PHYSICAL REVIEW C VOLUME 8, NUMBER 3 SEPTEMBER 1973

Effect of Fine Structure on Nucleon Transfer to Analog States *

C. M. Vincent
University of Pittsburgh, Pittsburgh, Pennsylvania 15213

H. T. Fortune
University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 5 December 1972)

We extend our previous distorted-wave Born-approximation treatment of reactions of the form $A(d,p)B$ (in which B is an unbound state) to include the case where B is (a) dissolved into a fine structure of resonances that may overlap strongly, and (b) unstable against emission of other particles in addition to the transferred particle n. We assume that the reaction is strongly peripheral and goes through the $n+A$ channel of B via the np interaction. We discuss the cross section for an experiment in which only particle p is detected, while the decay products of B are not observed or identified. For an isobaric-analog resonance we find that the cross section is proportional to $\Gamma_n \cos 2(\delta_N+\phi)$, where Γ_n is the n escape width, δ_N is the nuclear background phase shift, and ϕ is the asymmetry phase. The cross section is entirely determined by the gross-structure parameters of the resonance, and does not depend on the nature of the fine structure. For the reaction $^{92}\text{Mo}(d,n)$ to $d_{5/2}$ (8.40-MeV), $s_{1/2}$ (9.33-MeV), and $d_{3/2}$ (9.91-MeV) analog states, we find that the effect of $\delta_N+\phi$ is negligible.

I. INTRODUCTION

In a previous paper[1] we described the distorted-wave Born approximation (DWBA) for the reaction $A(d,pn)A$. We assumed that only the particle p was detected, and that the reaction proceeded by direct n transfer to an intermediate state B, which could decay only by n emission.[2] For a given angular momentum transfer l,j, the expression derived for the DWBA cross section was[3]

$$\frac{d\sigma^{\text{DWBA}}(d,pn)}{d\Omega_p}=\frac{2\mu}{\pi\hbar^2}\int_{E_1}^{E_2} dE_n k_n \sin^2\delta_{lj} \times\left(\frac{d\sigma^F}{d\Omega_n}\right)_{\text{peak}}. \quad (1)$$

Here δ_{lj} is the phase shift for $A+n$ elastic scattering, $E_n=\hbar^2 k_n^2/2\mu$ is the energy of relative motion of A and n, and k_n is the corresponding wave number. The quantity $(d\sigma^F/d\Omega_n)_{\text{peak}}$, which depends smoothly on E_n, is the cross section that would result from a DWBA calculation in which the normalized bound final state is replaced by a resonant form factor that is asymptotically equal to an irregular Coulomb function.

The aim of the present work is to generalize this result to the family of reactions

$$d+A\to p+B,\quad B\to c+C \quad (2)$$

in which the resonant state B has fine structure and may decay into any possible fragments C and c. An important special case is where $c=n$ and C is an excited state of A. We shall retain the notation and conventions of Ref. 1. We again specialize our treatment to the case where only particle p is detected, and the decay products C and c of B are neither identified nor observed. The cross section of interest is therefore

$$\frac{d\sigma}{d\Omega_p}=\sum_c \frac{d\sigma(d,pc)}{d\Omega_p}. \quad (3)$$

Our approach will be as phenomenological as possible; that is, we shall try to identify those parameters of the resonance B that one might hope to pin down by analyzing experimental results. However, we shall avoid any attempt to relate the resonance parameters to more fundamental models of the nucleus. Phenomenological theories of analog resonances usually describe only the energy-averaged transition amplitude. Because we wish to treat states B that have overlapping fine structure, we must find some way to include the contribution of the fluctuations of the transition amplitude about its average. We must also include the contributions of all open channels c, as indicated in Eq. (3). Our final result, Eq. (21), takes both effects into account, and yet is very simple indeed. Thus the analysis of transfer to analog states is placed on a firm basis, and remains simple enough for routine use.

Lipperheide[4] has given a plane-wave description of the reaction which has some points of contact with the present work. He shows that in the plane-wave Born approximation the transfer cross section is proportional to the so-called off-shell total cross section for $A+n$ scattering. This relation is irretrievably destroyed when strong distortion effects are included. Our analysis, however, will exploit the strong peripherality that is typical of transfer reactions with good angular momentum matching and with strong absorption in initial and final channels. If the reaction is peripheral, only *on-shell* information regarding the $A+n$ scattering is of importance.

Levin[5] has given a distorted-wave description of the reaction to a single isolated resonance, including the possibility of alternate decay channels. However, he does not treat the fluctuation contribution.

II. DWBA FOR THE REACTION (d,pc)

We neglect all effects due to the identity of particles. The exact transition amplitude for the reaction (2) is

$$T_{d,pc}=\langle\psi_{cp}^{-}|V_i|\phi_d\rangle\,. \tag{4}$$

Here ψ_{cp}^{-} is the complete scattering state corresponding to $c+p$ incident on C (with incoming scattered waves), while ϕ_d is a plane wave representing d incident on A, and V_i is the corresponding ("prior") interaction. By using the Gell-Mann-Goldberger relation we can single out the interaction V_{np}, so that Eq. (4) becomes

$$T_{d,pc}=\langle\Phi_{Bc}^{-}\chi_p^{-}|V_n+\overline{V}_p-V_{np}|\phi_d\rangle$$
$$+\langle\Phi_{Bc}^{-}\chi_p^{-}|V_{np}+V_p-\overline{V}_p|\psi_d^{+}\rangle\,. \tag{5}$$

Here ψ_d^{+} is a complete scattering state with d incident on A, and Φ_{Bc}^{-} is a complete scattering state with c incident on C. (In a single-nucleon transfer reaction to a bound state, Φ_{Bc}^{-} would be replaced by the bound state of the final nucleus B.) The proton distorted wave χ_p^{-} is generated from the optical potential $\overline{V}_p$. The total interaction of p (or n) with all the particles of A is denoted by V_p (or V_n).

The first term of Eq. (5) can be shown to vanish identically. One has

$$V_n+\overline{V}_p-V_{np}=\overline{H}-H_{0d}\,,$$

where

$$(\overline{H}-E)\Phi_{Bc}^{-}\chi_p^{-}=0\,,\quad (H_{0d}-E)\phi_d=0\,.$$

At large distances, $\Phi_{Bc}^{-}\chi_p^{-}$ vanishes in the d channel, while ϕ_d vanishes in all other channels. Therefore, no surface term is introduced when $\overline{H}$ is taken to act to the left. For the same reason, the scalar product $\langle\Phi_{Bc}^{-}\chi_p^{-}|\phi_d\rangle$ is finite. The result then follows immediately from the fact that both wave functions belong to the same energy.

The second term of the exact Eq. (5) shows clearly that *all* the rapid dependence of the amplitude on E_B (the energy of B) is contained in Φ_{Bc}^{-}. The distorted wave χ_p^{-} does depend on E_B, but only slowly. The potentials and the function ψ_d^{+} do not depend on E_B at all.

In the second term of Eq. (5) we replace ψ_d^{+} by $\Phi_A\chi_d^{+}$ (where Φ_A is the target ground state and χ_d^{+} is a distorted elastic scattering deuteron wave function) and neglect the contribution of $V_p-\overline{V}_p$. The result is

$$T_{d,pc}^{\mathrm{DWBA}}=\langle\Phi_{Bc}^{-}\chi_p^{-}|V_{np}|\Phi_A\chi_d^{+}\rangle\,. \tag{6}$$

The remainder of our study will rely on Eq. (6), which has also been derived by Levin.[5] Figure 1 gives a graphical representation of this amplitude. The broken lines represent the effects of the p and d distortions. The three diagrams obtained by omitting one or both of these are included in Eq. (6). The vertex $t_{d,np}$ represents the breakup of the deuteron into n and p. The vertex $t_{An,Cc}$ represents the intermediate state B, which is formed in the $A+n$ channel and decays in the $C+c$ channel.

In the case of unbound states, rescattering between n and p is a potentially important correction to the DWBA. This effect has recently been estimated by Friedman.[6] From his analysis, he concludes that the correction to the cross section might be as large as 80% in the typical case of $^{16}O(d,p)^{17}O$ to the 5.08-MeV state. However, most of his correction depends smoothly on energy, and only contributes to the subtracted background. Friedman overestimates the remaining correction, which is proportional to the DWBA cross section, because he does not take into account the fact that typically only a very small range of p energies is accepted by the detectors, and within this energy range p is moving much faster than n, so that rescattering is unlikely.

We have not yet assumed that B is a narrow resonance. Equation (6) includes a contribution from the incident plane wave in Φ_{Bc}^{-}, corresponding (in the case $c=n$), to "direct breakup" as defined by Noble.[7]

III. FORM FACTORS FOR COMPETING DECAYS OF B

The transition amplitude (6) can be evaluated by integrating first with respect to Ω_A, the variables

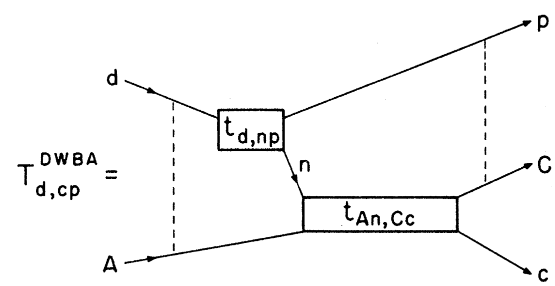

FIG. 1. Graphical representation of Eq. (6).

contained in Φ_A. The result can be expressed in terms of the form factor or channel wave function:

$$\chi_n^c(r_n) = \int d\Omega_A \Phi_A^* \Phi_{Bc}^- . \qquad (7)$$

Everywhere external to the target nucleus, the form factor χ_n^c can be expressed in terms of the asymptotic scattering matrix elements as

$$\chi_n^c = S_{nc}^* \left(\frac{\mu_n k_n}{\mu_c k_c}\right)^{1/2} H_n^- - \delta_{cn} H_n^* . \qquad (8)$$

Here the *nuclear* scattering matrix element is denoted by $S_{cc'}$. The symbols k_c, μ_c, and r_c, respectively, represent the channel wave number, reduced mass, and radius, while $H^+(H^-)$ is an outgoing (incoming) Coulomb wave function defined by

$$H_c^{\pm} = \frac{G_c \pm i F_c}{k_c r_c} , \qquad (9)$$

where G_c and F_c are Coulomb functions as defined by Messiah.[8]

The form factor $\chi_n^c(r_n)$ is defined for $0 \leq r_n < \infty$. It satisfies the equation

$$(E - K_n - V_n^{\rm Coul})\chi_n^c = \langle \Phi_A | V_n^{\rm nuc} | \Phi_{Bc}^- \rangle ,$$

where $V_n^{\rm nuc}$ and $V_n^{\rm Coul}$ are, respectively, the nuclear and Coulomb parts of V_n. The right-hand side of this equation vanishes for $r_n > R_A + R_0$, where R_A is the radius of the target and R_0 is the range of the two-nucleon force. Therefore at $r_n = R_A + R_0$, χ_n^c is already equal to its asymptotic form (8). It is worth noting that the rapidity of approach to the external form is independent of the channel c, and unaffected by the unbound nature of Φ_{Bc}^-. If there is strong absorption in the distorted waves and good angular momentum matching, the main contribution to the transition amplitude comes from the nuclear surface. This is fortunate, because this is just the region in which χ_n^c becomes external. Therefore only the external form of χ_n^c need be well known.

It was shown in Ref. 1 that a form factor equal to F_n produces a very small DWBA amplitude. This remains true even when the transferred particle is subject to strong Coulomb forces. Both for ^{16}O(d, p) and for ^{92}Mo(d, n) we have found that the F_n amplitude is at most 4% of the resonant amplitude. Accordingly, it is an excellent approximation[9] to subtract a multiple of F_n from χ_n^c, leaving only the part, $\overline{\chi}_n^c$, that is proportional to G_n in the exterior. The resulting form factor is

$$\overline{\chi}_n^c = -2it_{nc}^* \left(\frac{\mu_n k_n}{\mu_c k_c}\right)^{1/2} \chi_n^{Gc} , \qquad (10)$$

where χ_n^{Gc} is equal to G_n in the exterior. Here the nuclear transition amplitude $t_{cc'}$ (analogous to $e^{i\delta} \times \sin\delta$) is defined by

$$S_{cc'} = \delta_{cc'} + 2it_{cc'} . \qquad (11)$$

Equation (10) separates the form factor into a function χ_n^{Gc} with known *scattering* normalization and a number that depends on the spectroscopy of Φ_{Bc}^-. This separation is similar in spirit to what is done in defining the spectroscopic factor of a bound state. It is reasonable to hope that (just as for bound-state form factors) χ_n^{Gc} can be simply calculated without introducing much error. A suitable method is to solve the Schrödinger equation for a particle in a Woods-Saxon well with realistic geometrical parameters and a depth chosen to produce a resonance at the observed energy. In the bound-state case, this procedure is known as the "well-depth method."

We emphasize that the DWBA cross section depends on the *nuclear* transition amplitude t_{nc}, since this is the quantity that appears in Eq. (10). The total S matrix, $S^{\rm total}$, is related to the nuclear S matrix, S, by

$$S_{cc'}^{\rm total} = e^{i\sigma_c} S_{cc'} e^{i\sigma_{c'}} ,$$

where σ_c is the Coulomb phase shift.

IV. DWBA CROSS SECTION

In order to derive the cross section, we can directly apply Eq. (1) to the transition amplitude (6) with the form factor (10). It is only necessary to replace $\sin\delta_{lj}$ by $t_{nc}^*(\mu_n k_n/\mu_c k_c)^{1/2}$. For the cross section corresponding to events in which B has an energy E_B between E_1 and E_2, and the decay products of B are not observed or identified, the result is

$$\frac{d\sigma}{d\Omega_p} = \sum_c \frac{d\sigma(d, pc)}{d\Omega_p} = \frac{2}{\pi\hbar^2} \sum_c \sum_{lj} \int_{E_1}^{E_2} dE_B \mu_c k_c |t_{cn}(E_B)|^2 \frac{\mu_n k_n}{\mu_c k_c} \left(\frac{d\sigma_c^F}{d\Omega_p}\right)_{\rm peak} . \qquad (12)$$

Here $(d\sigma_c^F/d\Omega_p)_{\rm peak}$ is the (fictitious) cross section that would be calculated by an ordinary DWBA program using χ_n^{Gc} (asymptotically equal to G_n) in place of a normalized bound-state form factor. l and j are the angular momentum labels of channel n.

The slowly convergent radial integrals that arise in the calculation of $(d\sigma_c^F/d\Omega_p)_{\rm peak}$ can be evaluated by the contour-integration method described in Ref. 1. The only remaining problems are the integration over E_B and the summation over channels c.

V. TREATMENT OF FLUCTUATIONS AND ALTERNATIVE DECAY CHANNELS

The interesting energy dependence of the integrand of Eq. (12) is contained in the factor $|t_{cn}(E_B)|^2$. When B is an analog resonance, this factor is abnormally large in a range of E_B near the resonance energy. Within this range it may fluctuate rapidly, because of the presence of fine-structure resonances. Since conventional theories[10] of analog states show how to calculate only the energy-averaged t_{cn}, the problem of including the fluctuation contribution must now be solved.

The summation over c can be used to simplify Eq. (12). To do this, we must assume that the form factors χ_n^{Gc} can be replaced by a single form factor χ_n^G (independent of c). The χ_n^{Gc} may, of course, differ greatly in the nuclear interior, but they are guaranteed by definition to be equal outside the nuclear interaction region. The approximation will thus be a good one if the reaction is strongly peripheral, that is, if the nuclear interior makes only a small contribution to Eq. (6). This approximation ought to be used as far as possible, because it leads to major simplifications of procedure, as we see below.

The $(d\sigma_c^F/d\Omega_p)_{\text{peak}}$ can then be replaced by a single quantity $(d\sigma^F/d\Omega_p)_{\text{peak}}$, and it becomes possible to carry out the summation over c by using the unitarity of $S_{cc'}$, which may be written

$$\operatorname{Im} t_{nn} = \sum_c |t_{cn}|^2 . \tag{13}$$

The result is

$$\frac{d\sigma}{d\Omega_p} = \frac{2\mu_n}{\pi\hbar^2} \sum_{lj} \int_{E_1}^{E_2} dE_B k_n \operatorname{Im} t_{nn}(E_B) \left(\frac{d\sigma^F}{d\Omega_p}\right)_{\text{peak}} . \tag{14}$$

The validity of Eq. (14) (and hence of the DWBA without rescattering corrections) is apparently supported by data presented by Fuchs *et al.*[11] on the reaction $^{15}N(d,p)^{16}N$ to states in the continuum between 2.3 and 3.2 MeV above the neutron threshold. They find that the transfer cross section very closely follows the total cross section for $^{15}N + n$ scattering, which is related to $\operatorname{Im} t_{nn}$ through the optical theorem. It would be of interest to investigate this relationship quantitatively by carrying out detailed DWBA calculations. A similar experiment with proton transfer[2] could test the truth of our claim that it is the *nuclear* transition amplitude t_{nn} that determines the transfer cross section. In order to get unambiguous results, one should select a case with large Coulomb phase shifts for scattering of protons from the target.

Equation (14) permits the replacement of t_{nn} by an energy-averaged transition amplitude. To see this, suppose that the energy average is denoted by a bar and defined by

$$\bar{f}(E) = \int_{-I/2}^{I/2} dE' W(E') f(E + E') ,$$

$$\int_{-I/2}^{I/2} dE' W(E') = 1 ,$$

where I is the averaging interval and W is a real function. Consider an integral over a range (E_1, E_2) with $E_2 - E_1 \gg I$. Then it is easily shown that

$$\int_{E_1}^{E_2} dE \operatorname{Im} \bar{f}(E) = \int_{E_1}^{E_2} dE \operatorname{Im} f(E) , \tag{15}$$

provided that $f(E_1 + E') = f(E_2 + E')$ for all E' such that $|E'| < \frac{1}{2} I$. This condition is satisfied if f attains a constant background value near E_1 and E_2, so that end-point effects can be neglected. If this condition is reasonably well satisfied by the integrand of Eq. (14), we may then use the result (15) to justify the replacement of t_{nn} by $\bar{t}_{nn}$ in Eq. (14). This solves the problem of including the fluctuation contributions.

If the cross section $d\sigma(d, pn)/d\Omega_p$ were calculated neglecting both the fluctuation contribution and the possibility of decay of B into channels other than n, the result would contain $|\bar{t}_{nn}|^2$ in place of $\operatorname{Im} t_{nn}$. The average contribution of fluctuations and alternative decay modes is therefore proportional to the difference

$$\operatorname{Im} \bar{t}_{nn} - |\bar{t}_{nn}|^2 = \tfrac{1}{4}(1 - |\bar{S}_{nn}|^2) > 0 . \tag{16}$$

This quantity is related to the flux removed from the direct elastic $A + n$ channel. Part of this flux is compound elastic (i.e., fluctuation) and part is inelastic (i.e., due to alternative decay modes).

For an analog resonance, the averaging interval I may be chosen large enough to smooth out the fine structure, but small enough to reveal the gross structure. Let us assume that only the elastic $A + n$ channel is open. Then the root-mean-square magnitude of the fluctuations in the transfer cross section is proportional to

$$\mathcal{T}^2 - |\bar{t}_{nn}|^2 = \tfrac{1}{4}(1 - |\bar{S}_{nn}|^2) ,$$

where $\mathcal{T}^2$ is the energy average of $|t_{nn}|^2$. Therefore, the energy-averaged S matrix is sufficient to determine the magnitude of the fluctuations. Furthermore, the magnitude of the fluctuations must have the *same* energy dependence in the transfer cross section as in the $A + n$ total cross section. Nevertheless, the presence of a nonzero Coulomb phase shift may result in different energy dependences of the transfer cross section and the $A+n$ total cross section.

By using Eq. (14) it is trivial to precipitate results for any particular case, by inserting the appropriate form for $\mathrm{Im}t_{nn}$.

VI. APPLICATIONS TO RESONANCES

A. Isolated Resonance with Several Open Channels

For this case we can take the Breit-Wigner form:

$$t_{nn} \approx \frac{\frac{1}{2}\Gamma_n}{E_B - E_R - \frac{1}{2}i\Gamma}. \qquad (17)$$

Here E_R is the resonance energy, while Γ and Γ_n are its total and partial widths. If $(E_R - E_1)/\Gamma$ and $(E_2 - E_R)/\Gamma$ are large, E_1 and E_2 may be replaced by $-\infty$ and $+\infty$. If the other factors in the integrand of Eq. (14) vary little in an interval Γ, one may replace them by their values for $E_B = E_R$. With t_{nn} given by Eq. (17), the integral in Eq. (14) becomes

$$\int_{-\infty}^{\infty} dE_B \, \mathrm{Im}t_{nn}(E_B) = \tfrac{1}{2}\pi\Gamma_n .$$

If Eq. (1) is used to calculate $d\sigma(d,pn)/d\Omega_p$ with the transition amplitude (17), the result is

$$\frac{d\sigma(d,pn)}{d\Omega_p} = \frac{\mu_n k_n}{\hbar^2}\left(\frac{d\sigma^F}{d\Omega_p}\right)_{\mathrm{peak}} \frac{\Gamma_n{}^2}{\Gamma}. \qquad (18)$$

The interpretation of this result is that Γ_n/Γ is the probability that B decays through the n channel. Therefore the cross section for the (d,pn) reaction is Γ_n/Γ times the cross section (18) for the formation of B. It follows that the cross section for formation of B depends on the partial width Γ_n and *not* on the total width Γ.

B. Proton Transfer to Analog Resonances

Here the transferred particle (which we have hitherto denoted by n) is a proton. For a given partial wave, de Toledo Piza and Kerman[12] give the following parametrization of the proton elastic scattering matrix, averaged over an intermediate energy width I:

$$\overline{S}_{pp}^{\mathrm{total}} = e^{2i(\delta + i\eta)} - i\,\frac{e^{2i(\delta+\phi)}\Gamma_p}{E_B - E_R + \frac{1}{2}i\Gamma}. \qquad (19)$$

As already emphasized in Sec. III, it is the *nuclear* part of the scattering matrix that is relevant. This is

$$\overline{S}_{pp} = e^{2i(\delta_N + i\eta)} - i\,\frac{e^{2i(\delta_N+\phi)}\Gamma_p}{E_B - E_R + \frac{1}{2}i\Gamma}, \qquad (20)$$

where $\delta_N = \delta - \sigma_c$ is the *nuclear* optical background phase shift, and η describes the absorption. The proton escape width is Γ_p and the total width is Γ. The asymmetry phase is ϕ. All of these parameters are assumed to be independent of energy.

We shall calculate the excess of the transfer cross section (integrated over the experimentally accepted energy range) over its value for the background scattering in the absence of the resonance. We disregard the background term, since the calculation of the cross section involves $\mathrm{Im}\bar{t}_{nn}$, so that the contributions of the two terms of Eq. (20) simply add without interference. As for a single isolated resonance, we may extend the limits of integration to $\pm\infty$. The lemma (15) can be applied to Eqs. (14) and (20) to yield

$$\frac{d\sigma}{d\Omega_n} = \frac{\mu_p k_p}{\hbar^2}\left(\frac{d\sigma^F}{d\Omega_n}\right)_{\mathrm{peak}} \Gamma_p \cos 2(\delta_N + \phi). \qquad (21)$$

This result is seen to depend only on the gross-structure parameters of the resonance. Special effects that might have been expected from the presence of the fine structure are therefore absent. It is true that an improved dynamical theory of the fine structure might result in modifications to Eq. (19), with consequent changes in the form of Eq. (21). Nevertheless, we emphasize that Eq. (21) does not depend explicitly on the nature of the fine structure.

The cosine factor of Eq. (21) reduces the cross section and thereby takes account of the partial cancellation that arises from the asymmetry of the resonance. It is not related to the fluctuation contribution. To see this, consider the case of only one open channel, and suppose that the background S matrix and asymmetry phase can be ignored. In a calculation in which the fluctuation contribution is omitted (i.e., $|t_{nn}|^2$ is replaced by $|\bar{t}_{nn}|^2$), the resulting cross section is easily seen to be Γ_p/Γ times Eq. (21). Therefore the fluctuation contribution *enhances* the cross section by a factor Γ/Γ_p. In effect, the fluctuation contribution restores the part of the cross section that spreading removes.

It is remarkable that so simple a result as Eq. (21) includes contributions from all possible decay channels of B, and is valid even if B has strongly overlapping fine structure. The basic reason for this simple result is that the absorptive part of t_{nn} takes account of the flux lost to other channels, thereby eliminating the *squares* $|t_{nc}|^2$ and their attendant fluctuations. This fact is familiar in the statistical theory of reactions, particularly in its application to total cross sections.

Other recent work on nucleon transfer to analog states has been done by Agassi, Auerbach, and Maolem[13] and by Kawai, Kerman, and McVoy.[14] Both groups make assumptions about the individual fine-structure resonances, and advocate calculation of $d\sigma(d,pc)/d\Omega_n$ independently for each c.

Neither group makes explicit use of the unitarity of $t_{cc'}$. The labor of such detailed calculations seems justifiable only if the reaction is *not* strongly peripheral. For strongly peripheral reactions we believe that Eq. (21) yields a good approximation with trivial labor. Agassi *et al.* obtain for the cross section

$$\beta=\left(\frac{1+e^{-2\eta}}{2}\right)^2(1-\tan^2\phi)$$

times the cross section for a single-particle resonance. This disagrees with our Eq. (21).

The information provided by transfer to isolated unbound states is essentially the same as that provided by elastic scattering experiments, as appears clearly in Eq. (1). In the present (more general) case of transfer to a state with fine structure and several decay channels, the relevant information corresponds to a total-cross-section experiment with poor energy resolution. However, the information given by the transfer experiment differs in two ways: First, it effectively measures the total cross section in the *absence* of the Coulomb force, and second, the contribution of different partial waves (for given Γ_n) increases much more rapidly with l. These characteristics make the transfer reaction a useful spectrscopic tool. In practical analysis of transfer reactions to analog states, the experimental transfer cross section is compared with a DWBA calculation of $(d\sigma^F/d\Omega_n)_{\text{peak}}$ in order to deduce the width Γ_p for decay by emission of the transferred particle. The spectroscopic factor for the parent state may then be deduced from the value of Γ_p by means of one of the standard theories of analog resonances.[10]

VII. EFFECT OF BACKGROUND AND ASYMMETRY PHASES IN THE REACTION ^{92}Mo(d,n)

Experimental data[15] for the reaction ^{92}Mo(d, n) to the $d_{5/2}$ (8.40-MeV), $s_{1/2}$ (9.33-MeV), and $d_{3/2}$ (9.91-MeV) states of ^{93}Tc, have recently been analyzed by several authors.[16–18] In all of these analyses the background and asymmetry phases were neglected. We now estimate the effect of this neglect.

According to Auerbach *et al.*[19] the asymmetry phase ϕ in this mass range is always small, typically less than 0.2 rad. The simple optical-model calculations of the form factors that were used in Ref. 18 lead to the following values of the background phases: $\delta(d_{5/2})=0.05°$, $\delta(s_{1/2})=2°$, and $\delta(d_{3/2})=1°$. Even for the s-wave case, where the effect is largest, the correction factor $\cos 2(\delta_N+\phi)$ is quite unimportant. Therefore the conclusions of Refs. 16–18 are unaffected.

The correction will be important, however, whenever the background phase or the asymmetry phase is large.

VIII. CONCLUSIONS

Practical application of the present simple analysis of the transfer reaction to a state B with fine structure and with several open channels will yield on-shell information about B—typically, the width of B for escape of the transferred particle. By an application of unitarity, we have exploited the summation over the unobserved particles to trivially include the fluctuation contribution to the cross section. There is no need to be more suspicious of such analyses than of the more familiar bound-state analyses, provided that (a) the resonance width is small compared with its energy [so that $(d\sigma^F/d\Omega_p)_{\text{peak}}$ may be regarded as slowly varying], (b) the decay products of the resonance more slowly compared with the detected particle (so that rescattering is negligible), and (c) the background and asymmetry phases in the resonant partial wave are small.

ACKNOWLEDGMENTS

The authors are grateful to R. M. Drisko for helpful discussions, and to N. Austern for a critical reading of the manuscript.

*Work supported in part by the National Science Foundation.

[1] C. M. Vincent and H. T. Fortune, Phys. Rev. C 2, 782 (1970).

[2] As in Ref. 1, d, n, and p stand for a general projectile, transferred particle, and the remaining constituent of the projectile, respectively. In Secs. VI B and VII, n becomes a proton.

[3] C. M. Vincent and H. T. Fortune (see Ref. 1), Eq. (34).

[4] R. Lipperheide, Phys. Lett. 32B, 555 (1970).

[5] F. Levin, Ann. Phys. (N.Y.) 46, 41 (1968).

[6] W. A. Friedman, Phys. Rev. C 6, 87 (1972).

[7] J. V. Noble, Phys. Rev. C 1, 1742 (1970).

[8] A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1961), p. 480.

[9] This approximation for the first time involves the neglect of direct breakup.

[10] N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. 44, 48 (1972); D. Robson, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (Wiley, New York, 1969), p. 461; A. M. Lane, *ibid.*, p. 509.

[11]H. Fuchs, H. Homeyer, Th. Lorenz, and H. Oeschler, Phys. Lett. 37B, 285 (1971).
[12]A. F. R. de Toledo Piza and A. K. Kerman, Ann. Phys. (N.Y.) 43, 363 (1967).
[13]D. Agassi, N. Auerbach, and A. Maolem, Phys. Rev. C 6, 385 (1972).
[14]M. Kawai, A. K. Kerman, and K. W. McVoy, private communication.
[15]S. A. A. Zaidi, C. L. Hollas, J. L. Horton, P. J. Riley, J. C. L. Ford, and C. M. Jones, Phys. Rev. Lett. 25, 1503 (1970).
[16]S. A. A. Zaidi and W. R. Coker, Phys. Rev. C 4, 236 (1971).
[17]B. J. Cole, R. Huby, and J. R. Mines, Phys. Rev. Lett. 26, 264 (1971).
[18]H. T Fortune and C. M. Vincent, Phys. Rev. Lett. 27, 1664 (1971).
[19]N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin (see Ref. 10), pp. 118-119.