Momentum dependence of the nuclear mean field and multifragmentation in heavy-ion collisions

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We report the consequences of implementing momentum dependent interactions (MDI) on multifragmentation in heavy-ion reactions over entire collision geometry. The evolution of a single cold nucleus using static soft and soft momentum dependent equations of state demonstrates that inclusion of momentum dependence increases the emission of free nucleons. However, no heavier fragments are emitted artificially. The calculations performed within the framework of quantum molecular dynamics approach suggest that MDI strongly influence the system size dependence of fragment production. A comparison with ALADiN experimental data justifies the use of momentum dependent interactions in heavy-ion collisions.

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I. INTRODUCTION

Heavy-ion collisions have always played a fascinating role in exploring various aspects of nuclear dynamics such as fission-fusion [1–3], multifragmentation [4–12], collective flow [6,9,13–17], particle production [5,18,19], etc. The wider energy spectrum available because of modern accelerator technologies has become a powerful tool in experimental and theoretical studies in exploring the nature of hot and compressed nuclear medium via heavy-ion collisions.

It is well accepted that the outcome of a reaction depends not only on the density but also on the momentum space [18,20]. The momentum dependence of the equation of state can be extracted from the real part of the optical potential. This potential is expected to affect those nucleons from targets and projectiles that possess larger relative momenta. The momentum dependence of the nucleon-nucleon (*n-n*) potential is found to affect drastically the collective flow observables [21,22] and particle production [4,18,19]. It has been shown that observables related to the particle production, e.g., π, κ, and λ yields, n_d/n_p ratios, etc., are strongly influenced by the momentum dependence of the *n-n* interaction [18,19]. A strong influence of momentum dependent interactions was also observed on fragment flow for $E_{\text{lab}} \geqslant 400$ MeV/nucleon. In higher incident energy regimes, momentum dependent interactions (MDI) cause stronger reduction in the number of *n-n* collisions leading to more pronounced transverse flow. The momentum dependent interactions, therefore, increase the mean free path of nucleons and consequently affect the stopping and thermalization of nuclear matter [21,23].

On the contrary, very few attempts exist in the literature that shed light on the consequences of implementing momentum dependent interactions in fragmentation [10,22]. One of the basic problem with MDI is the strong repulsion created in the nuclear environment. As a result, nuclei propagating with MDI tend to be destabilized and start decaying via emission of free nucleons and clusters quite early during the reaction. Interestingly, the role of momentum dependent interactions depends crucially on the impact parameter of the reaction. It is found to enhance the energy of disappearance of flow in central collisions [15], whereas it reduces the energy of disappearance of flow in peripheral collisions [16,17]. Similarly, MDI reduce the production of fragments in central collisions whereas it enhances the same in peripheral collisions [22]. However, one always remains concerned about the stability of nuclei propagating with momentum dependent interactions. Even the use of a cooling procedure via Pauli potential is also reported in the literature [7]. Our present aim, therefore, is to investigate the stability of cold nuclei propagating under the influence of momentum dependent interactions and to see whether one can study fragmentation with MDI or not. An attempt to study the system size effects in the presence of momentum dependent forces is also made. We also confront our calculations with the multifragmentation data of the ALADiN group [11], which has a *rise and fall* variation with impact parameter. This study is carried out within the framework of quantum molecular dynamics (QMD) transport model [4,5]. The QMD model and implementation of momentum dependent potential are described in Sec. II. Our results are presented in Sec. III and summarized in Sec. IV.

II. QMD MODEL AND MOMENTUM DEPENDENT INTERACTIONS

The QMD model [4,19] is an *n*-body transport theory that simulates the heavy-ion (HI) reactions between 30 MeV/nucleon and 1 GeV/nucleon on an event by event basis. It includes quantum features like Pauli blocking, stochastic scattering, and particle production. Here each nucleon is represented by a Wigner distribution function of the form

$$f_i(\mathbf{r}, \mathbf{p}, \mathbf{t}) = \frac{1}{(\pi\hbar)^3} e^{-(\mathbf{r}-\mathbf{r}_i(t))^2/2L} . e^{-(\mathbf{p}-\mathbf{p}_i(t))^2 2L/\hbar^2}, \qquad (1)$$

where $\mathbf{r}_i(t)$ and $\mathbf{p}_i(t)$ define the classical orbit, the center of an ith Gaussian wave packet in phase space that evolves in time. The centers of these Gaussian wave packets propagate according to the classical equations of motion [4,6]. The interaction part used in the QMD model consists of local Skyrme interaction, a finite range Yukawa term, and an

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effective Coulomb interaction among protons, i.e.,

$$V^{\rm tot} = V^{\rm Sk} + V^{\rm Yuk} + V^{\rm Coul}, \tag{2}$$

with local Skyrme interaction consisting of two- and three-body interactions:

$$V^{\rm Sk} = t_1\delta(\mathbf{r}_i - \mathbf{r}_j) + t_2\delta(\mathbf{r}_i - \mathbf{r}_j)\delta(\mathbf{r}_i - \mathbf{r}_k). \tag{3}$$

Because the QMD model is an n-body theory, Eq. (3) can be reduced to a density dependent potential in the limit of infinite nuclear matter limit as

$$U^{\rm Sk} = \alpha\left(\frac{\rho}{\rho_o}\right) + \beta\left(\frac{\rho}{\rho_o}\right)^2. \tag{4}$$

The momentum dependence of the nuclear mean field is included in Eq. (2) via momentum dependent interaction as

$$V^{\rm MDI} = t_4 \ln^2\left[t_5(\mathbf{p}_i - \mathbf{p}_j)^2 + 1\right]\delta(\mathbf{r}_i - \mathbf{r}_j), \tag{5}$$

with parameters $t_4 = 1.57$ MeV and $t_5 = 5.0 \times 10^{-4}\ ({\rm MeV}/c)^{-2}$. This parametrization is deduced from the real part of the proton-nucleus optical potential that reproduces the experimental data up to 1 GeV/nucleon [4,24]. In an infinite nuclear matter limit, generalized n-n potential [Eqs. (2) and (5)] leads to the following density and momentum dependent potential (without Coulomb and Yukawa terms):

$$U(\rho, \mathbf{p}) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^\gamma + t_4\ln^2[t_5(\mathbf{p}^2+1)]\left(\frac{\rho}{\rho_o}\right). \tag{6}$$

The parameters α, β, and γ in Eq. (6) must be readjusted in the presence of momentum dependent interactions so as to reproduce the ground state properties of nuclear matter. The parameters corresponding to soft, hard, and their momentum dependent versions are labeled as S, H, SM, and HM, respectively. The constants α and β give the proper rms radii and binding energies of nuclei across the periodic table. Parameter γ gives us the possibility to examine the compressibility and hence the equation of state. The set of parameters corresponding to soft (S), hard (H), and their momentum dependent versions SM and HM, respectively, can be found in Ref. [5].

III. RESULTS AND DISCUSSION

A. Stability of cold QMD nuclei

To address the question of stability of the computational nucleus in the presence of momentum dependent interactions (MDI), we initialize a single cold projectile using a soft (S) equation of state and a soft momentum dependent equation of state (SM). Earlier theoretical attempts ranging from giant monopole resonances [25] to nucleosynthesis of heavy elements in mergers of neutron stars [26] could be explained if the equation of state (EoS) is relatively *soft* rather than if it is *stiff*. Another study concerning the linear momentum transfer occurring in central HI collisions also showed that a soft compressibility modulus is needed to explain the experimental data [27,28]. These observations motivated us in the choice

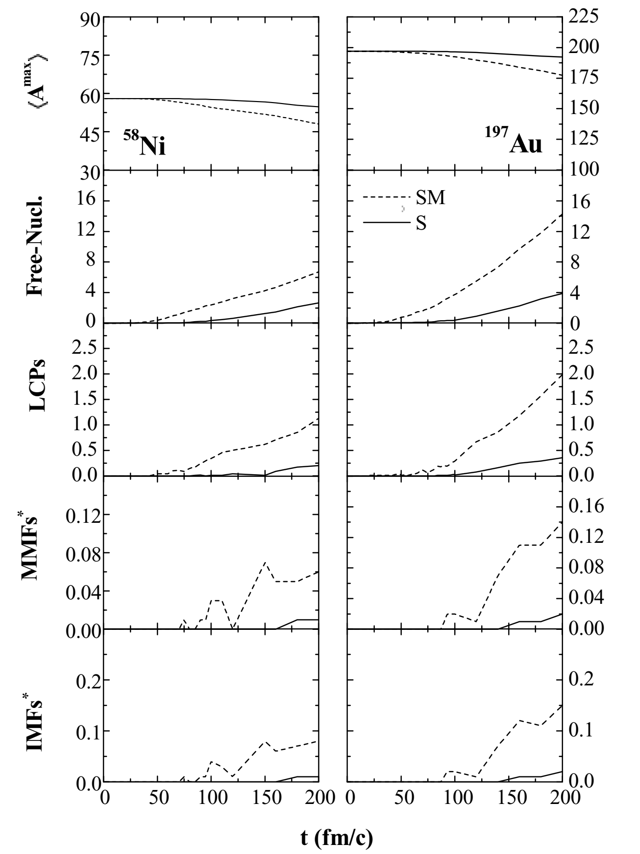

FIG. 1. The time evolution of heaviest fragment $\langle A^{\rm max}\rangle$, free nucleons, LCPs ($2 \leqslant A \leqslant 4$), MMFs* ($5 \leqslant A \leqslant 9$), and IMFs* ($5 \leqslant A \leqslant A_P/3$) ($A_P$ being the mass of projectile nucleus) emitted from a single cold nucleus of ^{58}Ni (left panel) and ^{197}Au (right panel). Results obtained with soft (S) equation of state are represented by solid lines whereas dashed lines show results with soft momentum dependent (SM) equation of state. A superscript asterisk indicates that the heaviest fragment has been excluded.

of comparatively softer EoS. We follow the cluster emission pattern and rms radii of a few computational nuclei. Figure 1 shows the time evolution of cold QMD nuclei, namely, ^{58}Ni and ^{197}Au initialized with S and SM interactions. The cluster emission is followed for the time span of 200 fm/c. Here, $A^{\rm max}$ denotes the size of the residual nucleus. This should be close to that of the parent nucleus if there is no destabilization of the nucleus. The sizes of parent nuclei ^{58}Ni and ^{197}Au reduce with the inclusion of momentum dependent interactions compared to nuclei propagating with static soft interactions alone. The SM interactions caused an enhanced emission of free nucleons and light charged particles (LCPs) ($2 \leqslant A \leqslant 4$). However, medium mass fragments (MMFs) ($5 \leqslant A \leqslant 9$) and intermediate mass fragments (IMFs) [($5 \leqslant A \leqslant A_P/3$); A_P being the mass of projectile] are almost insensitive toward momentum dependent interactions. A superscript asterisk indicates that the heaviest fragment has been excluded from the multiplicities of MMFs and IMFs. Only a small fraction is emitted as IMFs. It seems that nucleons close in space are emitted in bulk, therefore leading to an enhanced emission of light clusters. On the contrary, very few nucleons, LCPs, and heavier clusters are

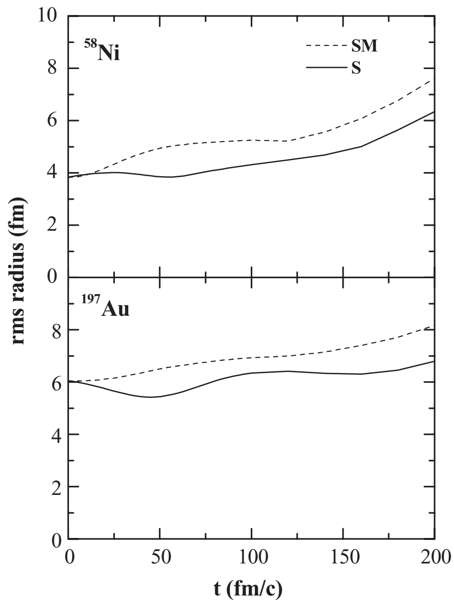

FIG. 2. The time variation of rms radii of single cold nuclei of ^{58}Ni (top panel) and ^{197}Au (bottom panel) using soft (S) equation of state (solid lines) and soft momentum dependent (SM) interactions (dashed lines).

emitted when propagating with soft EoS. The enhanced evaporation with MDI is also due to the repulsive nature of these interactions. Does this enhanced emission prohibit one from using MDI for fragmentation? If one looks carefully, the majority of mass that leaves the gold nucleus (for example, with MDI about 19 units are emitted and $\langle A^{\max} \rangle$ is close to 177) is in the form of free nucleons. In the above gold nucleus, out of 19 units about 15 are in terms of free nucleons. In other words, we see that nucleons from the surface are emitted and there is no contribution toward the emission of intermediate mass fragments. One sees that even with MDI, only 0.15 IMF is emitted on the average. Realizing that as many as 10–12 IMFs can be seen emitted in a Au + Au reaction [29], this number with MDI is negligible.

A survey of the time evolution of rms radii of a single QMD nucleus also depicts the same picture. In Fig. 2 we show the time evolution of rms radii of ^{58}Ni and ^{197}Au nuclei till 200 fm/c. The rms radius of nucleus with SM interactions increases gradually compared to that initialized with static soft interactions. This behavior reflects that MDI create additional repulsions among nucleons that lead to enhanced emission of free nucleons. The rms radii of gold and nickel nuclei in the soft case shows negligible deviation for the characteristic time of HI collision. As discussed above, this enhanced radius is due to the emission of free nucleons and not due to the IMFs. Therefore, one can study the fragmentation with MDI because the structure of IMFs is not altered by the inclusion of MDI.

B. Heavy-ion collisions and system size effects

After investigating the behavior of cold nuclei initialized with momentum dependent interactions, we study the effect of momentum dependent forces in heavy-ion reactions. One of

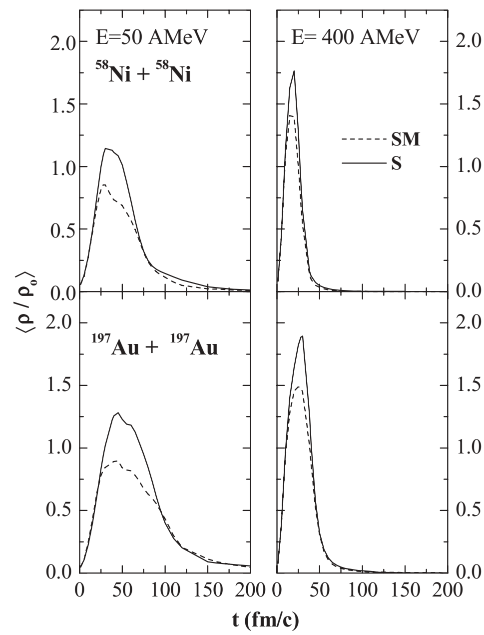

FIG. 3. The average nucleonic density $\langle \rho/\rho_o \rangle$ calculated in a central sphere of 2 fm radius versus reaction time for the central collisions of ^{58}Ni + ^{58}Ni (top panel) and ^{197}Au + ^{197}Au (bottom panel). The results obtained with soft (S) and soft momentum dependent (SM) interactions are compared at 50 A MeV (left) and 400 A MeV (right).

the observables linked with the compression and expansion of nuclear matter is the density of the fragmenting system. The total nuclear matter density is obtained as

$$\rho(\mathbf{r}, t) = \sum_{j=1}^{A_T + A_P} \frac{1}{(2\pi L)^{3/2}} e^{-(\mathbf{r}-\mathbf{r}_j(t))^2/2L}. \quad (7)$$

Here A_T and A_P stand for the target and projectile masses, respectively. In our approach, average nuclear matter density $\langle \rho/\rho_o \rangle$ is calculated in a sphere of 2 fm radius.

In Fig. 3, we display the time evolution of the average nucleon density $\langle \rho/\rho_o \rangle$ reached in the central region for the head-on collisions of ^{58}Ni + ^{58}Ni and ^{197}Au + ^{197}Au at incident energies of 50 and 400 A MeV. The maximal average density tends to reduce with inclusion of momentum dependent interactions. This happens because of additional n-n repulsions created in the system that prohibit compression of nuclear matter to a significant level. This difference in the behavior of $\langle \rho/\rho_o \rangle$ calculated using S and SM interactions diminishes at higher incident energies (400 A MeV). This is due to the fact that in central collisions at 400 A MeV most of the initial n-n correlations are already destroyed and matter is already scattered, and, therefore, repulsion generated due to MDI does not play any significant role. As a result, we do not see much difference in average central density reached at higher incident energy. Another important quantity related with the initial compression of nuclear matter is the rate of binary collisions. We have also checked the collision rate for these reactions and its behavior is found to be consistent with the density profile.

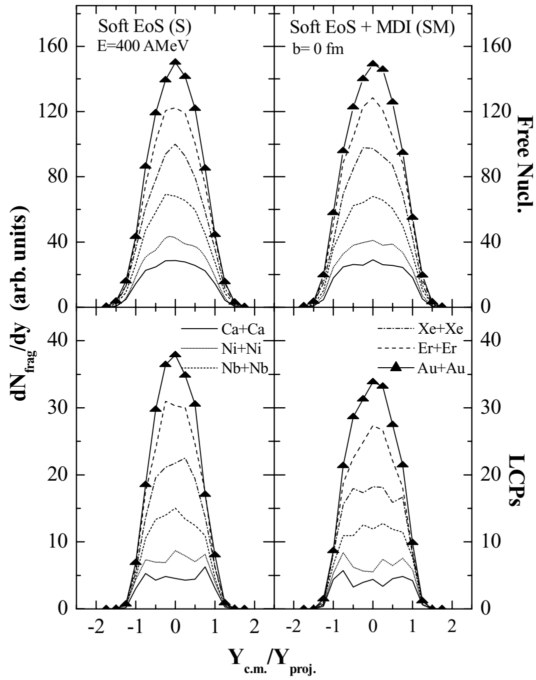

FIG. 4. The rapidity distribution dN_{frag}/dy of free nucleons and LCPs ($2 \leqslant A \leqslant 4$) as a function of scaled rapidity $Y_{\text{c.m.}}/Y_{\text{proj.}}$, with $Y_{\text{proj.}}$ being the rapidity of the projectile for the head-on collisions at 400 A MeV.

The rapidity distribution of nucleons is another useful tool to characterize the stopping and thermalization of the nuclear matter. We have displayed in Fig. 4 the fragment rapidity distribution dN_{frag}/dy of free nucleons and LCPs for central collisions of six different symmetric systems at 400 A MeV. The results are displayed here using soft EoS (left panels) and soft EoS including MDI (right panels). The rapidity distribution is more "isotropic" and nearly full stopping is achieved in heavier systems like Au + Au and Er + Er. In lighter systems, however, a larger fraction of particles is concentrated near target and projectile rapidities, resulting in a broad Gaussian shape. This feature can be seen in both S and SM cases. The lighter systems, therefore, exhibit larger *transparency* effect, i.e., less stopping. Such features are also observed in the experimental data of the FOPI group [30]. Based on the experimental observations and theoretical trends, one can say that the smaller the system the lesser is the stopping. With MDI, a slight increase in transparency effect is seen due to lesser stopping of particles in the longitudinal direction. This happens because of reduction in *n*-*n* collisions that deflect the fragments in the transverse direction. As a results, one obtains less particles being stopped in the longitudinal direction.

The system size effects in the production probability of different kinds of fragments has been studied and predicted by our group [8]. Here we extend the same study with reference to momentum dependent interactions. For this analysis, we simulated the central collisions of six symmetric systems, ^{40}Ca + ^{40}Ca, ^{58}Ni + ^{58}Ni, ^{93}Nb + ^{93}Nb, ^{131}Xe + ^{131}Xe, ^{168}Er + ^{168}Er, and ^{197}Au + ^{197}Au, at incident energies of 50 and 400 A MeV. We also parametrized the multiplicities

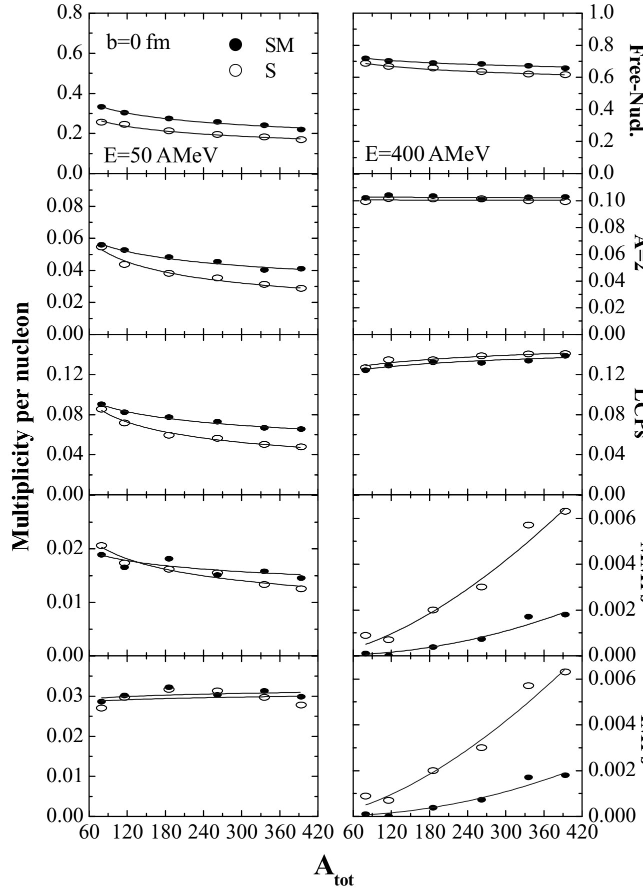

FIG. 5. The final state scaled multiplicity (calculated at 200 fm/c) of free nucleons, fragments with mass $A = 2$, LCPs ($2 \leqslant A \leqslant 4$), MMFs* ($5 \leqslant A \leqslant 9$), and IMFs* ($5 \leqslant A \leqslant \min\{A_P/3, 65\}$) as a function of total mass of the system A_{tot}. Results shown here are at incident energies of 50 A MeV (l.h.s) and 400 A MeV (r.h.s). Open circles depict the calculations with soft (S) interaction and solid circles are for soft momentum dependent (SM) interactions. A superscript asterisk means that the heaviest fragment has been excluded.

as a function of the total mass of the composite system using a power law of the form cA_{tot}^{τ}, with A_{tot} being the total mass of the system. Figure 5 displays the "reduced multiplicity," i.e., multiplicity per nucleon of various kinds of fragments. It is clear that the system size effects are more visible in the soft equation of state compared to the soft momentum dependent case. A negative slope obtained for the multiplicity of free nucleons, fragments of mass $A = 2$, and LCPs at 50 A MeV indicate their origin from the surface of interacting nuclei. As we move to the momentum dependent version, additional breakup of *n*-*n* correlations leads to enhanced emission of free nucleons and light charged particles. As a result, the multiplicity of MMFs* ($5 \leqslant A \leqslant 9$) and IMFs* ($5 \leqslant A \leqslant \min\{A_P/3, 65\}$) (excluding the largest fragment $A^{\max}$) gets reduced at 400 A MeV, indicating the vanishing of system size effect with MDI. In higher energy regimes, cluster production via emission of MMFs* and IMFs* is strongly suppressed in the presence of MDI. It is worth mentioning that earlier studies, e.g., see Ref. [31], also reported the momentum dependent potential to be more repulsive for high momentum

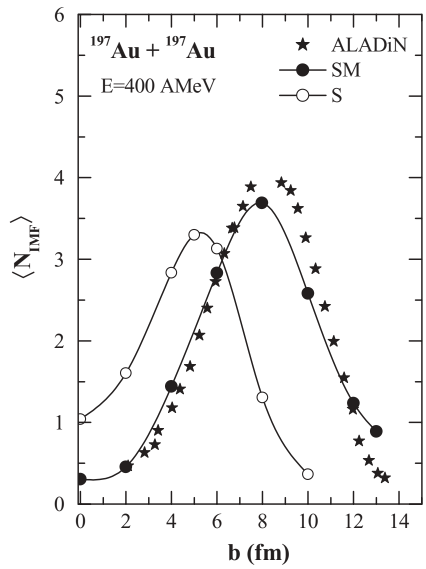

FIG. 6. The mean IMF multiplicity $\langle N_{\rm IMF}\rangle$ vs the impact parameter b for the reaction of ^{197}Au + ^{197}Au at 400 A MeV. The QMD calculations (at 300 fm/c) using soft EoS (open circles) and soft momentum dependent EoS (solid circles) are compared with the ALADiN experimental data (solid stars).

nucleons. This leads to enhanced emission of free nucleons and LCPs. A similar enhancement of the nucleon emission and light cluster production was predicted on inclusion of momentum dependent effective interactions in the isoscalar nuclear potential and symmetry potential [31,32]. Contrary to this, with a static soft equation of state, the production probabilities of MMFs and IMFs scale with the system size as power law: $cA_{\rm tot}^{\tau}$ with power factor τ close to 3/2.

Let us now try to confront our calculations with the experimental data of the ALADiN group [11]. The experimental data are very fascinating because it has been shown that there is a *rise and fall* of multiplicity of intermediate mass fragments with impact parameter [11]. However universality is observed with mass of the system and with incident energies exceeding 400 A MeV. In Fig. 6, we display the multiplicity of intermediate mass fragments as a function of impact parameter using soft (S) and soft momentum dependent (SM) equation of state. We see that the entire spectrum is very well reproduced by the momentum dependent interactions. One should also keep in the mind that for central impact parameters, different experimental groups like FOPI [9], ALADiN [10,11], and Miniball [29] differ significantly in the multiplicities of IMFs. Overall, we see a clear need for momentum dependent interactions in heavy-ion collisions.

IV. SUMMARY

We here present a detailed study on the consequences of employing momentum dependent potential in multifragment emission. Investigation of a single cold nucleus initialized with soft (S) and soft momentum dependent (SM) equations of state reveals that momentum dependent interactions act as a *destabilizing* factor, which results in enhanced emission of free nucleons only. However, no change is seen toward artificial emission of IMFs. Further, momentum dependent interactions are observed to weaken the system size effects studied at 50 and 400 A MeV. A comparison of model calculations with ALADiN data on Au + Au reactions favors strongly the use of momentum dependent equation of state in heavy-ion collisions.

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