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Nuclear Spreading Widths of Particle-Vibration $\frac{1}{2}^+$ Doorway Resonances in ^{207}Pb and ^{209}Pb

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The nuclear spreading widths of the ^{207}Pb and ^{209}Pb $\frac{1}{2}^+$ doorway resonances are calculated in a particle-vibration model. The hallway states are of the particle–two-phonon type. The fine structure in ^{207}Pb is essentially due to the availability of many hallways (some with large matrix elements) and their proximity to the doorway. The ^{207}Pb and ^{209}Pb $\frac{1}{2}^+$ doorway spreading widths are found to be in line with the experimental situation.

I. INTRODUCTION

In previous work[1,2] we have applied the weak-coupling (WC) particle-vibration (or particle-phonon) model to the calculation of the energies and escape widths of low-lying $\frac{1}{2}^+$ neutron resonances in the compound nuclei ^{207}Pb and ^{209}Pb. The only available $\frac{1}{2}^+$ particle-vibration doorway level Ψ_d with appreciable neutron escape width located below 1 MeV in each nucleus is $(2g_{9/2}\otimes 4^+)$. While these two states have similar calculated neutron escape widths and are in line with experiment,[3] the fine structure observed in this low-energy region differs drastically for the two nuclei. There is essentially no structure in ^{209}Pb while ^{207}Pb possesses many fine-structure resonances.[3] We have accounted for the fine-structure effects qualitatively in Ref. 2 by simply counting available one-particle–two-phonon background states, and in the present paper we give the results of a detailed quantitative study of the theoretical spreading widths of the $\frac{1}{2}^+$ doorway resonances.

In the particle-vibration doorway model the only levels that can be directly reached through the doorway are particle–two-vibration states.[4] We refer to these levels as hallway states $\{h\}$. These levels are the next stage in complication beyond the doorway and are assumed to provide the mechanism for forming the complex compound nucleus levels (we assume that these are one-particle–three-vibration etc.) observed in fine-structure experiments. While several authors[5] have studied the energies, electromagnetic transition rates, etc. of particle-multiphonon states in the bound region, there has to our knowledge been little attempt to apply the basic WC collective-excitation ideas to the continuum. The general theory will be given in Sec. II. Section III will present the results of our application to ^{207,209}Pb, and a discussion and conclusions will comprise Sec. IV.

II. THEORY

Feshbach, Kerman, and Lemmer[6] have shown that the spreading width $\Gamma_d^\downarrow$ (or width for decay into more complex states Φ_q) of a doorway Ψ_d at energy E_d is given by

$$\Gamma_d^\downarrow = I\sum_q \frac{|\langle\Phi_q|qHd|\Psi_d\rangle|^2}{(E_d-E_q)^2+\frac{1}{4}I^2}, \tag{1}$$

where the coupling interaction is qHd (q and d are the usual Feshbach projection operators), the energy of the q'th complicated level is $E_{q'}$, and the sum is over these complex states. The symbol I represents the Lorentzian energy-averaging interval used in converting the fine structure to intermediate structure.

We assume that the sum in Eq. (1) over complex states is restricted by the coupling interaction to only the hallway states $\{h\}$ defined in Sec. I and interpret I as the width of the hth hallway state Φ_h due to nuclear interactions. A more descriptive notation is to replace I by the symbol Γ_h. If all states Φ_h have a similar basic structure then Γ_h may be considered as a constant and Eq. (1) becomes

$$\Gamma_d^\downarrow = \Gamma_h\sum_h \frac{|\langle\Phi_h|hHd|\Psi_d\rangle|^2}{(E_d-E_h)^2+\frac{1}{4}\Gamma_h^2}, \tag{2}$$

where hHd is the coupling interaction (h being the operator that projects onto hallway states).[7] The particle-phonon model is used in this work so that

$$|\Psi_d\rangle = |n'\,l'\,j';N'=1,\,R';IM\rangle, \tag{3}$$

where $n'l'j'$ are the single-particle quantum numbers, $N'=1$ indicates one phonon with angular momentum R', and IM are the total angular momentum and z projection of the particle-phonon doorway state. The hth hallway state is of the particle-two-phonon type and is written as

$$|\Phi_h\rangle = |[nlj; N=2, R(J_1 J_2); IM]_h\rangle, \qquad (4)$$

where nlj are the single-particle quantum numbers, $N=2$ indicates two phonons with individual angular momenta J_1 and J_2 and total angular momentum R, and IM are the same as in Eq. (3).

The coupling interaction is taken as

$$hHd = r\frac{dV(r)}{dr}\sum_{\lambda\mu}\alpha_{\lambda\mu}Y_{\lambda\mu}, \qquad (5)$$

which was discussed by Mottelson.[8] The symbol $\alpha_{\lambda\mu}$ represents collective coordinates of the nuclear surface, $Y_{\lambda\mu}$ is a spherical harmonic, r is the particle coordinate, and $V(r)$ is taken as a Woods-Saxon potential. In many of the calculations[5] of matrix elements between the types of states represented by Eqs. (3) and (4) the quantity

$$-r\frac{dV(r)}{dr} = k \qquad (6)$$

is treated as a parameter. In the present work, however, the full radial dependence will be exploited and radial integrals calculated.

In second quantization Eq. (5) may be written as

$$hHd = -k\sum_{\lambda\mu}\left(\frac{\hbar\omega_\lambda}{2C_\lambda}\right)^{1/2}[b_{\lambda,\mu} + (-1)^\mu b^\dagger_{\lambda,-\mu}]\,Y_{\lambda\mu}, \qquad (7)$$

where $(\hbar\omega_\lambda/2C_\lambda)^{1/2}$ is a one-phonon vibrational amplitude, and b and $b^\dagger$ are phonon annihilation and creation operators, respectively. Inserting Eqs. (3), (4), and (7) into Eq. (2) gives

$$\Gamma_d^\downarrow = \Gamma_h\sum_h\Bigg[\delta_{J_2R'}(1+\delta_{J_1J_2}\delta_{R,\,\mathrm{even}})^{1/2}\hat{R}\langle n'l'j'|k|nlj\rangle \times\sum_\lambda \delta_{J_1\lambda}\left(\frac{\hbar\omega_\lambda}{2C_\lambda}\right)^{1/2}\langle l'j'\|Y_\lambda\|lj\rangle\begin{Bmatrix} I & j & R\\ \lambda & R' & j'\end{Bmatrix}\Bigg]_h^2\Bigg/\left[(E_d-E_h)^2+\tfrac{1}{4}\Gamma_h^{\,2}\right], \qquad (8)$$

where the subscript h in the numerator is a reminder of the particular state Φ_h being considered. The notation in the numerator requires some explanation. Specifically, $\delta_{J_2R'}$ is a shorthand notation indicating that the phonon state of angular momentum J_2 must be exaxtly identical to the phonon state of angular momentum R', i.e., the parities, energies, wave functions, etc. of the two states must be the same. Similarly $\delta_{J_1J_2}$ is zero unless the phonon state of angular momentum J_1 is exactly the same as the phonon state of angular momentum J_2. If $J_1=J_2$ then the angular momentum R must be even. The notation $\hat{R}$ means $(2R+1)^{1/2}$, and $\langle n'l'j'|k|nlj\rangle$ is a radial matrix element. The multipole order λ in the sum is determined by $\delta_{J_1\lambda'}$, the factor $\langle l'j'\|Y_\lambda\|lj\rangle$ is a reduced matrix element, and $\{\ \}$ is a $6j$ symbol. We note that the single particle need not be the same in Φ_h as in Ψ_d. Eliminating λ from Eq. (8) we finally have

$$\Gamma_\alpha^\downarrow = \Gamma_h\sum_h\Bigg[\delta_{J_2R'}(1+\delta_{J_1J_2}\delta_{R,\,\mathrm{even}})^{1/2}\hat{R}\langle n'l'j'|k|nlj\rangle\left(\frac{\hbar\omega_{J_1}}{2C_{J_1}}\right)^{1/2} \times\langle l'j'\|Y_{J_1}\|lj\rangle\begin{Bmatrix} I & j & R\\ J_1 & R' & j'\end{Bmatrix}\Bigg]_h^2\Bigg/\left[(E_d-E_h)^2+\tfrac{1}{4}\Gamma_h^{\,2}\right]. \qquad (9)$$

Equation (9) may be considered qualitatively as a sum of the square of overlap matrix elements between the doorway and the hallway states, where each matrix element squared is weighted off the tail of the Lorentzian nuclear spread of the hallway state.[9]

III. RESULTS

All possible combinations of one-particle–two-phonon states of the type of Eq. (4) are considered for the two nuclei in order to select the hallway states that can contribute to the spreading width Eq. (9). The energy, spin, and parity of each vibration are obtained from experiment.[10] The vibrational amplitudes $(\hbar\omega_{J_1}/2C_{J_1})^{1/2}$ can be directly obtained from the transition strength G_l (or β_l) which is extracted from the cross section for vibrational excitation in inelastic scattering experiments. The transition strengths of the "very weak" states in ^{206}Pb have not been extracted from experiment in the literature. For these states an arbitrary small value of ~0.5 is assumed for G_l (this being the lowest observed value in ^{206}Pb). We consider for the present purposes excitation energies up to ≈4.6 MeV in both the target nuclei ^{206}Pb and ^{208}Pb. The excited-level information is given in Table I. We emphasize that microscopic details are not known for many of the states listed in Table I. It is therefore an assumption in our model that these levels are all of the vibrational type.

The single neutron states used are $2g_{9/2}$, $1i_{11/2}$,

$3d_{5/2}$, $1j_{15/2}$, $4s_{1/2}$, $2g_{7/2}$, and $3d_{3/2}$. For ^{207}Pb $3p_{1/2}$ is also used. The energy of the $1j_{15/2}$ neutron state is not well known from (d, p) experiments. The lowest $\frac{15}{2}^-$ ^{209}Pb state at 1.44 MeV has a spectroscopic factor[11] of only about 0.46 single-particle units. The matrix element which involves the $1j_{15/2}$ state must therefore be multiplied by the square root of this number to calculate the corresponding contribution to the spreading width Eq. (9). However, hallways involving $1j_{15/2}$ are too far removed from the doorway of interest to be of importance. Proper Woods-Saxon potentials $V(r)$ with spin-orbit coupling were used to generate neutron radial wave functions and the quantity k [Eq. (6)] in both ^{207}Pb and ^{209}Pb. Radial integrals were performed using the code ABACUS.[12]

The doorway state of interest is [in the notation of Eq. (3)]

$$|\Psi_d\rangle = |\,2g_{9/2};\, N'=1,\, 4^+;\, \tfrac{1}{2}^+ M\rangle\,, \tag{10}$$

where the 4^+ state is the one labeled $4^+(1)$ in Table I. In ^{207}Pb this resonance is theoretically[2] (experimentally[3]) at 361 keV (~430 keV) above threshold while in ^{209}Pb the resonance is theoretically[2] (experimentally[3]) at 365 keV (500 keV) above threshold.

TABLE I. Available one-phonon states.

	E (MeV) [a]	J_I^π [b]	G_I (W.u.) [c]
^{206}Pb	0.803	2^+	6.2
	1.464	2^+	(0.5)
	1.682	4^+	2.6
	1.785	2^+	(0.5)
	1.997	4^+	0.8
	2.149	2^+	(0.5)
	2.197	7^-	1.7
	2.421	2^+	(0.5)
	2.526	3^-	(0.5)
	2.650	9^-	(0.5)
	2.653	3^-	35.0
	2.776	$5^-(1)$	(0.5)
	2.924	4^+	(0.5)
	3.010	$5^-(2)$	0.7
	3.128	6^+	(0.5)
	3.253	6^+	(0.5)
	3.404	$5^-(3)$	0.85
	3.452	3^-	0.5
	3.560	$5^-(4)$	1.0
	3.776	$5^-(5)$	9.3
	4.128	2^+	6.2
	4.368	$4^+(1)$	15.0 [d]
	4.386	6^+	6.2
	4.600	8^+	3.8
^{208}Pb	2.614	3^-	39.5
	3.198	5^-	14.0
	3.709	5^-	1.85
	4.070	2^+	8.00
	4.305	4^+	15.00 [d]
	4.405	6^+	5.5
	4.600	8^+	4.00

[a] Vibrational energy $\hbar\omega_{J_1}$ [Eq. (9)].

[b] The state $4^+(1)$ is the doorway constiuent. The various 5^- states are so labeled for later reference.

[c] The value (0.5) is arbitrarily assumed for those "very weak" levels whose G_I value has not been extracted from experiment in the literature. See text for details.

[d] The listed value of 15 W.u. for G_I represents the value used by the present authors in earlier work (Refs. 1 and 2), and also by Auerbach and Stein, Phys. Lett., 25B, 628 (1969).

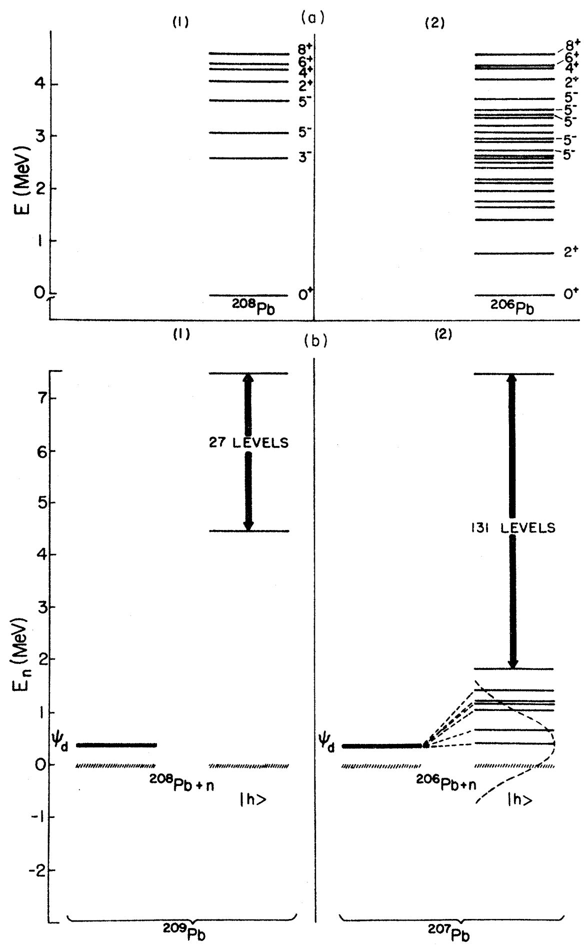

FIG. 1. (a) Parts (1) and (2), respectively, present the target ^{208}Pb and ^{206}Pb excited states listed in Table I. For ^{206}Pb, spins and parities are labeled for those states which are considered in constructing the relevant $\frac{1}{2}^+$ hallways (and doorways). (b) Parts (1) and (2) display the $(2g_{9/2}\otimes 4^+)$ doorway (thick line) and the available hallways in the compound nuclei ^{209}Pb and ^{207}Pb, respectively. A schematic representation of the effect of a Lorentzian spreading of the doorway is shown.

TABLE II. Percent contribution of hallways to $\Gamma_d^{\downarrow}$ of the ^{207}Pb, doorway $\Psi_d = (2g_{9/2} \otimes 4^+)_{1/2+}$, $E_d = 0.361$ MeV (above neutron threshold).

Hallway configuration $[(j_p \otimes 4^+) \otimes J_2]$	E_h (MeV) [a]	% contribution [b] to $\Gamma_d^{\downarrow}$	$\Gamma_d^{\downarrow}$ (keV) Theory [Eq. (9)] Γ_h (keV) 250	500	750	Experiment (Ref. 13)
$[(3p_{1/2} \otimes 4^+) \otimes 5_1^-]$ [c]	0.404	18				
$[(3p_{1/2} \otimes 4^+) \otimes 5_2^-]$	0.638	11				
$[(3p_{1/2} \otimes 4^+) \otimes 5_3^-]$	1.032	5	139	174	207	≈190
$[(2g_{9/2} \otimes 4^+) \otimes 2^+]$	1.164	43				
$[(3p_{1/2} \otimes 4^+) \otimes 5_4^-]$	1.188	3				
$[(3p_{1/2} \otimes 4^+) \otimes 5_5^-]$	1.404	20				

[a] Above neutron threshold.
[b] Corresponding $\Gamma_d^{\downarrow}$ result for $\Gamma_h = 500$ keV.
[c] The subscript for the various 5^- levels refers to the levels listed in Table I. Thus 5_1^- means $5^-(1)$, 5_2^- means $5^-(2)$, etc.

Figure 1(a) shows the energies and spins of the vibrations in ^{208}Pb and ^{206}Pb. In Fig. 1(b) are also presented the 1p-two-phonon states in compound nuclei ^{209}Pb and ^{207}Pb. For ^{209}Pb all the hallway states are positioned far away in energy and the lowest-energy hallway state $[(2g_{9/2} \otimes 4^+) \otimes 2^+]$ is at 4.4 MeV. Clearly this and other hallway states are too far away to contribute to $\Gamma_d^{\downarrow}$. In our model therefore we assign $\Gamma_d^{\downarrow} = 0$ for ^{209}Pb. This is consistent with the experimental determination of $\Gamma_d^{\downarrow} \ll \Gamma_d^{\downarrow}(=58$ keV) of Newson.[13] In contrast the ^{207}Pb hallways extend from about 400 keV to 7.5 MeV. Of these hallways only those that are between about 0 and 1.4 MeV (1 MeV above E_d) neutron energy are important in evaluating the spreading width. The effect of Lorentzian distribution is schematically shown in the lower right part of Fig. 1(b). A total of six hallway states are included within the Lorentzian tail and only these are considered in calculating expression (9). The calculated widths [Eq. (9)] are given in Table II (columns 4–6) along with the experimentally determined value (column 7). The six basic hallway states referred to earlier are enumerated in column 1 and their energies E_h are given in column 2. Since the width Γ_h is not known, we have performed the calculations for the reasonable values of 0.25, 0.50, and 0.75 MeV. (As a comparison the value of Γ_h usually taken for 3p-2h states is less than 1 MeV, 0.5 MeV being the number most often quoted.)[14]

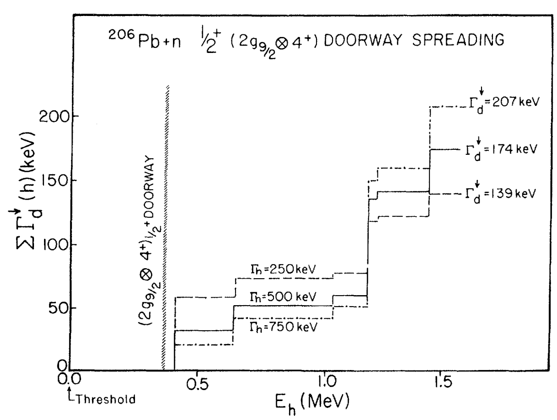

FIG. 2. Histogram plots of $\sum\Gamma_d^{\downarrow}(h)$ as function of hallway energy for three different values of $\Gamma_h = 250$, 500, and 750 keV are shown. The hatched bar represents the $\frac{1}{2}^+$ doorway. The number for each histogram at the saturation level represents the corresponding spreading width $\Gamma_d^{\downarrow} = \sum\Gamma_d^{\downarrow}(h)$.

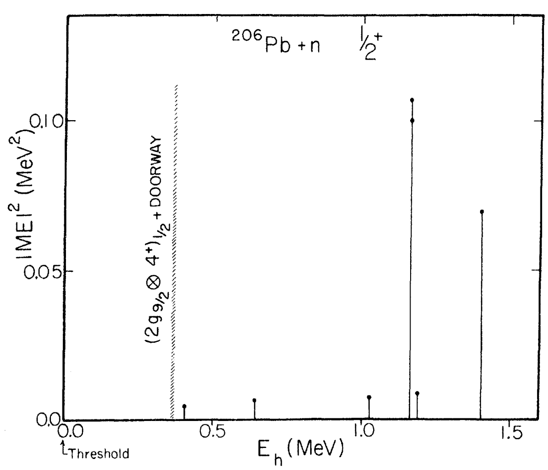

FIG. 3. Calculated matrix elements squared for possible hallway states vs energy. The hatched bar represents the doorway. Dots on one of the vertical bars indicate degenerate levels with different intermediate angular momenta.

IV. DISCUSSION AND CONCLUSIONS

We can see from Table II that for ^{207}Pb the spreading width is in excellent agreement with experiment. We note that there is also somewhat of a dependence on Γ_h and hence the Lorentzian distribution. This is clearly demonstrated in Fig. 2 where the actual contribution due to each of the hallway configurations is plotted in the form of a histogram. We note that the following hallways contribute most to the calculated spreading width: $[(3p_{1/2}\otimes 4^+)\otimes 5_1^-]$, $[(2g_{9/2}\otimes 4^+)\otimes 2^+]$, and $[(3p_{1/2}\otimes 4^+)\otimes 5_5^-]$. The first is important due to its proximity to the doorway and the other two have large matrix elements in addition to being near the doorway. The results for $\Gamma_d^{\downarrow}$ corresponding to the larger values 500 and 750 keV have better agreement than 250 keV.

The mixing in ^{207}Pb may be expected because the target ^{206}Pb has two holes in its ground state. The target ^{208}Pb on the other hand is doubly magic. The ^{206}Pb low-lying states have been explained by True[15] in terms of two neutron hole configurations.

There are over 100 other 1p–two-phonon hallways in ^{207}Pb [cf. Fig. 1(b)] but these are too far away to be of use. These hallways could, however, contribute to the spreading of higher-energy doorways (cf. Ref. 2) than the one considered here. The nucleus ^{209}Pb has 27 hallway states above 4.4 MeV. States based on the strong 3^- vibration, e.g. $[(1j_{15/2}\otimes 4^+)\otimes 3^-]$, occur at too high energies to be important. In Fig. 1(b) we note that the density of hallway states in ^{207}Pb is much greater than in ^{209}Pb.

The squares of the calculated matrix elements for each hallway configuration are displayed in Fig. 3. The hatched bar represents Ψ_d. One of the bars has more than one dot to indicate degenerate levels having different intermediate angular momenta. The experimentally observed fine structure in ^{207}Pb lies between 0.0- and 1-MeV neutron energy whereas the 1p–two-phonon states range from about 0.4–1.4 MeV [cf. Figs. 1(a) and 3] above threshold. The hallway configuration $[(2g_{9/2}\otimes 4^+)\otimes 2^+]$ has a large matrix element because of its similarity to Ψ_d.

In conclusion, the particle-vibration model accounts for the drastic difference in the spreading of the ^{207}Pb and ^{209}Pb $\frac{1}{2}^+$ doorways. In particular the particle–two-phonon description for the hallway states allows for a quantitative evaluation of the spreading widths for ^{207}Pb.

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