Critical comparison of Kramers' fission width with the stationary width from the Langevin equation

Jhilam Sadhukhan and Santanu Pal*

Physics Group, Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India

(Received 4 March 2009; published 17 June 2009)

It is shown that Kramers' fission width, originally derived for a system with constant inertia, can be extended to systems with a deformation-dependent collective inertia, which is the case for nuclear fission. The predictions of Kramers' width for systems with variable inertia are found to be in very good agreement with the stationary fission widths obtained by solving the corresponding Langevin equations.

DOI: 10.1103/PhysRevC.79.064606 PACS number(s): 24.75.+i, 24.10.Pa, 25.70.Gh, 25.85.Ge

I. INTRODUCTION

Fission of compound nuclei formed in heavy ion induced fusion reactions at energies above the Coulomb barrier has been investigated quite extensively, both experimentally and theoretically, during the last two decades. The multiplicities of pre-scission neutrons, light charged particles, and giant dipole resonance γs have been measured [1–8] and compared with the predictions of the statistical model of nuclear fission [6–11]. These investigations have revealed that the statistical model of nuclear fission based on the transition-state method [12] where effects due to nuclear dissipation are not considered is inadequate to describe fission of highly excited heavy nuclei, and consequently, dissipative dynamical models [13–15] are found to be essential to account for the experimental data. Similar conclusions are also reached while analyzing the evaporation residue cross sections of highly fissile compound nuclei [9,10,16]. Consequently, fission has become a useful probe to study the dissipative properties of the nuclear bulk.

A dynamical model for fission of a hot compound nucleus was first proposed by Kramers [17] based on its analogy to the motion of a Brownian particle in a heat bath. In this model, the collective fission degrees of freedom represent the Brownian particle while the rest of the intrinsic degrees of freedom of the compound nucleus correspond to the heat bath. The dynamics of such a system is governed by the appropriate Langevin equations or equivalently by the corresponding Fokker-Planck equation. Kramers analytically solved the Fokker-Planck equation with a few simplifying assumptions and obtained the stationary width of fission. Specifically, parabolic shapes were considered for the nuclear potential at the ground state and at the saddle region and the inertia of the fissioning system was assumed to be shape-independent and constant. Subsequently, the stationary width predicted by Kramers was found to be in reasonable agreement with the asymptotic fission width obtained from numerical solutions of the Fokker-Planck [18–25] and Langevin [26–30] equations in which harmonic oscillator potentials and constant inertia were used.

In the present work, we examine the applicability of Kramers' expression for stationary fission width for more realistic systems. Specifically, we use the finite range liquid drop model (FRLDM) potential [31,32] and shape-dependent inertia. To this end, we first approximate the FRLDM potential with suitably defined harmonic oscillator potentials in order to make use of Kramers' expression for fission width. Since the centrifugal barrier changes the potential profile as the nuclear spin increases, the frequencies of the harmonic oscillator potentials approximating the FRLDM potential also develop a spin dependence [33–35]. Though the oscillator potentials are fitted to closely resemble the FRLDM potential, it is instructive to compare Kramers' fission width with that obtained from the numerical solution of Langevin equations where the full FRLDM potential is employed. Considering the width from the Langevin equations to represent the true fission width, this comparison enables us to confirm the validity of Kramers' expression for systems described by realistic potentials over the entire range of compound nuclear spin populated in a heavy ion induced fusion reaction. We next extend Kramers' formulation of stationary width in order to include the variation of the collective inertia with deformation. The Kramers' formula was generalized earlier [36] for variable inertia where a factor $\sqrt{m_g/m_s}$ was introduced in the expression for the fission width. The inertias at the ground state and at the saddle point are denoted respectively by m_g and m_s here. In a Langevin calculation with variable inertia, Karpov *et al.* [26] however reported that Kramers' width (without the above mentioned factor) predicts the asymptotic fission width very accurately. We therefore address this issue here in some detail and show that the difference lies in different matching conditions. We draw our conclusions by comparing Kramers' predicted widths with the widths calculated from the Langevin equations.

The main motivation for this work concerns the use of Kramers' width in statistical-model calculations for the decay of compound nuclei. The strength of nuclear dissipation is often used as an adjustable parameter in these calculations [7–11], the value of which is obtained by fitting experimental data. To extract an unambiguous value of the dissipation parameter, it is thus essential to ensure that Kramers' width accurately corresponds to the dissipative dynamics of the compound nucleus. In particular, this issue becomes important when realistic fissioning systems are considered whose specifications go beyond the simplifying assumptions made by Kramers. In this context, we presently study a system described by the FRLDM potential and a shape-dependent collective inertia.

*santanu@veccal.ernet.in

In the next section, we present the necessary steps taken to include the effect of variable inertia in Kramers' expression for the stationary fission width. The Langevin equations for fission are given in Sec. III, while a comparison between the results from the Langevin calculation and Kramers' prediction is made in Sec. IV. A summary of the results is presented in the last section.

II. KRAMERS' APPROACH TO THE STATIONARY FISSION WIDTH

To introduce a shape-dependent collective inertia into the analytical formulation of stationary fission width, we follow the work of Kramers [17,37] very closely here. The Liouville equation describing the fission dynamics in one-dimensional classical phase space is

$$\frac{\partial\rho}{\partial t} + \frac{p}{m}\frac{\partial\rho}{\partial c} + \left\{K - \frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right)\right\}\frac{\partial\rho}{\partial p} = \eta p\frac{\partial\rho}{\partial p} + \eta\rho + m\eta T\frac{\partial^2\rho}{\partial p^2}, \tag{1}$$

where ρ denotes the phase-space density, c is the collective coordinate with p as its conjugate momentum, and m is the collective inertia. The conservative and dissipative forces are given as $K = -\partial V/\partial c$ and $-\eta p$, respectively, where V is the collective potential and η is the dissipation coefficient. T represents the temperature of the compound nucleus. In what follows, we consider fission as a slow diffusion of Brownian particles across the fission barrier. When quasiequilibrium is reached and a steady diffusion rate across the fission barrier has been established, Eq. (1) becomes

$$\frac{p}{m}\frac{\partial\rho}{\partial c} + \left\{K - \frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right)\right\}\frac{\partial\rho}{\partial p} = \eta p\frac{\partial\rho}{\partial p} + \eta\rho + m\eta T\frac{\partial^2\rho}{\partial p^2}. \tag{2}$$

We make the Werner-Wheeler approximation [38,39] for incompressible and irrotational flow to calculate the collective inertia. The FRLDM potential is obtained by double folding a Yukawa-plus-exponential potential with the nuclear density distribution using the parameters given by Sierk [32]. The calculated FRLDM potential and the collective inertia of the nucleus ^{224}Th are shown as functions of deformation in Fig. 1.

In nuclear fission, a compound nucleus that is at a temperature significantly less than the height of the fission barrier mostly stays close to its ground-state configuration except for occasional excursions toward the saddle region when it has picked up sufficient kinetic energy from the thermal motion and which may eventually result in fission. Evidently, we do not consider transients that are fast nonequilibrium processes and happen for nuclei with vanishing fission barriers. Therefore, in the present picture, the Brownian particles are initially confined in the potential pocket at the ground-state configuration with a fission barrier V_B and for $V_B \gg T$, they can be assumed to be in a state of thermal equilibrium described

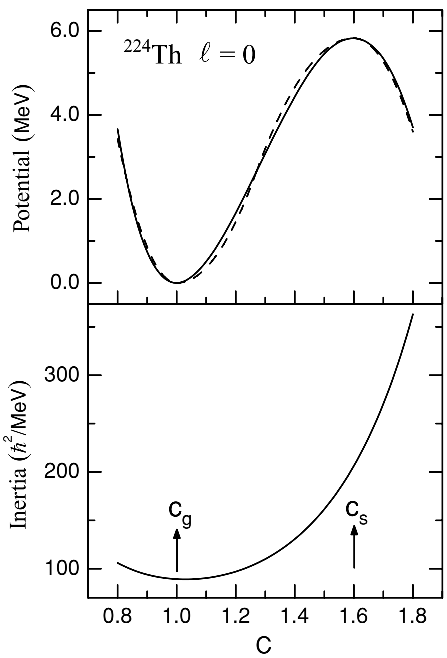

FIG. 1. FRLDM potential (solid line in upper panel) and the collective inertia (lower panel) of ^{224}Th. The dashed line in the upper panel is obtained by fitting the FRLDM potential with two harmonic oscillator potentials (see text). The ground-state (c_g) and saddle (c_s) configurations are also marked in the lower panel.

by the Maxwell-Boltzmann distribution

$$\rho = Ae^{-\left(\frac{p^2}{2m}+V\right)/T}, \tag{3}$$

where A is a normalization constant. We next seek a stationary solution of the Liouville equation which corresponds to a steady flow of the Brownian particles across the fission barrier. The desired solution should be of the form

$$\rho = AF(c,p)\,e^{-(\frac{p^2}{2m}+V)/T}, \tag{4}$$

such that $F(c,p)$ satisfies the boundary conditions

$$\begin{aligned} F(c,p) &\simeq 1 \quad \text{at} \quad c = c_g, \\ &\simeq 0 \quad \text{at} \quad c \gg c_s, \end{aligned} \tag{5}$$

where c_g and c_s define the ground state and the saddle deformations, respectively. The first boundary condition corresponds to a continuous change of both the potential and the inertia values with deformation. In this context, it may be pointed out that Hofmann *et al.* [36] considered discrete values of inertia for the saddle and ground-state configurations which resulted in the factor $\sqrt{m_g/m_s}$ in the stationary fission width expression. This factor, however, does not appear in the present work, since we consider a continuous variation of the inertia value.

Substituting Eq. (4) in the stationary Liouville equation, we obtain

$$m\eta T\frac{\partial^2 F}{\partial p^2} = \frac{p}{m}\frac{\partial F}{\partial c} + \frac{\partial F}{\partial p}\left\{-\frac{\partial V}{\partial c} + \eta p - \frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right)\right\}. \tag{6}$$

To assess the importance of the inertia derivative term in this equation, we estimate the magnitude of the term

$|\frac{p^2}{2}\frac{\partial}{\partial c}(\frac{1}{m})|$ with respect to ηp in the neighborhood of the fission barrier. Considering the inertia values as given in Fig. 1 for ^{224}Th and a temperature of 2 MeV, which gives the most probable momentum values, we find $\eta p > |\frac{p^2}{2}\frac{\partial}{\partial c}(\frac{1}{m})|$ for $\eta >$ 0.1 MeV/$\hbar$. Since a conservative estimate of the magnitude of nuclear dissipation η is about 2 MeV/$\hbar$ [29], we can neglect the inertia derivative term in Eq. (6). It may be pointed out here that though we neglect the inertia derivative term in Eq. (6) for F, the Boltzmann factor $\exp[-(\frac{p^2}{2m}+V)/T]$ of the density in Eq. (4) fully satisfies Eq. (2). This is the reason for not neglecting the inertia derivative term earlier in Eq. (2) for the full density function $\rho(c,p)$. In fact, we also retain the inertia derivative term in the Langevin equations, which we discuss in the next section.

Since we require the solution for F in the vicinity of the saddle point, we approximate the FRLDM potential in this region with a harmonic oscillator potential

$$V = V_B - \tfrac{1}{2}m_s{\omega_s}^2(c-c_s)^2, \tag{7}$$

where the frequency ω_s is obtained by fitting the FRLDM potential. Introduction of $X = c - c_s$ further reduces Eq. (6) to

$$m_s\eta T\frac{\partial^2 F}{\partial p^2} = \frac{p}{m_s}\frac{\partial F}{\partial X} + \frac{\partial F}{\partial p}\left(m_s\omega_s^2 X + \eta p\right). \tag{8}$$

Following Kramers [17], we next assume for F the form

$$F(X,p) = F(\zeta), \tag{9}$$

where $\zeta = p - aX$ and a is a constant. The value of a is subsequently fixed as follows. Substituting Eq. (9) for F in Eq. (8), we obtain

$$m_s\eta T\frac{d^2F}{d\zeta^2} = -\left(\frac{a}{m_s}-\eta\right)\left\{p - \frac{m_s{\omega_s}^2}{\frac{a}{m_s}-\eta}X\right\}\frac{dF}{d\zeta}. \tag{10}$$

To have consistency between Eqs. (10) and (9), we require

$$\frac{m_s{\omega_s}^2}{\frac{a}{m_s}-\eta} = a, \tag{11}$$

which leads to

$$\frac{a}{m_s} - \eta = -\frac{\eta}{2} + \sqrt{{\omega_s}^2 + \frac{\eta^2}{4}}, \tag{12}$$

where the positive root of a is chosen to satisfy the following boundary conditions: $F(X,p) \to 1$ for $X \to -\infty$ (assuming the ground state to be far on the left of the saddle point), and $F(X,p) \to 0$ for $X \to +\infty$. Equation (10) then becomes

$$m_s\eta T\frac{d^2F}{d\zeta^2} = -\left(\frac{a}{m_s}-\eta\right)\zeta\frac{dF}{d\zeta}. \tag{13}$$

The solution of Eq. (13) satisfying the above boundary conditions is

$$F(\zeta) = \frac{1}{m_s}\sqrt{\frac{(a-m_s\eta)}{2\pi\eta T}}\int_{-\infty}^{\zeta} e^{-\left(\frac{a}{m_s}-\eta\right)\zeta^2/2m_s\eta T}d\zeta. \tag{14}$$

Substituting for F according to this equation in Eq. (4), the stationary density in the saddle region is finally obtained.

We next obtain the net flux or current across the saddle as

$$\begin{aligned} j &= \int_{-\infty}^{+\infty}\rho(X=0,p)\frac{p}{m_s}dp \\ &= ATe^{-V_B/T}\sqrt{\frac{a-m_s\eta}{a}} \\ &= ATe^{-V_B/T}\left\{\sqrt{1+\left(\frac{\eta}{2\omega_s}\right)^2} - \frac{\eta}{2\omega_s}\right\}. \end{aligned} \tag{15}$$

The total number of particles in the potential pocket at the ground-state deformation is

$$n_g = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho\, dc\, dp = \frac{2\pi AT}{\omega_g}, \tag{16}$$

where we have approximated the FRLDM potential with the following harmonic oscillator potential near ground state,

$$V = \tfrac{1}{2}m_g{\omega_g}^2(c-c_g)^2, \tag{17}$$

in which the frequency ω_g is again obtained by fitting the FRLDM potential.

The probability P of a Brownian particle crossing the fission barrier per unit time is then

$$P = \frac{j}{n_g} = \frac{\omega_g}{2\pi}e^{-V_B/T}\left\{\sqrt{1+\left(\frac{\eta}{2\omega_s}\right)^2} - \frac{\eta}{2\omega_s}\right\}. \tag{18}$$

It is immediately noticed that this expression is exactly the same as the one obtained by Kramers using a shape-independent collective inertia. Equation (18), however, is obtained with different inertia values at the ground-state and saddle configurations, which consequently define the frequencies (ω_g and ω_s) in this equation. The fission width from Eq. (18) is

$$\Gamma = \hbar P = \frac{\hbar\omega_g}{2\pi}e^{-V_B/T}\left\{\sqrt{1+\left(\frac{\eta}{2\omega_s}\right)^2} - \frac{\eta}{2\omega_s}\right\}, \tag{19}$$

which we compare with the stationary width from Langevin equations in the following sections.

III. LANGEVIN EQUATIONS FOR FISSION

We use the "funny hills" shape parameters [40] to specify the collective coordinates for a dynamical description of nuclear fission. In the present study of fission dynamics in one dimension, the elongation parameter c is the relevant coordinate and the Langevin equations are [41,42]

$$\begin{aligned} \frac{dp}{dt} &= -\frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right) - \frac{\partial V}{\partial c} - \eta p + g\Gamma(t,) \\ \frac{dc}{dt} &= \frac{p}{m}, \end{aligned} \tag{20}$$

where $g\Gamma$ represents the random force, and its time-correlation property is assumed to follow the relation

$$\langle\Gamma(t)\Gamma(t')\rangle = 2\delta(t-t'),$$

while the strength of the random force is related to the dissipation coefficient through the fluctuation-dissipation theorem and is

$$g = \sqrt{mT\eta}.$$

The Langevin equations are numerically integrated in second order using a small time step of 0.001 $\hbar$/MeV [43]. Calculations are performed for a compound nucleus at specified values of its spin and temperature. The initial collective coordinate is that of a spherical nucleus, and its initial momentum distribution follows that of an equilibrated thermal system. A Langevin trajectory is considered to have undergone fission when it crosses the scission point. We choose the scission configuration to correspond to a neck radius of $0.3R$, where R is the radius of the initial shape of the compound nucleus [43,44]. The fission width is obtained from the time rate at which the Langevin trajectories cross the scission point using an ensemble of 10^6 trajectories for each calculation.

IV. COMPARISON OF FISSION WIDTHS FROM LANGEVIN DYNAMICS AND KRAMERS' FORMULA

Before we proceed to compare the fission widths from Langevin dynamics and Kramers' formula, we point out that the net flux leaving the potential pocket is calculated at different points in the two approaches, though both of them represent the time rate of fission. In a stochastic process such as nuclear fission, a fission trajectory can return to a more compact shape even after it crosses the saddle configuration due to the presence of the random force in the equations of motion. This back streaming is typical of Brownian motion and has been noted earlier by several authors [14,41,45]. The back streaming is described by the phase-space density for negative momentum values at the saddle point in Kramers' solution (Eq. (15)). If one considers outward trajectories passing a larger coordinate value, the probability of returning will approach zero as the potential becomes steeper beyond the saddle point. In numerical simulations of the Langevin dynamics, the scission point is usually so chosen such that the strong Coulomb repulsion beyond the scission point makes the return of a trajectory highly unlikely after it crosses the scission point. The calculated outgoing flux of the Langevin trajectories at the scission point then represents the net flux and hence corresponds to the net flux as defined in Kramers' approach. This feature is illustrated in Fig. 2, where fission trajectories crossing the saddle and the scission points are considered separately in order to obtain the time-dependent fission rates from the Langevin equations. Clearly, the stationary width calculated at the saddle point is higher than that obtained at the scission point, since the former does not include the back-streaming effects. In what follows, we therefore compare Kramers' width with the stationary widths from Langevin equations obtained at the scission configuration.

We first fit the numerically obtained FRLDM potentials [32] with harmonic oscillator potentials [Eqs. (7) and (17)] such that the latter match the FRLDM potential values at the ground state and at the saddle configuration and also at the midpoint between the ground state and the saddle point (Fig. 1). The fitted values of ω_g and ω_s are obtained both for a constant value of the collective inertia and for its different values at c_g and c_s. Figure 3 shows the compound nuclear spin dependence of the frequencies thus obtained.

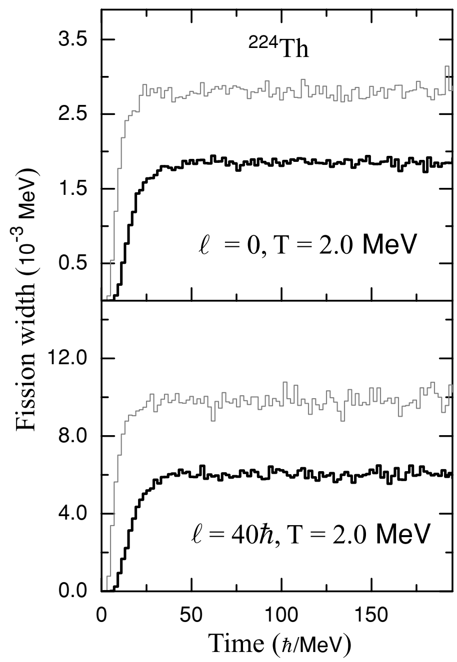

FIG. 2. Time-dependent fission widths from Langevin equations. The thick black and thin gray lines represent the fission rates obtained at the scission point and at the saddle point, respectively.

The Langevin equations [Eq. (20)] are next solved with a constant value of the inertia at all deformations. A constant value of $\eta = 5$ MeV/$\hbar$ is used in all the calculations. The time-dependent fission widths from the Langevin dynamics are displayed in Fig. 4 for different values of spin of the compound nucleus ^{224}Th. The corresponding Kramers' widths are also shown in this figure. A close agreement between the stationary widths from Langevin dynamics and those from Kramers' formula is observed for compound nuclear spin (ℓ) of 0 and $25\hbar$; while for $\ell = 50\hbar$, Kramers' limit underestimates the fission width by about 20%. The last discrepancy possibly reflects the fact that the condition $V_B \gg T$ required for validity of Kramers' limit is not met in this case, since the fission barrier is 1.64 MeV for $\ell = 50\hbar$, while the temperature of the compound nucleus is 2 MeV.

The Langevin equations are subsequently solved using shape-dependent values of the collective inertia (Fig. 1), and the calculated time-dependent fission widths are shown in Fig. 5. Kramers' widths are calculated using the frequencies ω_g and ω_s, which are obtained using the local values of the collective inertia at c_g and c_s, respectively. Kramers' widths, also shown in Fig. 5, are found to be in excellent agreement with the stationary widths from the Langevin equations. It is thus demonstrated that Kramers' formula [Eq. (19)] gives the correct stationary fission width even when the collective inertia of the system has a shape dependence.

We now study the stationary fission rate with a different type of shape dependence of collective inertia. We assume that the

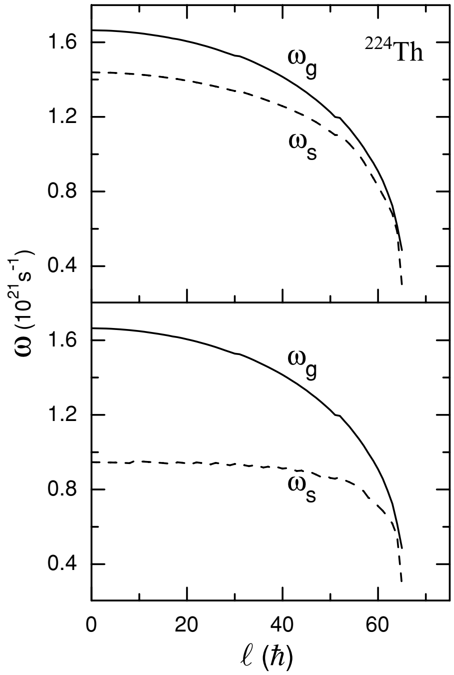

FIG. 3. Compound nuclear spin (ℓ) dependence of the frequencies of the harmonic oscillator potentials approximating the rotating FRLDM potential at the ground state (ω_g) and at the saddle point (ω_s). In the upper panel, the values of inertia at the ground state and at the saddle are taken to be the same, while the Werner-Wheeler approximation to the inertia is used in the lower panel.

value of the inertia remains constant at m_g for all deformations around the ground state; and at an intermediate deformation between the ground state and the saddle point, its value abruptly increases to m_s and remains so for all deformations in the saddle region. Such a system was considered by Hofmann *et al.* [36] and a modified version of Kramers' fission width was obtained as

$$\Gamma = \sqrt{\frac{m_g}{m_s}}\frac{\hbar\omega_g}{2\pi}e^{-V_B/T}\left\{\sqrt{1+\left(\frac{\eta}{2\omega_s}\right)^2}-\frac{\eta}{2\omega_s}\right\}. \tag{21}$$

We have solved the Langevin equations with the inertia defined as in the above, and the calculated fission widths are also plotted in Fig. 5. The modified Kramers' width from Eq. (21) is also shown for each case. The modified Kramers' width is found to predict satisfactorily the stationary fission width from dynamic calculations. This result shows that Kramers' width and the stationary width from the Langevin dynamical calculation remain in close agreement even under very distinct prescriptions of shape dependence of inertia. We, however, consider Kramers' width as given by Eq. (19) to be more appropriate for nuclear fission, since it is obtained for realistic and smooth dependence of inertia on deformation.

Before we conclude, we discuss the following expression for Kramers' width:

$$\Gamma = \frac{\hbar\omega_g}{T}\Gamma_{\rm BW}\left\{\sqrt{1+\left(\frac{\eta}{2\omega_s}\right)^2}-\frac{\eta}{2\omega_s}\right\}, \tag{22}$$

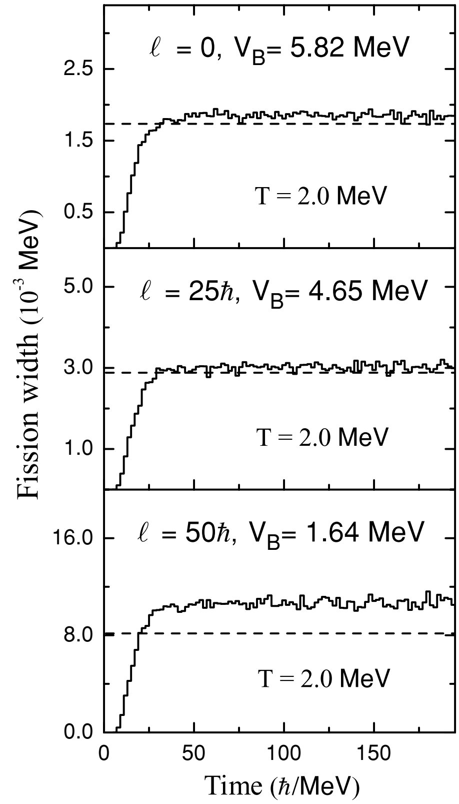

FIG. 4. Time-dependent fission widths (solid lines) from Langevin equations with no shape dependence of collective inertia. Results for different values of compound nuclear spin (ℓ) at a temperature (T) of 2 MeV are shown. The corresponding values of Kramers' width are indicated by the dashed lines.

which is often used in the literature [7,27,28,35]. $\Gamma_{\rm BW}$ in this equation is the transition-state fission width due to Bohr and Wheeler [12], and it is introduced in Eq. (22) in the following manner. According to Bohr and Wheeler, the transition-state fission width is given as

$$\Gamma_{\rm BW} = \frac{1}{2\pi\rho_g(E_i)}\int_0^{E_i-V_B}\rho_s(E_i-V_B-\epsilon)d\epsilon, \tag{23}$$

where ρ_g is the level density at the initial state (E_i, ℓ_i) and ρ_s is the level density at the saddle point. Under the condition $V_B/E_i \ll 1$ and assuming the level-density parameter for the ground state and at the saddle point to be the same and further assuming a simplified form of the level density as $\rho(E) \sim \exp(2\sqrt{aE})$, the Bohr-Wheeler width reduces to

$$\Gamma_{\rm BW} = \frac{T}{2\pi}e^{-V_B/T}. \tag{24}$$

Substituting for the exponential factor $e^{-V_B/T}$ from Eq. (24) in Eq. (19), Eq. (22) is obtained. Thus Eq. (22) represents Kramers' fission width only when the approximate form of $\Gamma_{\rm BW}$ is used in this equation. Consequently, it is not appropriate to obtain Kramers' width from Eq. (22) where the transition-state fission width $\Gamma_{\rm BW}$ is calculated from Eq. (23) using a shape-dependent level-density parameter. This observation follows from the fact that while the density of quantum mechanical microscopic states are explicitly taken

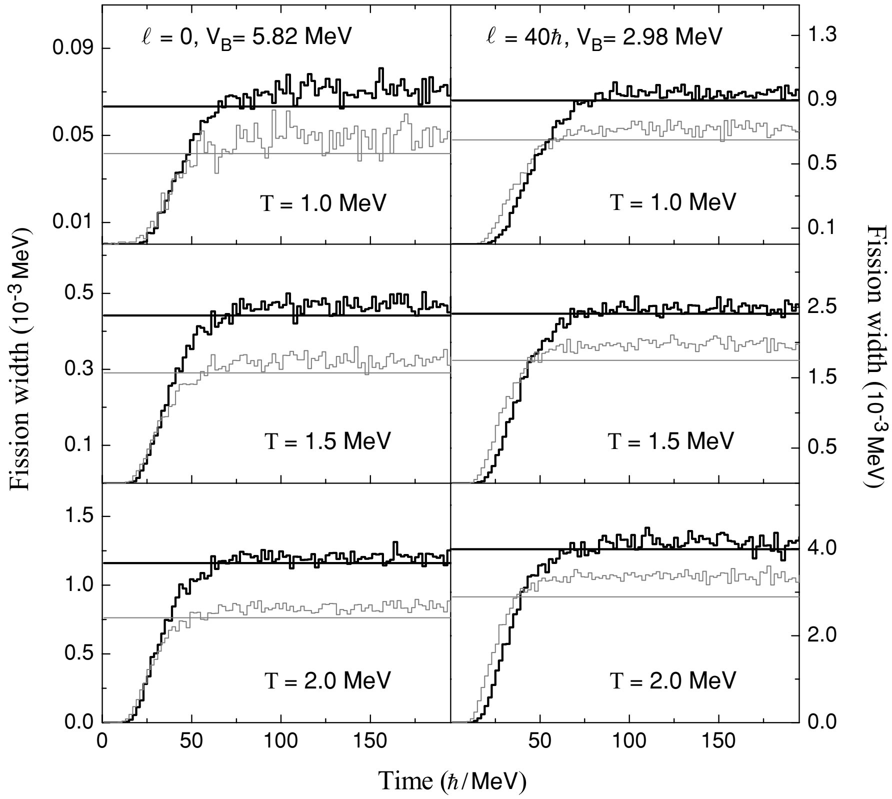

FIG. 5. Time-dependent fission widths from Langevin equations with shape-dependent collective inertia. Results for different values of compound nuclear spin (ℓ) and temperatures (T) are shown. The shape dependence is continuous for the histograms in thick black lines. The corresponding values of Kramers' width [Eq. (19)] are indicated by the horizontal thick black lines. The histograms in thin gray lines are obtained with discrete values of inertia in the ground state and saddle regions (see text) and the horizontal thin gray lines represent the corresponding stationary limits [Eq. (21)].

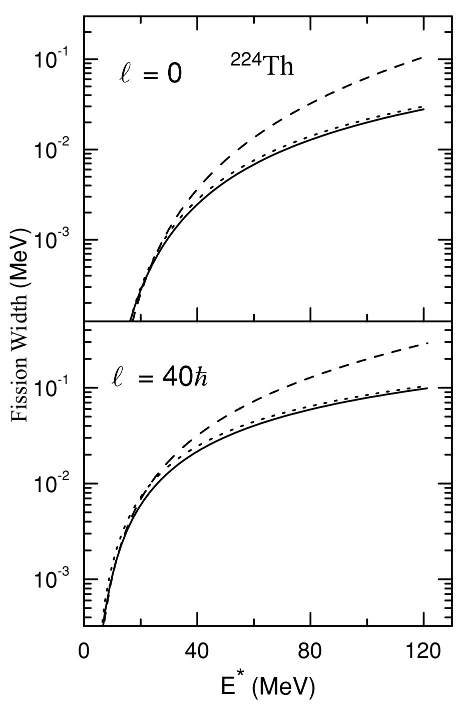

FIG. 6. Bohr-Wheeler fission width at different excitations. The solid lines are the approximate widths from Eq. (24); the short-dashed lines are obtained from Eq. (23) with shape-independent parameters of the level-density formula. The long-dashed line represents the widths obtained with shape-dependent parameters of the level-density formula.

into account in the work of Bohr and Wheeler [12], Kramers' work [17] essentially concerns the classical phase space of the collective motion. Since there is no scope of introducing any detailed information regarding density of states apart from the nuclear temperature in dissipative dynamical models of nuclear fission, Kramers' fission width cannot be connected to the Bohr-Wheeler expression where detailed density of states are employed. In fact, the magnitude of the fission width obtained from the simplified version of the Bohr-Wheeler expression [Eq. (24)] differs substantially from that calculated using Eq. (23) when the following standard form of the shape-dependent level-density formula [7] is used:

$$\rho(U,\ell) = \frac{2\ell+1}{12I^{3/2}}\sqrt{a}\,\frac{\exp(2\sqrt{aU})}{U^2}, \tag{25}$$

where U is the intrinsic excitation of the compound nucleus. I and a are, respectively, the rigid body moment of inertia and the level-density parameter, the values of which depend upon the shape of the compound nucleus and hence are different at the ground-state and saddle configurations. Figure 6 shows the Bohr-Wheeler fission width calculated under different conditions. It is immediately noticed in this figure that the width from Eq. (23) agrees reasonably well with that calculated from Eq. (24) when the parameters of the level-density formula are chosen to be shape independent. However, the widths calculated with shape-dependent parameters differ substantially from those obtained from Eq. (24), particularly at high excitation energies where the dissipative effects are important. Therefore, the use of the Bohr-Wheeler fission width obtained with a shape-dependent level density in Eq. (22) does not correspond to dissipative dynamics, as envisaged in Kramers' formula. Further, it can introduce an energy dependence in the dissipation coefficient when Eq. (22) is employed to fit experimental data. This can be one of the contributing factors leading to the inference of very large values of nuclear dissipation [29].

It may be worthwhile to discuss at this point the distinguishing features of the transition-state fission width Γ_K^{tr} which was obtained by Kramers [17] and is given as

$$\Gamma_K^{tr} = \frac{\hbar\omega_g}{2\pi} e^{-V_B/T}. \tag{26}$$

This width differs by a factor of $\hbar\omega_g/T$ from the approximate form of the Bohr-Wheeler fission width as given by Eq. (24). This difference arises because the accessible phase spaces are considered differently in the two approaches as we pointed out earlier. Strutinsky [46] introduced a phase-space factor in the Bohr-Wheeler transition-state fission width to account for the collective vibrations around the ground-state shape and obtained the same width as given in Eq. (26). It is important to recognize here that while the Bohr-Wheeler expressions [Eqs. (23) and (24)] represent the low-temperature limit ($T \ll \hbar\omega_g$) of fission width, Kramers width [Eq. (19)] corresponds to fission at higher temperatures ($T \gg \hbar\omega_g$). At low temperatures of a compound nucleus, quantal treatment of the collective motion is required, since the energy available to the collective motion is also very small [38]. Consequently, in the low-temperature limit, the collective motion is restricted to one state, namely, the zero-point vibration. Therefore, the Bohr-Wheeler width based upon density of quantum mechanical intrinsic nuclear states alone represents the low-temperature limit of nuclear fission width. On the other hand, the phase space for collective vibrations increases with increasing temperature, and the Strutinsky-corrected width thus becomes the high-temperature limit of transition-state fission width. At higher temperatures, however, the nuclear collective motion also turns out to be dissipative in nature. Thus Kramers' expression [Eq. (19)] should be considered as the high-temperature limit of the width of nuclear fission.

V. SUMMARY AND CONCLUSIONS

In the preceding sections, we considered the applicability of Kramers' formula to the stationary fission width of a compound nucleus that is described by a realistic collective potential and a shape-dependent collective inertia. It is shown that for a system with a deformation-dependent collective inertia, the stationary fission width retains the form as originally obtained by Kramers for constant inertia. The accuracy of the various approximations in deriving the above fission width is tested by comparing its values with the stationary fission widths obtained by solving the Langevin equations. Both approaches are found to be in excellent agreement with each other. The present work thus extends the applicability of Kramers' formula for stationary fission width to more realistic systems.

We further compare the strength of the statistical-model fission width obtained under different simplifying assumptions and point out the constraints in interpreting Kramers' width in terms of the statistical-model fission width of Bohr and Wheeler.

[1] H. Rossner, D. Hilscher, D. J. Hinde, B. Gebauer, M. Lehmann, M. Wilpert, and E. Mordhorst, Phys. Rev. C **40**, 2629 (1989).

[2] D. J. Hinde, D. Hilscher, H. Rossner, B. Gebauer, M. Lehmann, and M. Wilpert, Phys. Rev. C **45**, 1229 (1992).

[3] D. J. Hinde, H. Ogata, M. Tanaka, T. Shimoda, N. Takahashi, A. Shinohara, S. Wakamatsu, K. Katori, and H. Okamura, Phys. Rev. C **39**, 2268 (1989).

[4] H. Rossner, D. J. Hinde, J. R. Leigh, J. P. Lestone, J. O. Newton, J. X. Wei, and S. Elfström, Phys. Rev. C **45**, 719 (1992).

[5] R. Butsch, M. Thoennessen, D. R. Chakrabarty, M. G. Herman, and P. Paul, Phys. Rev. C **41**, 1530 (1990).

[6] D. J. Hofman, B. B. Back, and P. Paul, Phys. Rev. C **51**, 2597 (1995).

[7] I. Dioszegi, N. P. Shaw, I. Mazumdar, A. Hatzikoutelis, and P. Paul, Phys. Rev. C **61**, 024613 (2000).

[8] I. Dioszegi, N. P. Shaw, A. Bracco, F. Camera, S. Tettoni, M. Mattiuzzi, and P. Paul, Phys. Rev. C **63**, 014611 (2000).

[9] K.-T. Brinkmann, A. L. Caraley, B. J. Fineman, N. Gan, J. Velkovska, and R. L. McGrath, Phys. Rev. C **50**, 309 (1994).

[10] B. B. Back, D. J. Blumenthal, C. N. Davids, D. J. Henderson, R. Hermann, D. J. Hofman, C. L. Jiang, H. T. Penttilä, and A. H. Wuosmaa, Phys. Rev. C **60**, 044602 (1999).

[11] A. Gavron, A. Gayer, J. Boissevain, H. C. Britt, J. R. Nix, A. J. Sierk, P. Grange, S. Hassani, H. A. Weidenmuller, J. R. Beene, B. Cheynis, D. Drain, R. L. Ferguson, F. E. Obenshain, G. A. Petitt, and C. Butler, Phys. Lett. **B176**, 312 (1986).

[12] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

[13] Y. Abe, S. Ayik, P.-G. Reinhard, and E. Suraud, Phys. Rep. **275**, 49 (1996).

[14] P. Fröbrich and I. I. Gontchar, Phys. Rep. **292**, 131 (1998).

[15] A. V. Karpov, P. N. Nadtochy, D. V. Vanin, and G. D. Adeev, Phys. Rev. C **63**, 054610 (2001).

[16] C. R. Morton, D. J. Hinde, J. R. Leigh, J. P. Lestone, M. Dasgupta, J. C. Mein, J. O. Newton, and H. Timmers, Phys. Rev. C **52**, 243 (1995).

[17] H. A. Kramers, Physica (Amsterdam) **7**, 284 (1940).

[18] P. Grange, Nucl. Phys. **A428**, 37c (1984).

[19] P. Grange, Li Jun-Qing, and H. A. Weidenmüller, Phys. Rev. C **27**, 2063 (1983).

[20] K. H. Bhatt, P. Grange, and B. Hiller, Phys. Rev. C **33**, 954 (1986).

[21] D. Boilley, E. Surad, Y. Abe, and S. Ayik, Nucl. Phys. **A556**, 67 (1993).

[22] J. R. Nix, A. J. Sierk, H. Hofmann, F. Scheuter, and D. Vautherin, Nucl. Phys. **A424**, 239 (1984).

[23] F. Scheuter and H. Hofmann, Nucl. Phys. **A394**, 477 (1983).

[24] B. Jurado, K.-H. Schmidt, and J. Benlliure, Phys. Lett. **B553**, 186 (2003).

[25] B. Jurado, C. Schmitt, K.-H. Schmidt, J. Benlliure, and A. R. Junghans, Nucl. Phys. **A747**, 14 (2005).

[26] A. V. Karpov, P. N. Nadtochy, E. G. Ryabov, and G. D. Adeev, J. Phys. G **29**, 2365 (2003).
[27] N. D. Mavlitov, P. Fröbrich, and I. I. Gontchar, Z. Phys. A **342**, 195 (1992).
[28] I. I. Gontchar and P. Fröbrich, Nucl. Phys. **A551**, 495 (1993).
[29] S. G. McCalla and J. P. Lestone, Phys. Rev. Lett. **101**, 032702 (2008).
[30] G.-R. Tillack, Phys. Lett. **B278**, 403 (1992).
[31] H. J. Krappe, J. R. Nix, and A. J. Sierk, Phys. Rev. Lett. **42**, 215 (1979); Phys. Rev. C **20**, 992 (1979).
[32] A. J. Sierk, Phys. Rev. C **33**, 2039 (1986).
[33] M. Thoennessen, Nucl. Phys. **A599**, 1c (1996).
[34] J. P. Lestone, Phys. Rev. C **51**, 580 (1995).
[35] J. Sadhukhan and S. Pal, Phys. Rev. C **78**, 011603(R) (2008); **79**, 019901(E) (2009).
[36] H. Hofmann, F. A. Ivanyuk, C. Rummel, and S. Yamaji, Phys. Rev. C **64**, 054316 (2001).
[37] S. Chandrasekhar, Rev. Mod. Phys. **15**, 1 (1943).
[38] J. R. Nix, Nucl. Phys. **A130**, 241 (1969).
[39] K. T. R. Davies, A. J. Sierk, and J. R. Nix, Phys. Rev. C **13**, 2385 (1976).
[40] M. Brack, J. Damgard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. **44**, 320 (1972).
[41] G. Chaudhuri and S. Pal, Phys. Rev. C **63**, 064603 (2001).
[42] T. Wada, Y. Abe, and N. Carjan, Phys. Rev. Lett. **70**, 3538 (1993).
[43] S. Pal, G. Chaudhuri, and J. Sadhukhan, Nucl. Phys. **A808**, 1 (2008).
[44] P. N. Nadtochy and G. D. Adeev, Phys. Rev. C **72**, 054608 (2005).
[45] Jing-Dong Bao and Ying Jia, Phys. Rev. C **69**, 027602 (2004).
[46] V. M. Strutinsky, Phys. Lett. **B47**, 121 (1973).