which are equivalent on the energy shell were found to lead to very small changes[1-3] and substantial changes[4-6] in the triton binding energy. Recently, Kharchenko, Shadchin, and Storozhenko[1] noted that this could be attributed rather to various degrees of coincidence of the compared potentials at intermediate and large internucleon distances, and not to the shape of the short-range core interaction. Usually, these investigations make use of finite and hard-core interactions[1,2] or separable ones.[6] However, different potentials should be studied with a variety of core interactions, so that model-independent conclusions may be reached.

Working within the framework of the Faddeev formalism, the present paper aims at doing this using *NN* potentials of a nonlocal-square-well (NLSW) form[7] for the core repulsion combined with an outside attraction of a local-square-well (LSW) form.[8] In particular, the NLSW-core interaction provides a generalization of the hard- and finite LSW-core interactions as well as the separable ones. Further, the use of a nonlocal-core interaction reflects the basic notion of nonlocality of the potential at short internucleon distances.[9]

As a preliminary to the study of the off-shell effects in the three-body system, we investigate in this paper the off-shell variation of the triton binding energy for a family of such potentials. These are taken, for simplicity, to be central, spin- and isospin-independent s-wave two-body interactions which match an average of singlet-triplet *NN* data.[10-13] They have the form[8]:

$$V(r,r') = \frac{V_0\beta^2}{\sinh\beta r_0}\begin{cases}\sinh\beta r\,\sinh\beta(r'-r_0) & \text{for} \quad 0\le r<r'\le r_0\\ \sinh\beta(r-r_0)\,\sinh\beta r' & \text{for} \quad 0\le r'<r\le r_0\end{cases}, \tag{1a}$$

$$V(r,r') = \delta(r-r') \quad V_1<0 \quad \text{for} \quad r_0\le r\le r_1, \tag{1b}$$

$$V(r,r') = \delta(r-r') \quad V_2=0 \quad \text{for} \quad r\le r_1, \tag{1c}$$

where r_0 is the range of the repulsive NLSW-core interaction, V_0 defines its strength ($V_0>0$), and β is a measure of the degree of nonlocality. The outside LSW has a range r_1 and depth V_1.

Various sets of parameters determined for the above potential are presented in Table I. The five potentials so given produce an average of singlet-triplet low-energy *NN* data[10-12] and s-wave phase shifts up to 340-MeV lab energy.[13] Such average *NN* data are given in Tables II and III, where we also list those for the hard-core square well of Kim and Tubis[10] and Fuda.[11] The two-nucleon scattering length a_0, effective range r_e, binding energy E_d, and s-wave phase shifts are calculated by a standard procedure[14] and are given elsewhere.[15]

It should be remarked that our potentials differ from each other only at quite short distances of order of 0.5 fm. Besides allowing us to isolate the off-shell effects associated mainly with the short-range repulsion, this choice of distance closely approximates the radius of the innermost core region ($r\lesssim0.7$ fm) of the *NN* interaction[16] and excludes the intermediate ($0.7\lesssim r\lesssim1.5$ fm) and outermost ($r\gtrsim1.5$ fm) regions, both of which are known to be dominated by local components.

Under the assumption of an average singlet-triplet s-wave *NN* interaction, the triton ground-state wave function would be completely symmetric. The justification of this averaging procedure

TABLE I. Parameters of NLSW-core *NN* potentials.

Potential	r_0 (fm)	V_0 (MeV)	$1/\beta$ (fm)	r_1 (fm)	V_1 (MeV)
$P1$	0.51	248.82	0.18	1.68	−63.5
$P2$	0.44	829.40	0.10	1.68	−63.5
$P3$	0.38	4105.5	0.04	1.68	−63.5
$P4$	0.35	99 528	0.01	1.68	−63.5
$P5$	0.34	2 571 140	0.001	1.68	−63.5

TABLE II. Low-energy *NN* parameters and triton binding energies.

Potential	a_0 (fm)	r_e (fm)	E_d (MeV)	E_t (MeV)
$P1$	10.8	2.29	−0.414	−8.12 ± 0.04
$P2$	10.3	2.25	−0.414	−8.14 ± 0.04
$P3$	10.5	2.23	−0.414	−8.16 ± 0.04
$P4$	10.1	2.21	−0.414	−8.11 ± 0.04
$P5$	10.8	2.24	−0.414	−8.11 ± 0.04
HCSW [a]	10.8	1.95	−0.435	−8.13 ± 0.04

[a] The HCSW has a core radius of 0.4 fm and an outside attraction which is essentially similar to our NLSW-core potentials, namely an outer radius of 1.737 fm and a depth of −63.85 MeV (Refs. 10 and 11).

in the triton is given by Blatt and Weisskopf[17] in calculations of its binding energy. For the completely symmetric three-nucleon bound-state problem (corresponding to the case of three identical bosons), the two-variable Faddeev equation is[18,19]:

$$\psi(p,q)=\frac{8}{\sqrt{3}\pi q}\int_0^\infty q_2dq_2\int_{|2q-q_2|/3^{1/2}}^{(2q+q_2)/3^{1/2}}p_2dp_2 \times\frac{t_0(p,p_1;S-q^2)}{p_2^{\,2}+q_2^{\,2}-S}\psi(p_2,q_2)\,, \tag{2}$$

where same notation as that of Kim[19] is used, with S being the total center-of-mass energy of the three-particle system, p is proportional to the relative momentum of a pair of particles, and q is proportional to the magnitude of the momentum of the third particle in the three-particle center-of-mass system. Further, $t_0(p,p_1;S-q^2)$ is the complete off-shell t matrix with $p_1^{\,2}=p_2^{\,2}+q_2^{\,2}-q^2$.

An analytic expression of the two-body s-wave off-shell t matrix was previously derived[8] by the author for the potential combination in Eq. (1). Since our off-shell t matrix[8] is nonseparable, we therefore proceed to solve the above two-variable Faddeev equation. This is not an easy task due to the variable limits of integration involved. The problem was recently tackled, however, by some authors.[12,19,20] In this paper, the approximate-product integration technique due to Kim[19] is employed. This method for solving the two-variable Faddeev equation has proved to be quite accurate and practical in a number of situations.[10,21]

The method is discussed in detail by Kim[19] and by Kim and Tubis,[10] and only a few details about it will be given here. After discretizing the p and q variables with N_p and N_q points, then using N_q-point Gaussian quadrature for the q_2 integration, the method is essentially based on expanding $\psi(p,q)$ in a set of linearly independent functions [which are taken in the present work to be of the same form as that used by Kim and Tubis[10]: Eq. (4.9) in their paper] for fixed q to deal with the p_2 integration. Once this is made, the difficulties associated with the variable limits of integration are avoided and the N_p points selected in the p variable are then taken over the entire range $(0,\infty)$. In such a way, the two-variable Faddeev Eq. (2) is directly transformed into an $N\times N$ ($N=N_p\times N_q$) matrix equation, the vanishing of whose determinant yields the triton ground-state energy E_t.

The values obtained for E_t using the five potentials in Table I are listed in the last column of Table II. At the bottom of Table II, we also quote the value of E_t obtained by Kim and Tubis[10] using the hard-core square well (HCSW). In getting our values, both N_p and N_q were varied until the solutions were found stable under further increase. As such, all values of E_t in Table II are obtained with $N_q=10$ and $N_p=8$.

The values of E_t so obtained are quite close to each other and indicate the insensitivity of E_t to the off-shell differences associated with the various potentials considered. The conclusion to be drawn from these results is that the triton binding energy is very weakly dependent on the form of the

TABLE III. Average s-wave phase shifts (δ_0 in radians) and experimental (Ref. 13) values for comparison.

E_{lab} (MeV)	Experimental δ_0 Singlet	Triplet	NLSW-core δ_0 $P1$	$P2$	$P3$	$P4$	$P5$	HCSW δ_0
40	0.808 ± 0.015	0.746 ± 0.002	1.02	1.03	1.03	1.02	1.02	0.97
180	0.296 ± 0.036	0.177 ± 0.005	0.219	0.223	0.223	0.221	0.219	0.13
260	0.102 ± 0.037	−0.019 ± 0.006	−0.033	−0.029	−0.029	−0.032	−0.034	−0.14
340	−0.059 ± 0.039	−0.192 ± 0.008	−0.221	−0.219	−0.219	−0.221	−0.224	−0.34

short-range NN repulsion. This is consistent with the results of Kharchenko, Shadchin, and Storozhenko[1] and Harper, Kim, and Tubis.[2]

The present results are of interest for the following reasons. First, they show that the above conclusion can still be drawn for more generalized core interactions. Second, they are "exact" in the sense of using an analytic expression for the off-shell t matrix and solving the two-variable Faddeev equation involved. Third, the off-shell effects due to the short-range NN repulsion are quite well isolated since the various potentials considered differ only for $r \leq 0.5$ fm and agree at intermediate and long distances for $r \gtrsim 0.5$ fm.

Calculations with more realistic NN potentials would, of course, be desirable. Our purpose, however, was mainly to explore the off-shell variation of E_t in a situation which is moderately realistic and yet allows main conclusions to be drawn.

We also need to investigate the off-shell variation of other properties of the three-particle system (not only the bound state but also the continuum one) which would be more sensitive to the short-range NN repulsion than the triton binding energy. The author is currently studying such possibilities.

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