Anisotropic transverse flow introduction in Monte Carlo generators for heavy ion collisions

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Anisotropic transverse flow patterns that are observed in relativistic heavy ion collisions can be added to the available microscopic Monte Carlo event generators as a final state modification to the azimuthal angles of the particles, which are generated isotropically. The method proposed for this purpose by A. M. Poskanzer and S. A. Voloshin [Phys. Rev. C **58**, 1671 (1998)] is valid only for small values of the Fourier coefficients v_n and therefore it is not suitable for simulations with large values of anisotropy such as the ones predicted for Pb-Pb collisions at the LHC. We present here a possible solution to treat the cases of large anisotropies.

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It is a well-established experimental fact that in heavy ion collisions with nonzero impact parameter, the azimuthal distribution of the produced particles in the transverse plane is anisotropic with respect to the reaction plane [1–4]. The particle azimuthal distributions relative to the reaction plane are usually written in the form of a Fourier series as

$$\frac{dN}{d\varphi} = \frac{1}{2\pi}\left(1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\varphi - \Psi_{\rm RP})]\right), \tag{1}$$

where $\Psi_{\rm RP}$ is the angle defined by the impact parameter vector in the transverse plane. The v_n coefficients are given by $v_n = \langle\cos[n(\varphi - \Psi_{\rm RP})]\rangle$. The first two coefficients (v_1 and v_2) are called directed and elliptic flow, respectively.

The elliptic flow in particular is a powerful probe to investigate the matter produced in relativistic heavy ion collisions at the CERN Super Proton Synchrotron (SPS), the BNL Relativistic Heavy Ion Collider (RHIC), and the CERN Large Hadron Collider (LHC) accelerators. At these energies, the observed elliptic anisotropy is attributed to the presence of a collective motion (flow) of the particles produced in the collision. For noncentral collisions, the geometrical anisotropy of the almond-shaped overlap region of the colliding nuclei causes a larger pressure gradient in the reaction plane than in the direction orthogonal to it [5,6]. If enough particle rescatterings occur, the initial spatial anisotropy is converted into a momentum anisotropy, which can be observed in the particle azimuthal distribution. Thus, elliptic flow brings information on the degree of thermalization, on the time needed to reach the equilibrium, and on the viscosity of the system created in the nuclear collision [7–10].

The experimental measurement of these anisotropic patterns is based on the analysis of the azimuthal correlations among the reconstructed particles. It is therefore sensitive to various sources of particle correlations (such as particle decays, jet production, momentum conservation) that have nothing to do with the collective motion and are generally called nonflow correlations. It is therefore important to have proper Monte Carlo simulations of the physical events to study the appropriate corrections for nonflow effects. This is of crucial importance at the LHC energy where jets will be copiously produced, thus increasing the effect of nonflow correlations on the observed v_2.

Some of the Monte Carlo microscopic generators commonly used to generate heavy ion collisions, however, do not include collective effects and the resulting distributions of particle azimuthal angles relative to the reaction plane are isotropic. In HIJING [11] anisotropy is actually present when the parton energy loss is activated. However, this anisotropy does not come from a collective (hydrodynamic like) motion, but it is a consequence of the radiative energy loss that depends on the path length of the parton in the medium.

It is, however, possible to introduce anisotropy in a second step after particle generation following the prescriptions described in Ref. [1]. This method, used among others by the ALICE [12] and ATLAS [13] experiments, is not based on a dynamical description of the system evolution from the thermalization time up to thermal freeze-out, but it just consists in changing in a suitable way the momentum components of the final hadrons. This is obtained by modifying the azimuthal angle of each particle according to

$$\varphi_0 \to \varphi = \varphi_0 + \sum_n \frac{-2}{n}\tilde{v}_n \sin[n(\varphi_0 - \Psi_{\rm RP})], \tag{2}$$

where φ_0 is the original azimuthal angle (distributed isotropically) given to the particle by the Monte Carlo generator, $\tilde{v}_n$ are the input values of the Fourier coefficients, which can be functions of transverse momentum, rapidity, and particle species.[1] $\Psi_{\rm RP}$ is the reaction plane angle in the transverse plane, i.e., the direction of the added flow. The resulting azimuthal distribution of particles is anisotropic with flow coefficients given by

$$v_n = \langle\cos[n(\varphi - \Psi_{\rm RP})]\rangle \approx \tilde{v}_n, \tag{3}$$

which, as pointed out in Ref. [1], agree with the input values $\tilde{v}_n$ only for small values of $\tilde{v}_n$.

To simplify the formulas and the plots, in the following we consider the case in which only elliptic flow is present. The general discussion is, however, valid for all the harmonics and

[1]The preservation of correlations, which is of crucial importance, is discussed in more detail at the end of the article.

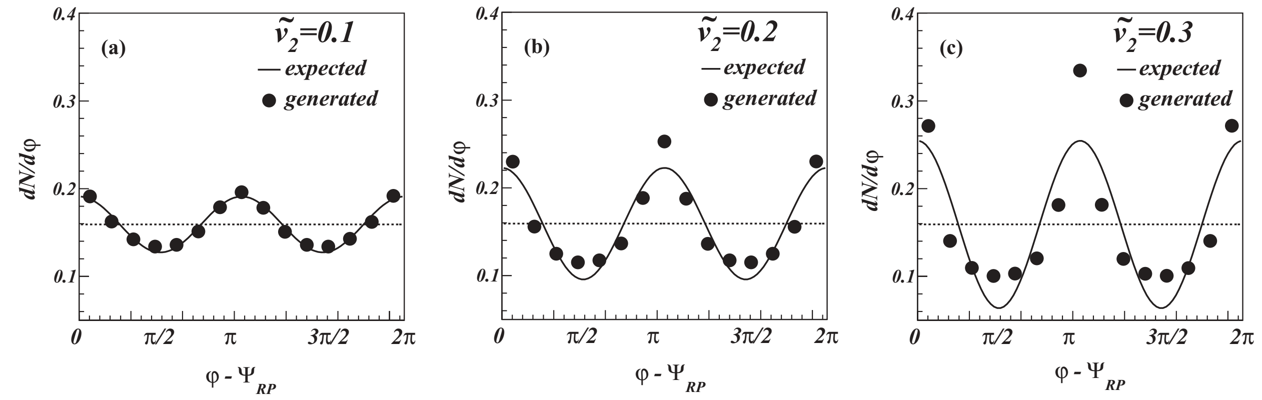

FIG. 1. Azimuthal distributions of particles generated isotropically and transformed with Eq. (4) for three values of input $\tilde{v}_2$. The expected $1 + 2\tilde{v}_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]$ trend is superimposed. The dotted line indicates azimuthal isotropic distribution of the particles ($dN/d\varphi = 1/2\pi$).

the extension of the formulas to the other terms of Fourier series is straightforward. If we assume that all v_n coefficients except v_2 are zero, Eq. (2) reduces to

$$\varphi_0 \rightarrow \varphi = \varphi_0 - \tilde{v}_2 \sin[2(\varphi_0 - \Psi_{\mathrm{RP}})]. \tag{4}$$

The resulting $dN/d\varphi$ obtained by changing the azimuthal angle of particles generated with isotropic azimuthal distributions are shown as solid circle markers in Fig. 1 for three values of $\tilde{v}_2$ (namely, 0.1, 0.2, and 0.3) together with the expected $1 + 2\tilde{v}_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]$ trend. It is evident that the expected $1 + 2v_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]$ modulation is reproduced only for small values of input $\tilde{v}_2$ (which is the case of particles with small transverse momentum). For large values of input anisotropy (i.e., $\tilde{v}_2 > 0.1$) a significant deviation from the expected trend is observed.

In Fig. 2(a) we report the v_2 values calculated starting from the generated $dN/d\varphi$ distributions, obtained by applying the transformation in Eq. (4), as a function of the input value $\tilde{v}_2$. The generated value of Ψ_{RP} (uniformly distributed in $[-\pi, \pi]$) has been used to define the particle azimuthal angles relative to the reaction plane ($\varphi - \Psi_{\mathrm{RP}}$). The resolution on the experimental estimation of Ψ_{RP} is not taken into account because only the azimuthal anisotropic pattern generation is being investigated. The v_2 values have been extracted both as $\langle\cos[2(\varphi - \Psi_{\mathrm{RP}})]\rangle$ (solid triangles) and from a fit to the expected trend $1 + 2v_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]$ with v_2 as a free parameter (open squares). In Fig. 2(b) the relative difference $(\tilde{v}_2 - v_2)/\tilde{v}_2$ is plotted as a function of $\tilde{v}_2$. It can be seen that for large values of $\tilde{v}_2$ (>0.1) the resulting anisotropy v_2 differs from the input value $\tilde{v}_2$ and a correction is needed, as stated in Ref. [1]. The values of v_2 extracted from the fit to the generated distribution are further affected by the fact that the fitting function does not reproduce the features of the generated distribution. It should be noted that the values of v_2 measured at RHIC for transverse momenta $\gtrsim 1$ GeV/c [3,4] as well as the ones predicted for the LHC [14,15] fall in the range where the method is not giving the proper azimuthal distribution to the generated particles.

The discrepancy between the input value $\tilde{v}_2$ and the one extracted from the generated distribution is due to the

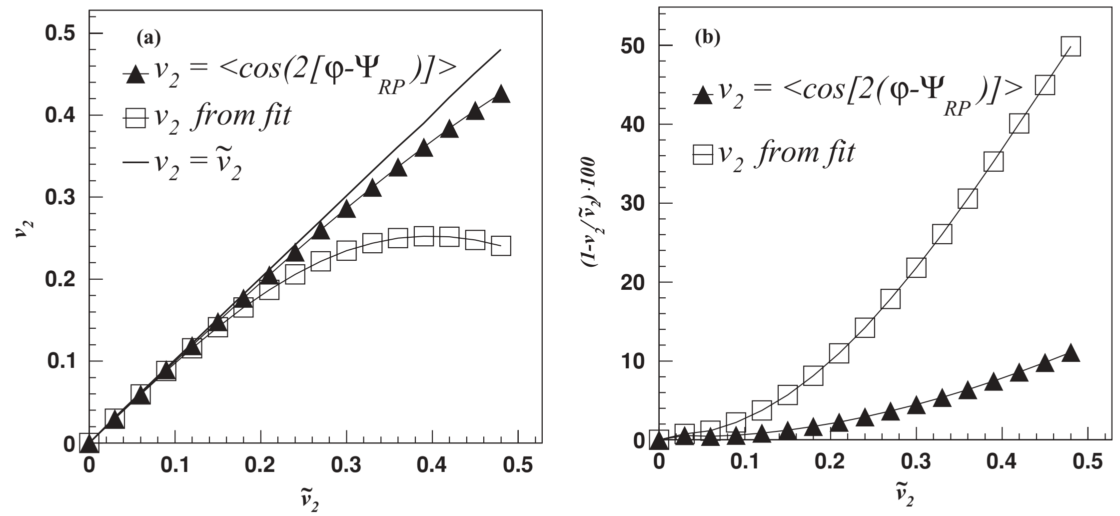

FIG. 2. (a) v_2 extracted from generated azimuthal distributions as a function of input $\tilde{v}_2$ in the case of anisotropy applied via Eq. (4). (b) Relative difference between resulting v_2 and input $\tilde{v}_2$.

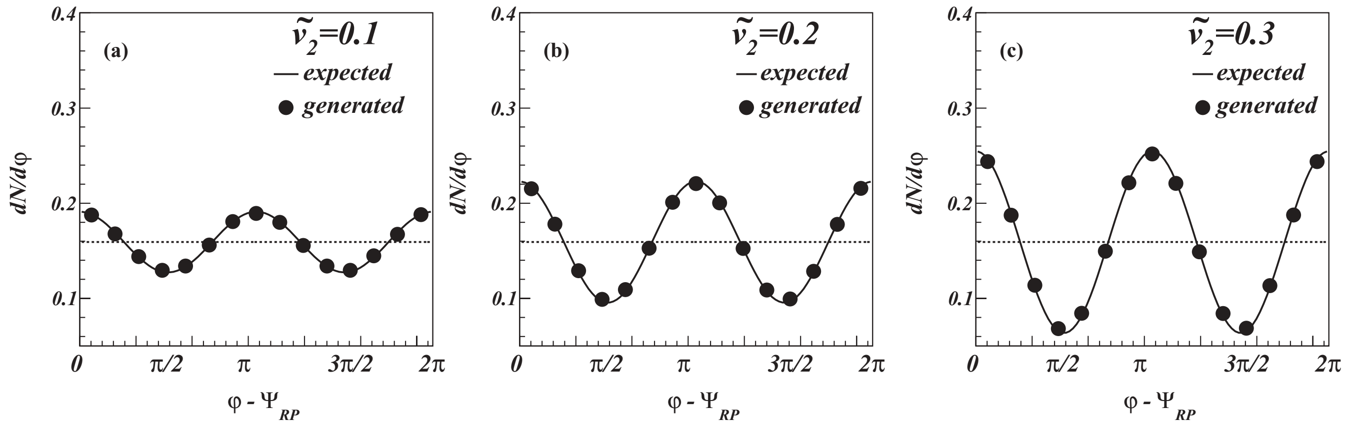

FIG. 3. Azimuthal distributions of particles generated isotropically and transformed with Eq. (8) for three values of input $\tilde{v}_2$. The expected $1 + 2\tilde{v}_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]$ trend is superimposed. The dotted line indicates azimuthal isotropic distribution of the particles ($dN/d\varphi = 1/2\pi$).

approximations used to calculate the formula for the angular shift. The starting point is a distribution, which is isotropic in the original angle φ_0:

$$\frac{dN}{d\varphi_0} = \frac{1}{2\pi}. \tag{5}$$

A proper transformation of the azimuthal angle $\varphi_0 \to \varphi$ should give a distribution:

$$\frac{dN}{d\varphi} = \frac{1}{2\pi}\{1 + 2\tilde{v}_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]\}. \tag{6}$$

The requirement is therefore to have

$$\frac{dN}{d\varphi} = \frac{dN}{d\varphi_0}\frac{\partial\varphi_0}{\partial\varphi} = \frac{1}{2\pi}\frac{\partial\varphi_0}{\partial\varphi}, \tag{7}$$

which gives as solution

$$\varphi = \varphi_0 - \tilde{v}_2 \sin[2(\varphi - \Psi_{\mathrm{RP}})]. \tag{8}$$

The difference between Eqs. (4) and (8) is just the presence of φ instead of φ_0 in the argument of the sine. Equation (4), being a result of a first order iteration, is a good approximation of the correct transformation if $\varphi \approx \varphi_0$; i.e., if the modification of the azimuthal angle is small, which is the case only for small values of the input anisotropy $\tilde{v}_2$. For large values of anisotropy, Eq. (8) must be used. Because it is a transcendental equation, the solution for φ should be found numerically.

The resulting azimuthal distribution of particles obtained by solving Eq. (8) with the Newton method are shown as solid circles in Fig. 3 together with the expected $1 + 2\tilde{v}_2 \cos[2(\varphi - \Psi_{\mathrm{RP}})]$ trend. It can be seen that the produced azimuthal distributions follow the expected modulation also for large values of input anisotropy.

In Fig. 4(a) we report the calculated v_2 (obtained both as $\langle\cos[2(\varphi - \Psi_{\mathrm{RP}})]\rangle$ and from fits to the particle azimuthal distributions) as a function of the input $\tilde{v}_2$. In this case, the $dN/d\varphi$ distributions have been obtained with the corrected formula [Eq. (8))] for the azimuthal angle transformation. In Fig. 4(b) the relative difference between the input value $\tilde{v}_2$ and the resulting anisotropy v_2 is displayed: the systematic deviation observed in Fig. 2 is no longer present.

To conclude, the transformation of the azimuthal angles that should be used to create an anisotropic azimuthal distribution starting from an isotropic one in case of large values of v_n is given by the following transcendental equation:

$$\varphi + \sum_n \frac{2}{n}\tilde{v}_n \sin[n(\varphi - \Psi_{\mathrm{RP}})] = \varphi_0, \tag{9}$$

where φ_0 is the original (isotropically distributed) value of the particle azimuthal angle. The resulting distribution $dN/d\varphi$ is described by Fourier coefficients $v_n = \langle\cos[n(\varphi - \Psi_{\mathrm{RP}})]\rangle$ that agree with the input values $\tilde{v}_n$ also for large values of

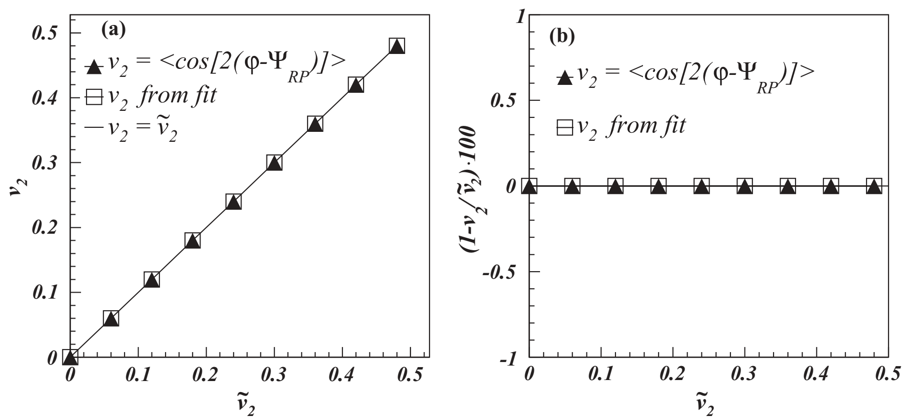

FIG. 4. (a) v_2 extracted from generated azimuthal distributions as a function of input $\tilde{v}_2$ in the case of anisotropy applied via Eq. (8). (b) Relative difference between resulting v_2 and input $\tilde{v}_2$.

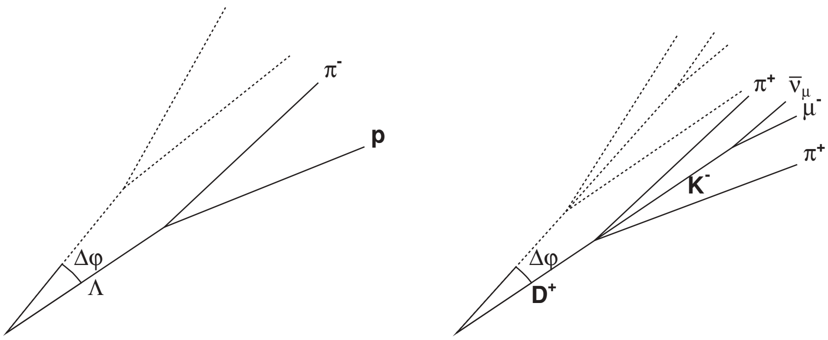

FIG. 5. Scheme of $\Lambda \to p\pi^-$ and $D^+ \to K^-\pi^+\pi^+$ decays. Dashed lines show the effect of the azimuthal angle modifications used to introduce elliptic flow.

input anisotropy. This method can be easily added as a second step (after burner) treatment of particles produced by any Monte Carlo generator. The solution of the transcendental equation with Newton algorithm is fast. In the case of the HIJING MC generator the time required for the rotation of particles produced in Pb-Pb collisions at $\sqrt{s} = 5500$ GeV (in average 35 000 particles in full phase space) is the $\sim 5\%$ of the particles' production time. It is important to note that because of the shifting of the azimuthal angles, this transformation does not alter existing multiparticle correlations, due to quantum statistics, resonances, mini and real jet production, etc. However, one has to be careful with the correlations due to global momentum conservation, because as a consequence of the applied shift, particles systems may obtain an uncompensated transverse momentum (see Ref. [16] for a discussion about the importance of correlations due to global transverse momentum conservation). Another important source of correlations comes from the decay of unstable particles. In this respect, it should be noted that special care must be taken to introduce anisotropy in decay chains where the mother particle participates in the collective motion and it decays after decoupling from the system. This is typically the case of nonstable long-lived particles (such as Λ's, D mesons, etc.) that are experimentally reconstructed from their decay products. In these cases, the azimuthal angles of the daughter products should not be modified independently using Eq. (9) and the $\tilde{v}_n$ coefficients of their own species. On the contrary, the azimuthal angle of the mother particle should be transformed with Eq. (9) before it decays. However, the decay products are usually provided by the Monte Carlo generator; thus, it is sufficient to modify coherently the azimuthal angle of the daughter products by using the same angular shift ($\Delta\varphi = \varphi - \varphi_0$) computed for their mother. As a consequence, the space coordinates of the decay vertex need to be properly shifted, as sketched in Fig. 5.

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