Systematic study of multiquark states: qqq-$q\bar{q}$ configuration

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Group theoretic method for the systematic study of five-quark states with meson-baryon ($q\bar{q}$-q^3) configuration is developed. The calculation of matrix elements of many-body Hamiltonian is simplified by transforming the physical bases (meson-baryon quark cluster bases) to symmetry bases (group chain classified bases), where the fractional parentage expansion method can be used. Three quark models, the Glashow-Isgur naive model, the Salamanca chiral quark model, and the quark delocalization color screening model, are used to show the general applicability of the method and general results of constituent quark models for five-quark states are given. The method is also useful in the study of the five-quark components effect in baryon structure, the calculation of meson-baryon scattering, and the meson-baryon open channel coupling effect on baryon resonances. The physical contents of different model configurations for the same five-quark system can also be compared through the transformation between different physical bases to the same set of symmetry bases.

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I. INTRODUCTION

After 40 years of quark model study, the idea about baryon and meson is about to go beyond the naive picture: baryon q^3 and meson $q\bar{q}$. The proton spin puzzle could be explained by introducing $q^3q\bar{q}$ component in quark models [1]. To understand baryon spectroscopy better, five-quark component of baryon was proposed [2]. The baryon resonance is certainly coupled to meson-baryon-scattering states and should be studied by coupling q^3 with q^3-$q\bar{q}$ scattering channel in a quark model approach. The recent progresses on meson spectroscopy called for the $q\bar{q}$ and $q\bar{q}q\bar{q}$ coupling [3]. Although the pentaquark states claimed by experimental groups few years ago might be questionable (LEPS Collaboration insists on the existence of pentaquark Θ^+ [4]) and multiquark states might be hard to be identified, the multiquark study is indispensable for understanding the low-energy quantum chromodynamics (QCD), because multiquark states can provide information unavailable for $q\bar{q}$ meson and q^3 baryon, especially the property of hidden color structure. Generally a multibody interaction multichannel coupling calculation is needed in multiquark study. Therefore a powerful method is necessary.

There are various methods for a few quark problems available in the literature. Notably, the Gaussian expansion method, developed by Kamimura and Hiyama, was applied to study four- and five-quark systems [5]. The stochastic variational approach, developed many years ago [6], was applied to the four-quark system by Janc and Rosina [7]. The expansion of the antisymmetric wave function of the few-body system using a hyperspherical harmonic basis to facilitate the evaluation of matrix elements has been applied to four-quark systems by J. Vijande *et al.* [8]. However, the matrix elements of the Hamiltonian have to be calculated channel by channel in these approaches; therefore, a systematic study of n-quark system is very time-consuming. The resonating-group method (RGM) [9], widely used to study the nucleus-nucleus interaction, has been successfully applied to study baryon-baryon interaction [10]. The combination of RGM and generator-coordinate method (GCM) paved a way to use the group theory method to do a systematic quark cluster model calculation, especially the fractional parentage (fp) expansion technique, which was developed in 1960s and has been proven as a powerful method for few-body problems. The quark cluster model uses the group chain classified wave function to describe the hadrons. This special feature makes the quark cluster model calculation a very suitable field to employ the group theory method.

However, to fully use the power of the group theoretic method for quark cluster model calculations of a multiquark system, one needs not only the group chain classified multiquark wave function and the fractional parentage expansion coefficients of these multiquark wave function as usual but also a relation between various quark cluster model states (hereafter called physical bases) and the group chain classified states (hereafter called symmetry bases). Such a method had been developed and successfully applied in the systematic study of baryon-baryon (B-B) effective interaction, dibaryon search [11–13], B-B scattering [14], and a five-quark states study with Jaffe-Wilczek diquark configuration [15], where the physical bases were transformed to the symmetry bases first and then the many-body matrix elements calculation of Hamiltonian (with two body interaction) on the symmetry bases was done by means of fractional parentage expansion, i.e., the many-body matrix elements can be reduced to an overlap and two-body matrix elements. At last the matrix elements on the symmetry bases were transformed back to the physical bases.

The group chain classified $5q$ states had been discussed before [15,16]. The present work is to provide the transformation coefficients between physical bases and symmetry bases of five-quark systems to facilitate the calculation of many-body Hamiltonian matrix elements. The physical bases discussed in this article are the meson-baryon bases. The Jaffe-Wilczek (JW) diquark bases have been reported before in Ref. [15]. Any meson-baryon state, bound or scattering, can be expanded in terms of these meson-baryon bases. The colorless meson-baryon states used in the normal hadron degree of freedom description are part of the Hilbert space of the $5q$ systems. The confinement interaction restricts the asymptotic states to be these colorless meson-baryon states. The genuine $5q$ bound state will include the hidden color meson-baryon states. If the energy of these bound states are higher than the corresponding meson-baryon threshold, these states will decay into colorless meson-baryon asymptotic states through color recoupling.

The method is applied to the quark cluster model calculation of five-quark systems with three quark models: the naive quark model (NQM), i.e., the Glashow-Isgur model [17], the Salamanca version of chiral quark model (ChQM) [18], and the quark delocalization color screening model (QDCSM) [12–14,19]. The general applicability of this method is verified through these model calculations. A systematic study of the general properties of the $5q$ systems has been done that shows the power of this method.

Even though the multiquark system might be hard to identify, the method discussed here is still useful in the study of the multiquark components effect in hadron, the meson-baryon scattering and the meson-baryon open channel coupling effect in baryon resonances in the framework of quark model. The transformation between physical bases and symmetry bases is also useful in the study of the physical contents of different model approaches for the same multiquark system.

In Sec. II, the physical bases and symmetry bases are introduced and the transformation between them is derived. A typical set of transformation coefficients is listed in a sample table and an explanation on how to use the table is described. The complete set of transformation coefficients are collected in arXiv:0802.2891 [hep-ph]. Section III explains three quark models and the corresponding meson and baryon wave functions we used. How to express a five-quark bound and scattering state in terms of the physical bases is explained in detail. The fractional parentage technique applied to calculate the matrix elements on the symmetry bases had been explained before [12] and so have not described in detail. The results of the systematic adiabatic calculation of five-quark system with GCM and dynamical calculation with RGM in the u, d, s three-flavor world are given in Sec. IV. The last section gives the summary.

II. PHYSICAL BASES AND SYMMETRY BASES

The physical bases are constructed as follows. First, the wave function of each quark cluster (both colorless and colorful mesons and baryons as shown in Table I below) is constructed based on the group chain classification

$$\begin{array}{ccccccccc} [1^n] & & [\nu] & & [\tilde{\nu}] & & [c] & & [\mu] \quad [f] \quad I \quad Y \quad J \end{array}$$
$$\mathrm{SU}_{36} \supset \mathrm{SU}_2^x \times \left\{\mathrm{SU}_{18} \supset \mathrm{SU}_3^c \times \left[\mathrm{SU}_6 \supset \left(\mathrm{SU}_3^f \supset \mathrm{SU}_2^\tau \times \mathrm{U}_1^Y\right)\mathrm{SU}_2^\sigma\right]\right\}, \tag{1}$$

[the Young diagrams or quantum numbers for each group are also shown in Eq. (1)], and then the meson and baryon quark cluster wave functions are coupled to overall color-singlet, spin, and isospin quantum numbers by Clebsch-Gordan (CG) coefficients of color SU_3^c, spin SU_2^σ, and isospin SU_2^τ group and, finally, antisymmetrized. For meson-baryon ($q\bar{q}$-q^3) configuration, the five-quarks are separated into two clusters: the separation $\mathbf{S}$ between the meson and baryon clusters is the generating coordinate of the meson-baryon cluster wave function. The physical basis (meson-baryon basis) is defined as

$$\Psi_{\alpha k}(q^4\bar{q}) = \mathcal{A}\left[\psi_3(q_1q_2q_3)\psi_2(q_4\bar{q}_5)\right]^{[c]IJ}_{WM_IM_J}, \tag{2}$$

where $\mathcal{A}$ is a normalized antisymmetrization operator for four-quarks. ψ_3 and ψ_2 are quark cluster wave functions of baryons and mesons, respectively. The details of the baryon ψ_3 and meson ψ_2 wave functions used in this study are explained in Sec. III. The generating coordinate $\mathbf{S}$ is included implicitly through the single quark orbital wave functions, which are the bases of SU_2^x (see below). [ ] means coupling in terms of the SU_3^c, SU_2^τ, SU_2^σ CG coefficients so that it has color symmetry $[c]W$, isospin IM_I, and spin JM_J. $\alpha = (YIJ)$, k represents the quantum numbers $\nu_3, \nu_2, c_3, \ldots, J_2$, which specify the baryon and meson states. We emphasize that the $\Psi_{\alpha k}(q^4\bar{q})$ is a function of the generating coordinate $\mathbf{S}$. To relate the physical bases to the symmetry bases, the four-quark cluster basis is introduced

$$\Psi_{\alpha_4 k_4}(q^4) = \mathcal{A}\left[\psi_3(q_1q_2q_3)\psi_1(q_4)\right]^{[c_4]I_4J_4}_{W_{c_4}M_{I_4}M_{J_4}}, \tag{3}$$

where $\alpha_4 = (Y_4I_4J_4)$ and k_4 represents the quantum numbers $\nu_3, \nu_1, c_3, \ldots, J_1$. Coupling the antiquark state to four-quark basis Eq. (3) in terms of SU_3^c, SU_2^τ, SU_2^σ CG coefficients gives the five-quark basis

$$\left[\Psi_{\alpha_4k_4}(q^4)\psi_{[\bar{c}]\bar{I}\bar{J}}(\bar{q}_5)\right]^{[c]IJ}_{WM_IM_J}.$$

TABLE I. Indices of baryons and mesons. The subscript 8 means color octet; baryons 1–8 are ordinary baryons; 9–12 with flavor symmetry [21] and spin 1/2; 13–16 with flavor symmetry [21] and spin 3/2; 17–20 with flavor symmetry [3] and spin symmetry 1/2. Λ_{s8} is the flavor singlet Λ.

B	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Baryon	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω	N_8	Λ_8	Σ_8	Ξ_8	N_8^*	Λ_8^*	Σ_8^*	Ξ_8^*	$\Delta_8^{\frac{1}{2}}$	Σ_8'	Ξ_8'	$\Omega_8^{\frac{1}{2}}$	Λ_{s8}
M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Meson	π	K	$\bar{K}$	η	η'	ρ	K^*	$\bar{K}^*$	ω	φ	π_8	K_8	$\bar{K}_8$	η_8	η_8'	ρ_8	K_8^*	$\bar{K}_8^*$	ω_8	φ_8	

It relates to the physical basis through the SU_3^c, SU_2^τ, SU_2^σ Racah coefficients,

$$\begin{aligned}\Psi_{\alpha k}(q^4\bar{q}) = {} & U(c_3c_1cc_{\bar{1}};c_4c_2)U(I_3I_1II_{\bar{1}};I_4I_2) \\ & \times U(J_3J_1JJ_{\bar{1}};J_4J_2) \\ & \times \left[\Psi_{\alpha_4k_4}(q^4)\psi_{[\bar{c}]\bar{I}\bar{J}}(\bar{q}_5)\right]^{[c]IJ}_{WM_IM_J},\end{aligned} \tag{4}$$

where U's are Racah coefficients, defined as

$$U(c_3c_1cc_{\bar{1}};c_4c_2) = \sum_{\text{all } W's} C^{[c_4]W_{c_4}}_{[c_3]W_{c_3},[c_1]W_{c_1}} C^{[c]W_c}_{[c_4]W_{c_4},[c_{\bar{1}}]W_{c_{\bar{1}}}} C^{[c_2]W_{c_2}}_{[c_1]W_{c_1},[c_{\bar{1}}]W_{c_{\bar{1}}}} C^{[c]W_c}_{[c_3]W_{c_3},[c_2]W_{c_2}}.$$

The Racah coefficients can be found in Ref. [20].

The symmetry basis of four-quark system can be defined as,

$$\Phi_{\alpha_4K_4}(q^4) = \left| \begin{matrix} [\nu_4]W_{\nu_4} \\ [c_4]\ W_{c_4}[\mu_4]\ [f_4]Y_4I_4M_{I_4}J_4M_{J_4} \end{matrix} \right\rangle, \tag{5}$$

which is the basis vector belonging to the irreducible representations of group chain Eq. (1) with $n=4$. In Eq. (5) K_4 stands for $\nu_4\mu_4f_4$. Coupling the antiquark to Eq. (5), the symmetry bases for five-quark is obtained

$$\Phi_{\alpha K}(q^4\bar{q}) = \left[\Phi_{\alpha_4K_4}(q^4)\psi_{[\bar{c}]\bar{I}\bar{J}}(\bar{q}_5)\right]^{[c]IJ}_{WM_IM_J}. \tag{6}$$

To relate the symmetry basis to physical basis, first we express the four-quark cluster basis Eq. (3) in terms of the symmetry basis Eq. (5) [11,12,21],

$$\Psi_{\alpha_4k_4}(q^4) = \sum_{\tilde{\nu}_4\mu_4f_4} C^{[\tilde{\nu}_4][c_4][\mu_4]}_{[\tilde{\nu}_3][c_3][\mu_3],[\tilde{\nu}_1][c_1][\mu_1]} C^{[\mu_4][f_4][J_4]}_{[\mu_3][f_3][J_3],[\mu_1][f_1][J_1]} C^{[f_4]Y_4I_4}_{[f_3]Y_3I_3,[f_1]Y_1I_1}\Phi_{\alpha_4K_4}(q^4), \tag{7}$$

C's are the isoscalar factors (ISFs) of $\mathrm{SU}_{mn} \supset \mathrm{SU}_m \times \mathrm{SU}_n$, which can be obtained from Chen's book [20]. Then the physical bases and symmetry bases for five-quark can be transformed to each other by

$$\begin{aligned}\Psi_{\alpha k}(q^4\bar{q}) &= \sum_K C_{kK}\Phi_{\alpha K}(q^4\bar{q}) \\ &= \sum_{\tilde{\nu}_4\mu_4f_4} \{UUU\}_{\text{Racah}}\{CCC\}_{\text{ISF}}\Phi_{\alpha K}(q^4\bar{q}).\end{aligned} \tag{8}$$

where $\{UUU\}_{\text{Racah}}$ are the Racah coefficients in Eq. (4) and $\{CCC\}_{\text{ISF}}$ are the isoscalar factors in Eq. (7). All the transformation coefficients are tabulated in the e-print archive [22]. Here only one table is given (Table II). As an example, we also give an expression to show how to read the table. The

TABLE II. Transformation coefficients between physical bases and symmetry bases. The column labels are $[\nu_4]$, $[\mu_4]$, $[f_4]$, $[\sigma_{J_4}]$, $[I_4]$. For first four labels, 1 stands for the symmetry [4]; 2, [31]; 3, [22]; 4, [211], and, for the last one, 1 stands for isospin 2; 2, $\frac{3}{2}$; 3, 1; 4, $\frac{1}{2}$; 5, 0. The row labels are B (baryon index) and M (meson index), where indices B and M are listed in Table I. The transformation coefficients should be the square root of the entries, and a negative sign means to take the negative square root.

BM	$Y=1\ I=\frac{3}{2}\ J=\frac{1}{2}\ Y_4=\frac{4}{3}$										
	12223	12233	21223	22223	22233	12121	22121	23223	24233	24223	23131
1 1	$-\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$	0	0	0	0	0	0
1 6	$-\frac{1}{36}$	$-\frac{1}{4}$	$-\frac{1}{6}$	$-\frac{1}{18}$	$-\frac{1}{2}$	0	0	0	0	0	0
5 6	$-\frac{5}{36}$	0	$\frac{5}{24}$	$-\frac{5}{18}$	0	$\frac{1}{8}$	$\frac{1}{4}$	0	0	0	0
5 9	$\frac{1}{12}$	0	$-\frac{1}{8}$	$\frac{1}{6}$	0	$\frac{5}{24}$	$\frac{5}{12}$	0	0	0	0
9 11	$\frac{1}{3}$	$\frac{1}{12}$	0	$-\frac{1}{6}$	$-\frac{1}{24}$	0	0	$\frac{1}{4}$	$\frac{1}{8}$	0	0
9 16	$\frac{1}{9}$	$-\frac{1}{4}$	0	$-\frac{1}{18}$	$\frac{1}{8}$	0	0	$\frac{1}{12}$	$-\frac{3}{8}$	0	0
13 16	$\frac{1}{9}$	0	0	$-\frac{1}{18}$	0	0	0	$-\frac{1}{3}$	0	$-\frac{1}{2}$	0
17 11	$-\frac{5}{96}$	$\frac{5}{96}$	0	$\frac{5}{192}$	$-\frac{5}{192}$	$-\frac{3}{16}$	$\frac{3}{32}$	$\frac{5}{32}$	$-\frac{5}{64}$	$-\frac{15}{64}$	$-\frac{3}{32}$
17 15	$\frac{1}{32}$	$-\frac{1}{32}$	0	$-\frac{1}{64}$	$\frac{1}{64}$	$-\frac{5}{16}$	$\frac{5}{32}$	$-\frac{3}{32}$	$\frac{3}{64}$	$\frac{9}{64}$	$-\frac{5}{32}$
17 16	$-\frac{5}{288}$	$-\frac{5}{32}$	0	$\frac{5}{576}$	$\frac{5}{64}$	$-\frac{1}{16}$	$\frac{1}{32}$	$\frac{5}{96}$	$\frac{15}{64}$	$-\frac{5}{64}$	$\frac{9}{32}$
17 19	$\frac{1}{96}$	$\frac{3}{32}$	0	$-\frac{1}{192}$	$-\frac{3}{64}$	$-\frac{5}{48}$	$\frac{5}{96}$	$-\frac{1}{32}$	$-\frac{9}{64}$	$\frac{3}{64}$	$\frac{15}{32}$

TABLE III. Model parameters.

	QDCSM	ChQM	NQM
$m_{u,d}$ (MeV)	313	313	313
m_s (MeV)	450	450	450
b (fm)	0.518	0.518	0.518
a_c (MeV fm^{-2})	56.75	46.938	56.75
Δ (fm^2)	−1.398	−1.348	−1.398
α_0	1.916	1.916	1.916
Λ_0 (fm^{-1})	0.113	0.113	0.113
μ_0	36.976	36.976	36.976
$\frac{g_{ch}^2}{4\pi}$	0.54	0.54	–
$\theta_p(^\circ)$	−15	−15	
m_π (fm^{-1})	0.7	0.7	–
m_K (fm^{-1})	2.51	2.51	–
m_η (fm^{-1})	2.77	2.77	–
Λ_π (fm^{-1})	4.2	4.2	–
$\Lambda_K = \Lambda_\eta$ (fm^{-1})	5.2	5.2	–
m_σ (fm^{-1})	–	3.42	–
Λ_σ (fm^{-1})	–	4.2	–
μ	0.45	–	–

TABLE IV. Masses of baryons and mesons (unit: MeV).

	QDCSM	ChQM	NQM
N	939	939	1107
Λ	1037	1037	1165
Σ	1122	1122	1200
Ξ	1179	1179	1245
Δ	1204	1204	1252
Σ^*	1260	1260	1293
Ξ^*	1316	1316	1339
Ω	1373	1373	1389
π	501	523	525
K	647	669	653
$\bar{K}$	647	669	653
η	645	667	525
η'	781	803	765
ρ	727	749	719
K^*	779	801	778
$\bar{K}^*$	779	801	778
ω	679	701	719
φ	840	862	845

first row of Table II reads

$$
\begin{aligned}
[N\pi]^{I=\frac{3}{2},J=\frac{1}{2}}
&= -\sqrt{\frac{1}{12}}\left[\left|\begin{matrix}[4]RRRL\\ [211]W_{c_4}\ [31][31]\frac{4}{3}1M_{I_4}1M_{J_4}\end{matrix}\right\rangle \times \psi_{[11]\frac{1}{2}\frac{1}{2}}(\bar{q}_5)\right]^{I=\frac{3}{2},J=\frac{1}{2}} \\
&\quad + \sqrt{\frac{1}{12}}\left[\left|\begin{matrix}[4]RRRL\\ [211]W_{c_4}\ [31][31]\frac{4}{3}1M_{I_4}0M_{J_4}\end{matrix}\right\rangle \times \psi_{[11]\frac{1}{2}\frac{1}{2}}(\bar{q}_5)\right]^{I=\frac{3}{2},J=\frac{1}{2}} \\
&\quad - \sqrt{\frac{1}{2}}\left[\left|\begin{matrix}[31]RRRL\\ [211]W_{c_4}\ [4][31]\frac{4}{3}1M_{I_4}1M_{J_4}\end{matrix}\right\rangle \times \psi_{[11]\frac{1}{2}\frac{1}{2}}(\bar{q}_5)\right]^{I=\frac{3}{2},J=\frac{1}{2}} \\
&\quad - \sqrt{\frac{1}{6}}\left[\left|\begin{matrix}[31]RRRL\\ [211]W_{c_4}[31][31]\frac{4}{3}1M_{I_4}1M_{J_4}\end{matrix}\right\rangle \times \psi_{[11]\frac{1}{2}\frac{1}{2}}(\bar{q}_5)\right]^{I=\frac{3}{2},J=\frac{1}{2}} \\
&\quad + \sqrt{\frac{1}{6}}\left[\left|\begin{matrix}[31]RRRL\\ [211]W_{c_4}[31][31]\frac{4}{3}1M_{I_4}0M_{J_4}\end{matrix}\right\rangle \times \psi_{[11]\frac{1}{2}\frac{1}{2}}(\bar{q}_5)\right]^{I=\frac{3}{2},J=\frac{1}{2}}, \qquad (9)
\end{aligned}
$$

where $R(L)$ means right(left) single-quark orbital wave function (see below).

III. QUARK MODELS AND MODEL WAVE FUNCTIONS

The models used in the calculations include the naive quark model, the Salamanca chiral quark model, and the quark delocalization color screening model. The Hamiltonian of these models have been given in the first article of this series [15]. The model parameters and the calculated masses of baryons and mesons are shown in Tables III and IV (the same size parameter b is used for baryon and meson).

There are only two quark clusters q^3-$q\bar{q}$ now instead of three in the qq-qq-$\bar{q}$ configuration that we studied before. Therefore there is only one generating coordinate, $\mathbf{S}$, the separation between two clusters, that is used. The baryon and meson quark cluster wave functions ψ_3 and ψ_2 are frozen as usual in the quark cluster model approaches: The color, flavor, and spin part wave function of the meson and baryon are fixed to be the usual $SU_{18}^{cf\sigma} \supset SU_3^c \times (SU_6^{f\sigma} \supset SU_3^f \times SU_2^\sigma)$ classified wave function. For the naive and chiral quark model the orbital wave function of baryon and meson are chosen to be the three-particle product of ϕ_R and two-particle product of ϕ_L, respectively,

$$
\begin{aligned}
\phi_R(\mathbf{r}) &= \left(\frac{1}{\pi b^2}\right)^{3/4} \exp\left[-\frac{\left(\mathbf{r}-\frac{2}{5}\mathbf{S}\right)^2}{2b^2}\right], \\
\phi_L(\mathbf{r}) &= \left(\frac{1}{\pi b^2}\right)^{3/4} \exp\left[-\frac{\left(\mathbf{r}+\frac{3}{5}\mathbf{S}\right)^2}{2b^2}\right].
\end{aligned} \qquad (10)
$$

Here b is a parameter denoting the hadron size and it is fixed to be 0.518 fm in this study for both baryon and meson, $\mathbf{S}$ is the separation of the reference centers $\frac{2}{5}\mathbf{S}$ of baryon and $-\frac{3}{5}\mathbf{S}$ of meson and is the generating coordinate in our five-quark cluster wave function. The baryon and meson cluster wave functions are thus frozen to be,

$$\psi_3(q_1q_2q_3) = \chi_c(B)\eta_{I_1S_1}(B)\prod_{\alpha=1}^{3}\phi_R(\mathbf{r}_\alpha), \tag{11}$$

$$\psi_2(q_4\bar{q}_5) = \chi_c(M)\eta_{I_2S_2}(M)\prod_{\beta=4}^{5}\phi_L(\mathbf{r}_\beta), \tag{12}$$

where χ and η are the SU_3^c and $SU_6^{f\sigma} \supset SU_3^f \times SU_2^\sigma$ color and spin-flavor wave function. For QDCSM, the orbital wave function of baryon and meson are chosen to be the three-particle product of the delocalized single-particle orbital wave function ψ_R and two-particle product ψ_L given below,

$$\begin{aligned}\psi_R(\mathbf{r}) &= (\phi_R + \epsilon_1\phi_L)/N(\epsilon_1),\\ \psi_L(\mathbf{r}) &= (\phi_L + \epsilon_2\phi_R)/N(\epsilon_2),\end{aligned} \tag{13}$$

$$N(\epsilon_1) = \sqrt{1+\epsilon_1^2+2\epsilon_1\langle\phi_R|\phi_L\rangle}, \tag{14}$$

$$N(\epsilon_2) = \sqrt{1+\epsilon_2^2+2\epsilon_2\langle\phi_R|\phi_L\rangle}, \tag{15}$$

where ϕ_L, ϕ_R are the left- and right-centered Gaussian function given in Eq. (10), ϵ_1 denotes the quark in baryon orbit delocalized into meson orbit, whereas ϵ_2 is from meson to baryon. They are determined by minimizing the diagonalized five-quark Hamiltonian with respect to the delocalization parameters ϵ_1 and ϵ_2 for every generating coordinate $S = |\mathbf{S}|$ and so they are functions of S, which describes the adiabatic mutual distortion of meson and baryon in the course of interacting process. The $\psi_3(q_1q_2q_3)$ and $\psi_2(q_4\bar{q}_5)$ fixed above is used in this study to construct the physical bases Eq. (2).

Any bound or scattering state is expanded in terms of these physical bases. We combine the RGM and GCM to do this expansion. Based on the cluster representation of the many-body problem, we choose the RGM wave function to express the bound or scattering five-quark state,

$$\Psi(5q) = \mathcal{A}[\Psi_B\Psi_M]^{[c]IS} \otimes \chi(\mathbf{R}). \tag{16}$$

Here $\mathcal{A}$ is the normalized antisymmetrization operator and Ψ_B and Ψ_M are the three-quark baryon and $q\bar{q}$ meson internal wave function. The color-flavor-spin part is the usual $SU_{18}^{cf\sigma} \supset SU_3^c \times (SU_6^{f\sigma} \supset SU_3^f \times SU_2^\sigma)$ wave function. The orbital part is

$$\begin{aligned}\Phi_B &= \left(\frac{1}{\sqrt{3}\pi b^2}\right)^{3/2}\exp\left[-\frac{\boldsymbol{\xi}_1^2}{4b^2}-\frac{\boldsymbol{\xi}_2^2}{3b^2}\right],\\ \boldsymbol{\xi}_1 &= \mathbf{r}_1-\mathbf{r}_2, \quad \boldsymbol{\xi}_2 = \frac{\mathbf{r}_1+\mathbf{r}_2}{2}-\mathbf{r}_3,\end{aligned} \tag{17}$$

$$\Phi_M = \left(\frac{1}{2\pi b^2}\right)^{\frac{3}{4}}\exp\left[-\frac{\boldsymbol{\xi}_3^2}{4b^2}\right], \quad \boldsymbol{\xi}_3 = \mathbf{r}_4-\mathbf{r}_5. \tag{18}$$

$[]^{[C]IS}$ means coupling meson and baryon wave functions to be overall color singlet and spin S and isospin I. $\chi(\mathbf{R})$ is the wave function of the relative motion between meson and baryon, where $\mathbf{R}$ is the relative coordinate between the center-of-mass coordinates of meson $\mathbf{R}_2$ and baryon $\mathbf{R}_3$,

$$\mathbf{R} = \mathbf{R}_3-\mathbf{R}_2, \quad \mathbf{R}_2 = \frac{\mathbf{r}_4+\mathbf{r}_5}{2}, \quad \mathbf{R}_3 = \frac{\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3}{3}. \tag{19}$$

Then expand the relative motion wave function $\chi(\mathbf{R})$ by Gaussians centered at different $\mathbf{S}_i$,

$$\chi(\mathbf{R}) = \sum_i C_i\left(\frac{6}{5\pi b^2}\right)^{3/4} e^{-\frac{3}{5b^2}(\mathbf{R}-\mathbf{S}_i)^2}, \tag{20}$$

and inserting an overall center-of-mass motion Gaussian function,

$$\Phi_C(\mathbf{R}_C) = \left(\frac{5}{\pi b^2}\right)^{3/4} e^{-\frac{5}{2b^2}\mathbf{R}_C^2}, \tag{21}$$

where the $\mathbf{R}_c = \frac{2\mathbf{R}_2+3\mathbf{R}_3}{5}$, the ansatz, Eq. (16), can be rewritten as

$$\begin{aligned}\Psi_{5q} &= \mathcal{A}\sum_i C_i\prod_{\alpha=1}^{3}\phi_R(\mathbf{r}_\alpha)\prod_{\beta=4}^{5}\phi_L(\mathbf{r}_\beta)\\ &\quad\times[\eta_{I_1S_1}(B)\eta_{I_2S_2}(M)]^{IS}_{M_IM_S}[\chi_c(B)\chi_c(M)]^{[c]}_W\\ &= \mathcal{A}\sum_i C_i[\psi_3(q_1q_2q_3,\mathbf{S}_i)\psi_2(q_4\bar{q}_5,\mathbf{S}_i)]^{[C]IS}_{WM_IM_S}.\end{aligned} \tag{22}$$

Equation (22) shows that any bound or scattering five-quark state can be expanded in terms of physical bases Eq. (2). Here the generating coordinate $\mathbf{S}_i$ dependence of the physical bases is shown explicitly. To obtain this result, a special property of the Gaussian function with the same size parameter b plays a vital role: the product of single Gaussian functions of different particle coordinates $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ can be transformed into the product of individual cluster wave functions of Gaussian with Jacobian coordinates $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2$, etc., the wave function of Gaussian with relative coordinate of individual cluster centers $\mathbf{R}_m$, $\mathbf{R}_n$, etc., and the overall center-of-mass motion Gaussian function and vice versa. This is also the reason why we choose the same size parameter b for meson and baryon even it is not good in describing the meson and baryon internal structure.

Through a partial-wave expansion and coupling the spin $\mathbf{S}$ and orbital angular momentum $\boldsymbol{l}$ of the relative motion wave function $\chi(\mathbf{R})$, the ansatz Eq. (22) can be expressed as

$$\begin{aligned}\Psi_{5q} &= \mathcal{A}\sum_{i,l} C_{i,l}\int\frac{d\Omega_{S_i}}{\sqrt{4\pi}}\prod_{\alpha=1}^{3}\phi_R(\mathbf{r}_\alpha)\prod_{\beta=4}^{5}\phi_L(\mathbf{r}_\beta)\\ &\quad\times\left[[\eta_{I_1S_1}(B)\eta_{I_2S_2}(M)]^{IS}Y^l(\hat{\mathbf{S}}_i)\right]^J[\chi_c(B)\chi_c(M)]^{[\sigma]}.\end{aligned} \tag{23}$$

From the above derivation one can see that in naive and chiral quark model cases, even though we use the physical bases, which include the overall center-of-mass coordinate, the center-of-mass wave function can be factorized and it does not enter into the Hamiltonian matrix elements and therefore there is no spurious center-of-mass motion while in QDCSM the center-of-mass wave function cannot be factorized and so there is spurious center-of-mass motion. If we use a size parameter b' for meson different from the size parameter b for baryon then even for the simple left and right Gaussian

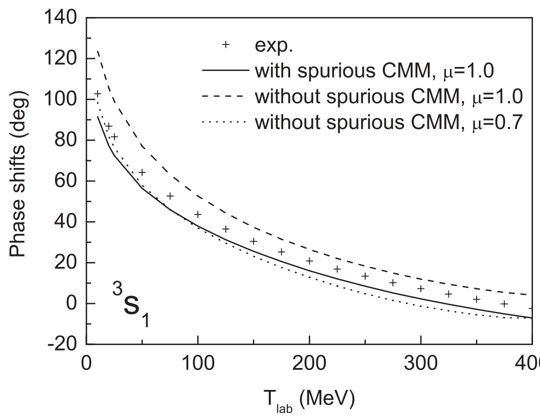

FIG. 1. The NN-scattering phase shifts with/without center-of-mass motion in QDCSM.

function used in naive and chiral quark models there is also the spurious center-of-mass motion.

A projection method had been developed and the spurious center-of-mass motion in the NN scattering has been studied for QDCSM and the effect can be canceled by readjusting the color screening parameter μ (see Fig. 1) [23].

In the study of five quark exotic states we first do a general survey of the meson-baryon effective potentials consisting of ground-state mesons and baryons, which are listed in Table I, within the u, d, s three-flavor world. Here an adiabatic approximation based on the physical bases Eq. (2) has been done. A GCM five-quark wave function with quantum number set $\alpha = (YIJ)$ is expressed as a channel-coupling wave function, i.e., a linear combination of the physical bases Eq. (2),

$$\Psi_\alpha(q^4\bar{q}) = \sum_k C_k \Psi_{\alpha k}(q^4\bar{q}). \tag{24}$$

The channel-coupling coefficients C_k is determined by the diagonalization of the five-quark Hamiltonian and the lowest state for every quantum number set and every generating coordinate $\mathbf{S}$ is fixed in this way.

The effective potential between two hadron clusters at separation S is defined as [24]

$$V_{eff}(S) = E(S) - E(\infty), \tag{25}$$

where $E(S)$ is the lowest energy obtained from diagonalizing the system Hamiltonian on the physical bases $\Psi_{\alpha k}$ at the generating coordinate S.

For special interesting channels dynamical calculations with RGM have been done. The RGM dynamical equation

$$\int H(\mathbf{R},\mathbf{R}')\chi(\mathbf{R}')\,d\mathbf{R}' = E\int N(\mathbf{R},\mathbf{R}')\chi(\mathbf{R}')d\mathbf{R}', \tag{26}$$

is obtained from the variation of the

$$\langle\Psi(5q)|H - E|\Psi(5q)\rangle = 0 \tag{27}$$

with respect to the relative motion wave function $\chi(\mathbf{R})$ but fixed the internal wave function Ψ_B and Ψ_M, where the $\Psi(5q)$ is the RGM wave function, Eq. (16). After expanding the RGM wave function, Eq. (16), as Eq. (23), the integrodifferential RGM equation changed to be a set of linear algebraic equations [12–14],

$$\sum_{j,l} C_{j,l} H_{i,j}^{l,l'} = E\sum_j C_{j,l'} N_{i,j}^{l'}, \tag{28}$$

where $N_{i,j}^{l'}$, $H_{i,j}^{l,l'}$ are the overlaps and Hamiltonian matrix elements of the wave function Eq. (2), respectively, and i, j correspond to the generating coordinates S_i and S_j. This equation can be used both for bound- and scattering-state calculations if a bound- or scattering-state boundary condition is used, respectively. An interesting point is worth mentioning here: if a bound-state boundary is set at quite a large distance, the solution of Eq. (28) corresponding to energy E larger than the channel threshold still show the scattering asymptotic behavior and a scattering phase shift obtained from such a solution is the same as those obtained from the solution with the scattering boundary condition within the numerical uncertainty.

The main advantage of the group theory method proposed in this report is that the Hamiltonian matrix elements $H_{i,j}$ with the physical bases Eq. (2) can be calculated easily through the matrix elements with the symmetry bases and the transformation between the physical bases and symmetry bases. Storing the needed fp, CG, Racah U coefficients and isoscalar factors in a computer program, the properties of the whole bunch of 60 five-quark states consisted of the 20 meson and 21 baryon states listed in Table I in the u, d, s three-flavor world can be obtained at one blow, one only needs to input the two-body matrix elements of the choosing model Hamiltonian.

The shortcoming of this method is the internal orbital wave function of hadrons are limited to be Gaussian with the same size parameter. For bound-state calculation this shortcoming is not serious, because the physical bases constructed in this article are mere bases of Hilbert space. However, for the scattering calculation, the channel wave function (we use the RGM wave function in this study) is expected to be as realistic as possible, i.e., the internal wave functions of interacting hadrons are better to be the realistic solutions of the model Hamiltonian. For NN scattering, the group theory method as explained above has been quite successful. For meson-baryon scattering, if one use a single Gaussian approximation to describe the internal orbital wave function of a meson, the size parameter b' of the meson is usually different from the baryon size parameter b. In this case the spurious center-of-mass motion appears and a center-of-mass motion correction must be done. The center-of-mass correction method used in Ref. [23] can be used here. However, a good meson wave function is in general a multiple Gaussian expansion [25]. In this case the spurious center-of-mass motion correction in principle can be done as before but in practice will be more involved and the question of how to apply the group theory method developed here needs further study.

IV. RESULTS AND DISCUSSIONS

To take into consideration of the effects of various color structures, the hidden color channels are included in this calculation, which is different from our dibaryon calculation

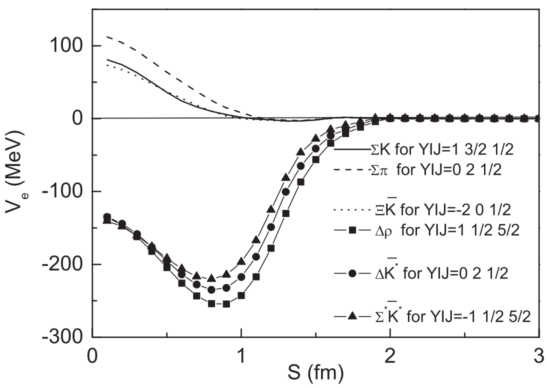

FIG. 2. Effective potentials vs. cluster separation in QDCSM.

where only color singlet channels are included [12,13]. Like the systematic study of the JW diquark configuration, all possible states consisted of ground-state mesons and baryons within the u, d, s three-flavor world have been calculated in this baryon-meson configuration. Both adiabatic single-channel and channel-coupling calculations have been carried out in three quark models. Dynamical calculations with RGM have been done for some interesting cases.

The threshold checking is made as follows. For adiabatic calculation, the energy $E(\infty)$ is found to be the sum of two-hadron masses and the relative-motion kinetic energy. For example, the total energy of KN channel at separation $S =$ 3 fm is 1902/1924 MeV in QDCSM/ChQM, which is the sum of the theoretic masses of N (939 MeV) and K (647/669 MeV) and the relative-motion kinetic energy $\frac{3\hbar^2}{4\mu_{\rm th}b^2} = 316$ MeV, $\mu_{\rm th}$ is the theoretic reduced mass of N and K. A relative motion kinetic energy appears because the GCM wave function Eq. (2) or Eq. (24) is always with a Gaussian relative motion [see Eq. (20)]. For dynamical calculation, it is found that if the lowest eigenenergy is smaller than the sum of the theoretical masses of two hadrons, the wave function shows bound-state behavior, otherwise, it shows scattering behavior. For KN channel, the wave function of the lowest eigenenergy (1572/1577 MeV in QDCSM/ChQM) shows bound-state behavior because the energy is smaller than the theoretic threshold: 1586/1608 MeV. When the energy is higher than these thresholds, the wave functions beyond the interaction range oscillate as the spherical Bessel function and the phase shifts obtained from these wave functions are the same as those obtained from the scattering solution within the numerical uncertainty.

To save space, only several general features and the comparison between the JW diquark and meson-baryon configurations and among three models are given below.

(i) Generally there exist effective attractions for almost all the states both in the extended QDCSM and the chiral quark model, while there are about 10 channels that are pure repulsive in the naive quark model (NN interaction study already showed that the naive quark model is not realistic for hadron interaction, it cannot provide the intermediate range NN attraction). In QDCSM, the attraction between decuplet baryons and vector mesons are usually large (>200 MeV), those between octet baryons and pseudoscalar mesons are small (less than 10 MeV), and those between octet (decuplet) baryons and vector (pseudoscalar) mesons lie between. There are also several exceptions. This situation is similar to the effective baryon-baryon interactions where the attraction between decuplet baryons is large, that between octet baryons is small, and that between decuplet and octet baryons is in between. The physical mechanism of these general feature is the same as discussed for baryon-baryon effective interaction previously [12,13]. In the chiral quark model, the above general features are kept but with more exceptions. Figure 2 gives several examples. It is clear that $\Delta\rho$, $\Delta\bar{K}^*$, and $\Sigma^*\bar{K}^*$ channels have strong attractions, whereas $\Sigma\pi$, ΣK, and $\Xi\bar{K}$ show pure repulsive or very small attractions. More examples can be found in Figs. 4–7.

(ii) For chiral quark model and naive quark model, most states have very similar results (energy difference less than 30 MeV) for two configurations: qq-qq-$\bar{q}$ and qqq-$q\bar{q}$ (the two configurations have the same results when S, T in qq-qq-$\bar{q}$ go to zero and S in qqq-$q\bar{q}$ goes to zero). For these states, the separations between quark clusters corresponding to the minimum-energy are small (S and/or $T < 0.7$ fm). The differences

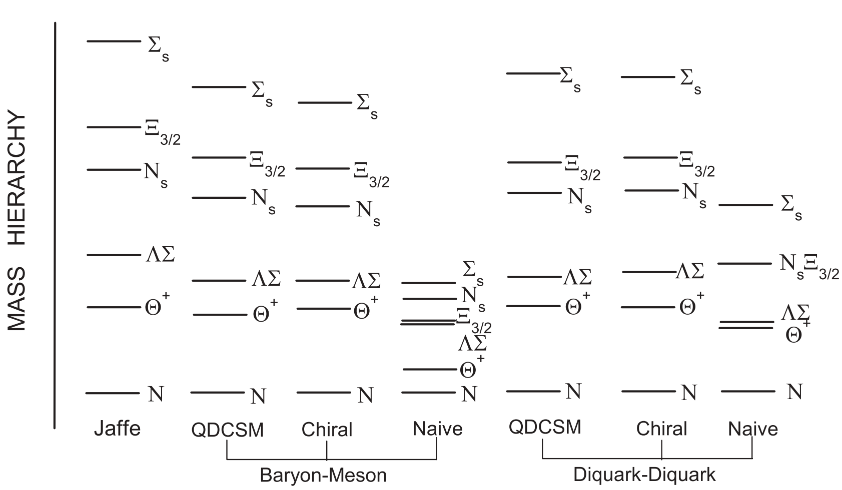

FIG. 3. Mass hierarchy in the $8_f + \overline{10}_f$ with $J^P = \frac{1}{2}^+$ in three quark models, compared with the one in diquark-diquark configuration and Ref. [26].

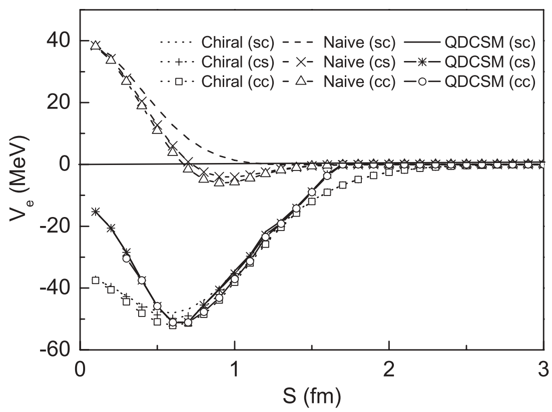

FIG. 4. Effective potentials for $(YIJ) = (20\frac{1}{2})$.

between two configurations tend to disappear after channel-coupling, the remaining minor differences come from the different quark orbital wave functions and the limited model spaces used. There are also several states, the energy differences are large (greater than 60 MeV, even 100 MeV) for two configurations. For these states, the separation between quark clusters corresponding to minimum-energy in baryon-meson configuration is a little larger (color-singlet clusters can stay far away), while the ones in JW diquark configuration are smaller (colorful clusters tend to stay closer), which makes the energies different. However, the extended QDCSM gives different results. The diquark configuration always gives lower energy than the baryon-meson configuration. The main reason is that the color confinement is screened between quark pairs in different clusters in QDCSM. The higher the number of clusters, the lower the model energy. The screened color confinement assumed for quark pairs in two-color singlet clusters was shown to be reasonable by the previous studies [12,13,19]; however, extends to multicolor clusters directly even with the parameters fixed in NN scattering is questionable. However, the direct extension of the color-dependent two-body confinement to multiquark system as used in the naive and chiral quark models is also questionable. Further investigation is needed.

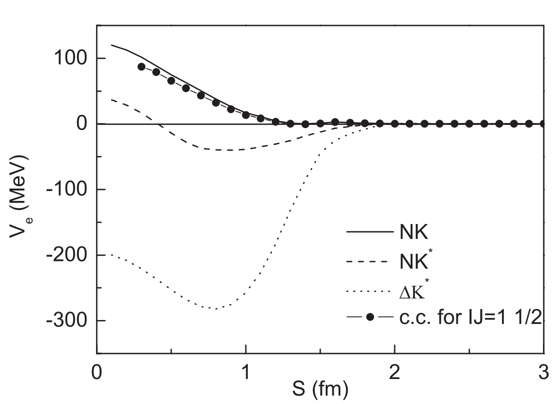

FIG. 5. Effective potentials for $(YIJ) = (21\frac{1}{2})$ in QDCSM.

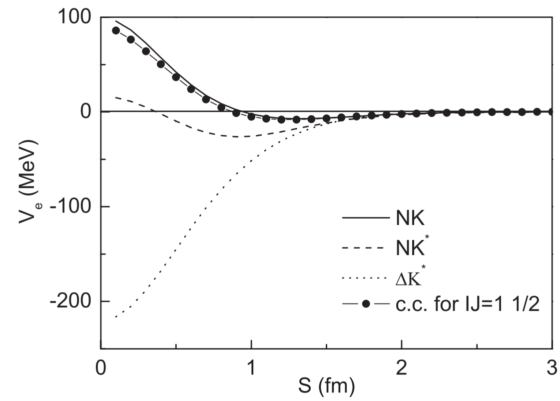

FIG. 6. Same as Fig. 4 for chiral quark model.

(iii) For the states in antidecuplet, 27-plet, and 35-plet the mass hierarchies obtained in the meson-baryon configuration are quite similar to that of JW diquark configuration. Here only the mass hierarchies of $8_f + \overline{10}_f$ states with $J^P = \frac{1}{2}^+$ of two configurations in three quark models are given in Fig. 3.

(iv) Figures 4–7 show the effect of channel-coupling (sc stands for single-channel, cs for color singlet channel coupling, and cc for full channel-coupling). For most channels, the effect of channel coupling is small, especially for QDCSM. It is worth noting that the hidden-color states have minor effect on the lowest energy of a channel. Figures 8 and 9 show that channel coupling effect for $(YIJ) = (00\frac{1}{2})$, where the channel-coupling contributes a considerable attraction. However, the contributions of hidden-color channels are still negligible, especially for QDCSM. We like to emphasize that this result is based on the assumption that the quark interaction used for hadron spectroscopy and color singlet hadron channels can be directly extend to hidden color channels and this assumption is questionable. In fact we have no information about the quark interaction between hidden color channels and colorless ones up to now.

(v) Generally there exist attractions for baryon-meson system; however, in most cases, the attraction is not enough to make the energy lower than the threshold, i.e.,

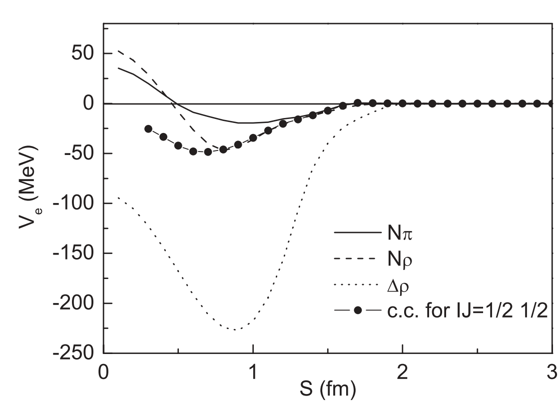

FIG. 7. Effective potentials $(YIJ) = (1\frac{1}{2}\frac{1}{2})$ in QDCSM.

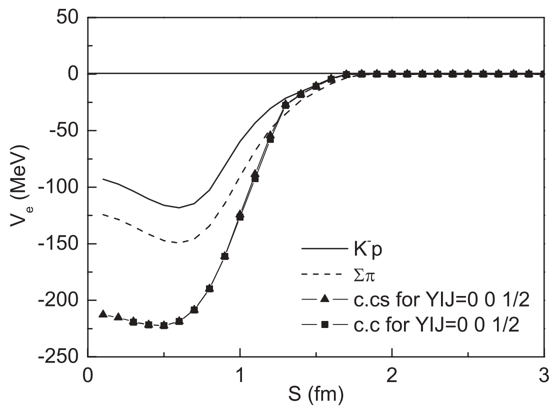

FIG. 8. Effective potentials for $(YIJ) = (00\frac{1}{2})$ in QDCSM.

the sum of corresponding baryon and meson masses. Therefore there will be no narrow resonances except there is special reason to prevent the decay. Taking Θ^+ as an example, the lowest energy state here is the KN state. The effective potential for the NK channels in three quark models is given in Fig. 4. For the $I = 0S$-wave KN state, although the naive quark model gives repulsive potentials, there is a moderate attraction ~51 MeV at a separation of 0.6 fm in the QDCSM and a little weaker attraction ~48 MeV at a separation of 0.6 fm in the chiral quark model. (The results differ from the previous calculation [27], where no attraction was obtained. The reason is the effect of σ exchange was reduced in Ref. [27] by using larger mass and ideal mixing of σ_0 and σ_8.) In QDCSM, the mass of Θ^+ is 1851 MeV. If we make a correction of the physical mass of kaon (the model kaon mass is 647 MeV for QDCSM), the energy of the state can be reduced to 1700 MeV, which is still too high to match the claimed value 1540 MeV. In the chiral quark model, the case is quite similar to QDCSM, the mass of Θ^+ is 1872 MeV, it will be reduced to 1699 MeV with the kaon mass correction.

(vi) Finally, there are several interesting states in QDCSM:

(a) The masses of the following states are lower than the lowest two- and three-body decay threshold and they might be promising pentaquark candidates. However, the results are sensitive to model details. For $(YIJ) = (1\frac{1}{2}\frac{5}{2})$ channel, the lowest state has mass around 1822 MeV, which is not only lower than the two-body decay thresholds ($\Delta\rho$) but also lower than the three-body decay thresholds ($N\pi\rho$). And for $(YIJ) = (00\frac{5}{2})$ channel, there is a state with mass around 2017 MeV, which is lower than the corresponding two-body decay threshold $\Sigma^*\rho$ and the three-body decay thresholds ($\Lambda\pi\rho$).

(b) K^-p state has been calculated dynamically to see if there is a resonance state $\Lambda(1405)$. The calculated mass of K^-p is 1560 MeV. If we make a correction of the physical mass of kaon, the energy of the state can be reduced to 1408 MeV, which is close to the observed mass of $\Lambda(1405)$.

(c) In Ref. [28], several quasibound states have been proposed, such as $I = 1\Delta K$, $I = \frac{1}{2}\Sigma K$, $I = \frac{1}{2}, \frac{3}{2}N\phi$, and so on. In QDCSM, strong attraction (>100 MeV) is obtained for all of these states in the adiabatic calculation. The dynamical calculated results are shown in Table V together with the results of Ref. [28]. Generally QDCSM obtains larger binding energies than Ref. [28]. The trend agrees with the previous calculation on dibaryons. QDCSM obtains lower energies for the states with small strangeness, while the chiral quark model of Zhang's group obtains lower energies for the states with large strangeness. To make things clear, the corresponding baryon-meson-scattering

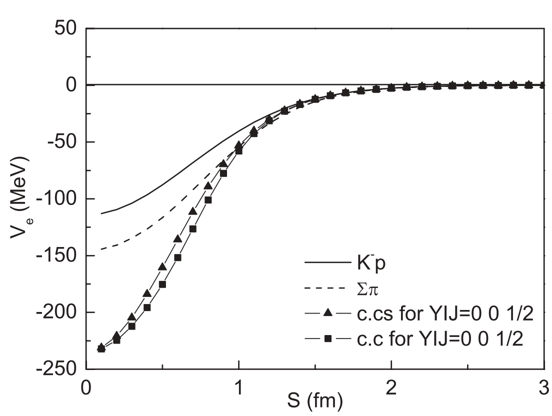

FIG. 9. Effective potentials for $(YIJ) = (00\frac{1}{2})$ in chiral quark model.

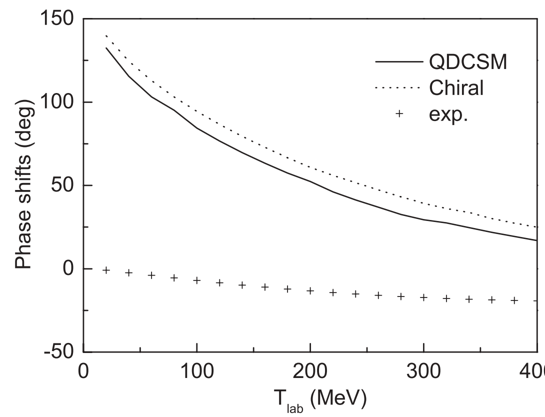

FIG. 10. The phase shifts of KN scattering.

TABLE V. The binding energies of several pentaquark-states (in MeV).

States	QDCSM	Ref. [28]
$N\phi$ $(S = 1/2)$	46	$1 \sim 3$
$N\phi$ $(S = 3/2)$	45	$3 \sim 9$
ΔK $(S = 1/2)$	56	$3 \sim 20$
ΣK $(S = 1/2)$	38	$18 \sim 44$
ΛK $(S = 1/2)$	Unbound	Unbound

calculation has been done. Figure 10 shows the phase shifts of KN obtained in a single-channel approximation. Here we directly take the model parameters fixed before for NN scattering. Clearly both the QDCSM and chiral quark models give too much KN attraction. This means that the model parameters fixed in NN scattering cannot be directly extended to KN scattering. Zhang *et al.* readjusted the model parameters as such in their KN-scattering calculation. But this will block the unified description of meson-baryon and baryon-baryon scattering. Maybe the too-rough approximation of the orbital wave function of the K meson is also responsible for this. To improve the calculation by using a more realistic meson orbital wave function and incorporating other states in baryon-meson scattering is in progress. This will be a further check to see if the group theory method proposed in this report is good enough to do the meson-baryon-scattering calculation.

V. SUMMARY

The multiquark system is complicated, our knowledge about multiquark systems is limited, and there is no experimentally well-established multiquark state. Nevertheless, its study is indispensable for understanding the properties of the abundant color structures allowed in QCD. The systematic study could provide a general features of the multiquark system. In this case, a powerful method is needed. This article (and the previous one [15]) reports a group theoretic method for five-quark calculation. The method is applied to a systematic study of all possible 60 five-quark states consisted of ground-state mesons and baryons within u, d, s three-flavor world in baryon-meson configuration with three quark models. The powerful feature of group theory method is shown by the large amount of spectroscopy data obtained at one blow in this approach (only very limited results have been shown in this report).

In the pentaquark studies few years ago various constituent quark models were proposed. Our transformation between different physical bases to the same set of symmetry bases shows these models are a choice of different parts of the Hilbert space of the five-quark system. Our calculation shows that if one uses a large-enough model space the differences of these different configuration choices tend to disappear.

In addition to the five-quark bound-state calculation, the baryon-meson scattering is another field where the technique developed in this article can be used. With the progress on the meson spectroscopy of nonrelativistic quark model [29], the unified description of hadron properties and hadron-hadron interaction is expected. There are lot of work devoted to baryon-baryon scattering, but baryon-meson scattering is less studied. The group theoretic method presented here will be a useful one to do the baryon-meson scattering within the framework of constituent quark models.

There are experimental indication and theoretical motivation that the pure q^3 baryon and $q\bar{q}$ meson configuration should be modified to include higher Fock components, i.e., unquench the bare quark model. The effect of the five-quark component $q^3q\bar{q}$ of baryon can be studied as well by the same technique developed here. Many baryon resonances' energies are higher than the related meson-baryon threshold and the resonance parameters are obtained from the corresponding scattering. To calculate the properties of these resonances, such as their energies and widths, a bare q^3 quark model calculation is not reliable. The open scattering channel-coupling gives rise not only to the width but also large energy shift. A q^3 and q^3-$q\bar{q}$ channel-coupling calculation is needed. The method developed in this report is also useful in these bound q^3 and open q^3-$q\bar{q}$ quark model calculations.

ACKNOWLEDGMENTS

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