Neutrino-induced coherent pion production off nuclei reexamined

T. Leitner, U. Mosel, and S. Winkelmann[*]

Institut für Theoretische Physik, Universität Giessen, Germany

(Received 23 January 2009; published 12 May 2009)

It is pointed out that so far all theoretical estimates of coherent pion production off nuclei induced by neutrinos rely on the "local approximation" well known in photonuclear physics. The effects of dropping this approximation are discussed. It is found that in a plane wave approximation for the pion, the local approximation overestimates the coherent neutrino-induced pion production on nuclei.

A precise determination of neutrino oscillation parameters from long baseline experiments requires a precise knowledge of the incoming neutrino energy and a detailed understanding of the neutrino interaction within the detection target. Since the targets in all present experiments involve heavy nuclei, from C to Fe, a precise knowledge of the interaction of neutrinos with nuclei is obviously required (for a collection of recent references, see Ref. [1]). In the few GeV region, this involves pion production channels, and a detailed study of this interaction is necessary to improve the accuracy of any determination of the neutrino energy spectrum. In this energy regime, the K2K Collaboration has observed a significant deficit in forward lepton direction when comparing their measurements with simulations [2]. Since the observed deficit is present at forward angles, i.e., small momentum transfers, charged-current (CC) coherent pion production is one candidate interaction that could explain the difference. More recent searches, however, found no evidence for CC coherent pion production in the low-GeV region [3,4]. The MiniBooNE Collaboration, on the other hand, has reported neutral-current (NC) coherent π^0 production [5] at neutrino energies less than 1 GeV. All these experimental analyses have in common that fact that the coherent fraction is not accessible directly but has to be extracted from data assuming specific models for incoherent pion production; the theoretical models for coherent scattering used in the Monte Carlo event simulations give too large contributions not seen experimentally [3–5].

This has triggered a large number of theoretical publications on this subject. The theories generally fall into two categories. In one class of theories, the partially conserved axial current (PCAC) is used from the outset, and the coherent neutrino-induced pion production is related to a pion forward scattering amplitude. This class of models assumes that the process is dominated by the axial current and that specific nuclear effects play no role besides providing nuclear size information. A very recent example of this type of approach is the work by Berger and Sehgal [6]. In a fully consistent calculation, the models of this first class should emerge as approximations of the second class of models, which start from a theoretical description of the nuclear structure and sum the pion production amplitude coherently over all target nucleon states. A very recent example of this approach is given by the work by Amaro *et al.* [7],

which also contains a rather complete list of references to both approaches, the PCAC based and the ones based on nuclear structure.

All these studies of neutrino-induced reactions are closely related to the experiments on coherent photon-induced pion and η production on nuclei [8–12]. While the latter depend only on vector currents, they are, of course, closely related to those involving neutrinos in the incoming channel. These reactions were often investigated theoretically by invoking the "local approximation." This simplifies the hadronic current significantly, because it allows one to pull the Δ propagator out of the pion production amplitude [13], which becomes local, and as a consequence allows one to separate out the nuclear form factor. Peters *et al.* [14,15] have discussed the accuracy and the limitations of the local approximation for the case of photon-induced coherent π^0 production as well as for coherent η production (see also Refs. [13,16,17]).

The local approximation is the starting point also used by the theoretical studies of neutrino-induced coherent nuclear pion production (see Ref. [7] and references therein). In all these studies, the pion production amplitude is factorized into a part that contains the pion production amplitude and one that contains the nuclear size information. Only via this approximation does the nuclear form factor emerge in the expressions for coherent pion production on nuclei. Since the results of both the PCAC-based models and the nuclear physics models rely on this factorization, it is of interest to investigate how good this approximation actually is for neutrino-induced processes. In this brief report, we present the results of such an investigation in which we compare the results of a calculation using the local approximation with the results of a full calculation without this approximation, both without pion final state interactions.

We now briefly discuss the full calculation and then show how the local approximation leads to the expressions commonly used for calculating the coherent pion production cross section.

We assume that the pions are dominantly created via the $\Delta(1232)$ resonance; in Refs. [7,18] it was shown that this is a very good approximation for coherent pion production. The hadronic current for a nucleon is then given by

$$J_{\text{nucleon}}^{\mu}(p, q)$$
$$= i \frac{f^*}{m_\pi} C^\Delta F(p_\Delta^2) \bar{u}(\vec{p}') k_\pi^\alpha G_{\alpha\beta}(p_\Delta) \Gamma^{\beta\mu}(p, q) u(\vec{p}), \quad (1)$$

---

[*]Present address: Physikalisches Institut, Universität Freiburg.

where u-channel diagrams have been neglected. Here f^* is the $N\Delta\pi$ coupling constant, $F(p_\Delta^2)$ a form factor depending on the invariant mass of the Δ, k_π the pion momentum, p' and p the nucleon's final and initial momenta, and q the transferred four-momentum. Correspondingly, the Δ momentum is given by $p_\Delta = p + q$. C^Δ contains isospin factors; it is given by

$$
C^\Delta = \begin{cases}
\text{CC} & p & \sqrt{3}\cos\theta_c \\
\text{CC} & n & \sqrt{\frac{1}{3}}\cos\theta_c \\
\text{NC} & p, n & \sqrt{\frac{2}{3}}.
\end{cases} \tag{2}
$$

The quantity $G_{\alpha\beta}$ represents the full Rarita-Schwinger (RS) propagator

$$
G_{\alpha\beta} = \frac{1}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} P_{\alpha\beta}, \tag{3}
$$

where $P_{\alpha\beta}$ is the usual RS projection operator. The vertex function $\Gamma^{\beta\mu}$ in Eq. (1) denotes the standard electroweak vertex structure with vector and axial contributions including the resonance excitation form factors for which we take the same set as applied in Ref. [19] (for details, see Refs. [20,21]). The propagator [Eq. (3)] does not include any in-medium changes of the Δ spectral function. Their effect is significant, however, they affect both the local approximation and the full calculation. In the target nucleus considered here, ^{12}C, only $s_{1/2}$ and $p_{3/2}$ states are occupied. The latter reach farther out than the former and have a higher degeneracy. The effect of in-medium changes of the Δ spectral function is thus expected to be smaller in the full calculation than in the one using the local approximation where the same spectral function is used for all states [cf. Eq. (6)].

The dyadic product of the currents in Eq. (1) yields the hadronic tensor, which, in turn, determines the $\nu + N \to \ell + N + \pi$ cross section.

In coherent pion production, the single-particle current [Eq. (1)] has to be summed over all occupied states of the target nucleus. This total current then yields the hadronic tensor for coherent π production on nuclei. This is in contrast to incoherent processes, where—in the spirit of the impulse approximation—one sums over the hadronic tensor of each nucleon.

Assuming a mean field model for the nucleons gives immediately a momentum distribution for the nucleon states. This necessitates the appearance of a momentum integral in the hadronic current

$$
J_{\text{nucleus}}^\mu(q) = \sum_{\text{nucleons}} \int d^3p \, J_{\text{nucleon}}^\mu(p, q). \tag{4}
$$

Here the sum runs over all occupied nucleon single-particle states in the target nucleus. The single-nucleon current on the right-hand side is obtained from that in Eq. (1) by replacing the free-particle spinors $u(\vec{p})$ by the momentum representations of the bound-state spinors $\psi_i(\vec{p})$ [same for $\bar{u}(\vec{p}')$]. In the following comparison, we obtain the latter from a Walecka-type mean field model for ^{12}C using parameters from Ref. [14]. We stress that in this most general expression (4) for the nuclear current, the momentum integration extends over the Δ propagator as well, since $p_\Delta = p + q$. Because of the

presence of the Δ propagator in the single-particle currents and the Lorentz structure of the vertex function $\Gamma^{\beta\mu}$, coherent pion production does not test the local vector density of the nucleus but instead the nonlocal structure of various other (tensor) densities. This is the method used successfully to describe coherent photon-induced π^0 production on nuclei [14].

The local approximation now consists of fixing the momentum of the initial nucleon state in the product $G_{\alpha\beta}(p_\Delta)\Gamma^{\beta\mu}(p, q)$ to some value. An often-used prescription is [7]

$$
\vec{p}^{\,0} = -(\vec{q} - \vec{k}_\pi)/2. \tag{5}
$$

As a consequence of this "freezing" of the initial nucleon's momentum in the transition operator, the momentum of the Δ resonance is also determined. The propagator of the Δ resonance can thus be moved out of the integral and then even out of the sum in Eq. (4). This approximation basically consists of suppressing the propagation of the Δ resonance and corresponds to the assumption of a very heavy Δ resonance. As a consequence, the $W, Z + N \to \pi + N$ vertex becomes local. Thus we are left with the current in the local approximation

$$
\begin{aligned}
\tilde{J}_{\text{nucleus}}^\mu(q) = {}& i\frac{f^*}{m_\pi} C^\Delta F(p_\Delta^{0\,2}) \frac{k_\pi^\alpha}{p_\Delta^{0\,2} - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \\
& \times \sum_i \int d^3p (\psi_i(\vec{p})\bar{\psi}_i(\vec{p} + \vec{q} - \vec{k}_\pi)) \\
& \times P_{\alpha\beta}(p_\Delta^0)\Gamma^{\beta\mu}(p^0, q),
\end{aligned}
$$

where i runs over all occupied nucleon states and $\vec{p}_\Delta^{\,0} = (\vec{q} + \vec{k}_\pi)/2$ according to the local approximation (5). In an r-space representation, this becomes

$$
\begin{aligned}
\tilde{J}_{\text{nucleus}}^\mu(q) = {}& i\frac{f^*}{m_\pi} C^\Delta F(p_\Delta^{0\,2}) \frac{k_\pi^\alpha}{p_\Delta^{0\,2} - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \\
& \times \int d^3r \, e^{i(\vec{q} - \vec{k}_\pi)\cdot\vec{r}} \text{tr}(\rho(\vec{r}, \vec{r}) P_{\alpha\beta}(p_\Delta^0)\Gamma^{\beta\mu}(p^0, q)).
\end{aligned} \tag{6}
$$

Here the trace is taken over the Dirac indices and $\rho(\vec{r}, \vec{r})$ is the diagonal element of the one-body density matrix. This is the final result in the local approximation. Equation (6) shows that the nuclear form factor has been factorized out; all the other (nonlocal) densities present in the full expression no longer appear.

In the following, we compare the full calculation, based on Eq. (4) with a propagating Δ, with the results of the local approximation [Eq. (6)] for the target nucleus ^{12}C. To isolate the effects of the local approximation, both calculations are done in the plane wave approximation in which the produced pion is taken to be a free particle. Both calculations use the same nuclear structure model, i.e., the density and momentum distributions are calculated consistently in the same relativistic mean field model. For the local approximation, the incoming nucleon's momentum has been set to $\vec{p} = 0$, following Ref. [19]. We note that when we use the same approximation,
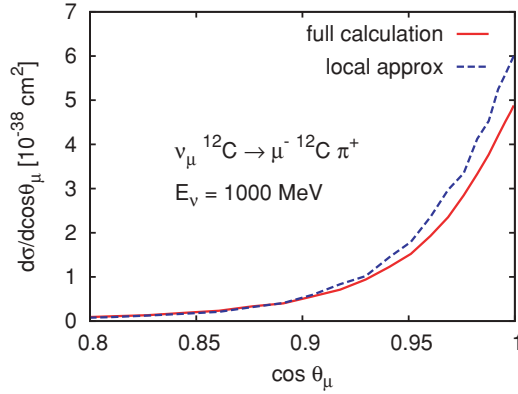
FIG. 1. (Color online) CC-induced pion angular distribution for a neutrino energy of 1 GeV and target ^{12}C. The dashed curve gives the result of the calculation using the local approximation [Eq. (6)]; the solid curve gives the result of a fully dynamic calculation [Eq. (4)]. All curves are without pion final state interactions.

we reproduce the local results obtained by Alvarez-Ruso *et al.* [19].

Figure 1 shows a comparison of the full calculation for the angular distribution of the produced pions for $E_\nu = 1$ GeV with the results obtained by using the local approximation. Over a wide angular range there is perfect agreement. However, at very forward angles, the local approximation gives a cross section that is about 20% larger than that obtained in the full calculation.

The difference between the full and the approximate calculation is larger at lower energies, as can be seen from Fig. 2, where we compare the full and the local approximation calculations for the angular distribution at the lower beam energy of 500 MeV. Here the difference between the two curves is drastic and amounts to a factor of ≈ 2 at $0°$.

The pion momentum distribution for $E_\nu = 1$ GeV is shown in Fig. 3. It is seen that the local approximation overestimates the full result by about 70% at the peak. The slight shift downward relative to the fully dynamical result is a consequence of the local approximation which, as discussed earlier, assumes a very heavy Δ thus minimizing any recoil
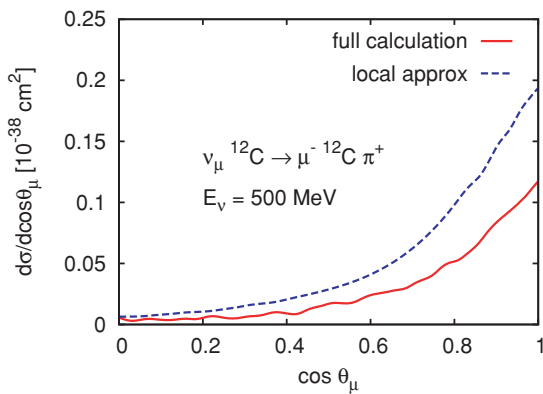

FIG. 2. (Color online) CC-induced pion angular distribution for an incoming neutrino energy of 500 MeV and target ^{12}C. Curves as in Fig. 1.
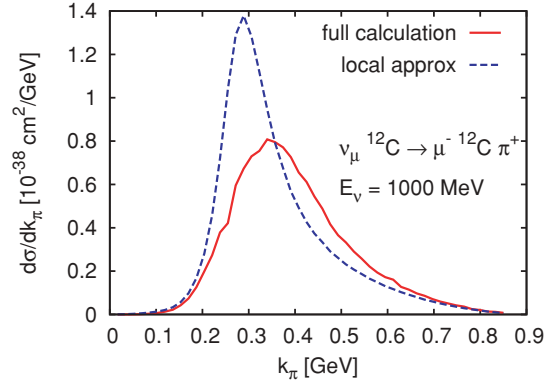

FIG. 3. (Color online) CC-induced pion momentum distribution for an incoming neutrino energy of 1 GeV and target ^{12}C. Curves as in Fig. 1.

effects. We find qualitatively similar results for NC-induced coherent pion production.

Recently, Amaro *et al.* [7] have criticized the assumption of a vanishing initial nucleon momentum as used in Ref. [19] (and also in our calculations for the local approximation). Therefore, in Fig. 4, we compare the results of our full calculations with the results obtained by Amaro *et al.* [7]. The latter calculation does not use an incoming momentum distribution $\sim\delta(\vec{p})$, but it still employs the local approximation from the start. Again, the curves shown do not contain any final state interactions of the pion. As already found above, there
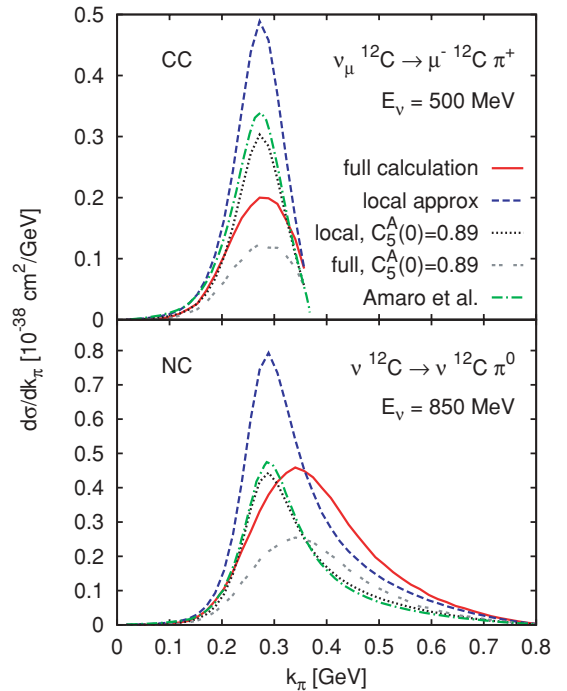

FIG. 4. (Color online) Pion momentum distribution for 500 (CC) and 850 (NC) MeV incoming neutrino energy. The solid and the dashed curves are as in the previous figures, the dotted (double-dashed) ones show the calculation with a modified axial coupling for the local approximation (full calculation) as detailed in the text; the dash dotted lines are taken from Ref. [7].

is again a drastic disagreement between our full calculation and our local approximation result (compare the solid and dashed curves). For example, in the CC case at 500 MeV (upper part of Fig. 4), the full calculation is less than 1/2 the local approximation result at the peak value. The result of Amaro *et al.* (dash-dotted curve), obtained in the local approximation, lies significantly lower than our "local" result. However, this difference between our local approximation and their result is mainly due to the use of a significantly reduced axial coupling $C_5^A(0)$ in the work of Amaro *et al.* [7], as can be seen by comparing their result with the dotted line results in which we also reduced $C_5^A(0)$ from 1.2 to 0.89 in our calculations. The small remaining differences between the two local approximation calculations can be attributed to a somewhat different nuclear structure input and the different treatment of the incoming nucleon's momentum distribution. The double-dashed line in Fig. 4 shows the full calculation with the reduced value of C_5^A. Again, there is a factor of ≈ 2 difference between it and the corresponding local curve (dotted).

So far the plane wave approximation has been used for the outgoing pions, while any observable cross section contains the effects of the strong pion final state interactions (FSI). These are known to not only lower the cross section by about 60% at the lower beam energies [7], but also affect the shape of the pion momentum distributions. In coherent events, the quasielastic pion-nucleon scattering and the absorption through the Δ resonance move the peak to lower momenta. We thus expect that the local approximation result, having more cross section at lower momenta, will be somewhat more affected by pion FSI than the full calculation result, so that the shapes, but not the magnitudes, will become closer to each other once pion FSI are included.

In conclusion, all available calculations for neutrino-induced coherent pion production rely on the local approximation for the elementary interaction vertex. Only this assumption allows one to factorize out the nuclear form factor. For the case of photonuclear reactions, this assumption had been scrutinized in many theoretical studies, starting in the 1980s. For neutrino-induced reactions, however, we do not know of any previous study of this effect. In the present paper, we show that the use of the local approximation involves errors in the pion momentum distribution that can reach up to 100% and lead to different shapes. The discrepancy decreases with the neutrino energy (more than a factor of 2 for 500 MeV, about 1.7 for 1 GeV, and less for 2 GeV). The differences for the pion angular distribution at forward angles also decrease with energy. They are of the order of 100% at 0.5 GeV and 20% at 1 GeV neutrino energy. There is thus a general tendency for the local approximation to overestimate the coherent neutrino-induced pion production. Full calculations, including pion FSI and medium modifications of the Δ, which go beyond this brief report, will yield a final clarification of this point.

[1] *Proceedings of the Fifth International Workshop on Neutrino-Nucleus Interactions in the Few-GeV Region, 30 May 2007 – 3 June 2007, Fermilab, Batavia, Illinois, USA*, AIP Conf. Proc. **967** (2007).

[2] E. Aliu *et al.* (K2K Collaboration), Phys. Rev. Lett. **94**, 081802 (2005).

[3] K. Hiraide *et al.* (SciBooNE Collaboration), Phys. Rev. D **78**, 112004 (2008).

[4] M. Hasegawa *et al.* (K2K Collaboration), Phys. Rev. Lett. **95**, 252301 (2005).

[5] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Lett. **B664**, 41 (2008).

[6] C. Berger and L. M. Sehgal, Phys. Rev. D **79**, 053003 (2009).

[7] J. E. Amaro, E. Hernandez, J. Nieves, and M. Valverde, Phys. Rev. D **79**, 013002 (2009).

[8] G. Koch *et al.*, Phys. Rev. Lett. **63**, 498 (1989).

[9] F. Rambo *et al.*, Nucl. Phys. **A660**, 69 (1999).

[10] J. Weiss *et al.*, Eur. Phys. J. A **11**, 371 (2001).

[11] U. Siodlaczek *et al.*, Eur. Phys. J. A **10**, 365 (2001).

[12] B. Krusche *et al.*, Phys. Lett. **B526**, 287 (2002).

[13] L. Tiator and L. E. Wright, Phys. Rev. C **30**, 989 (1984).

[14] W. Peters, H. Lenske, and U. Mosel, Nucl. Phys. **A640**, 89 (1998).

[15] W. Peters, H. Lenske, and U. Mosel, Nucl. Phys. **A642**, 506 (1998).

[16] L. J. Abu-Raddad, J. Piekarewicz, A. J. Sarty, and M. Benmerrouche, Phys. Rev. C **57**, 2053 (1998).

[17] L. J. Abu-Raddad, J. Piekarewicz, A. J. Sarty, and R. A. Rego, Phys. Rev. C **60**, 054606 (1999).

[18] L. Alvarez-Ruso, L. S. Geng, and M. J. Vicente Vacas, Phys. Rev. C **76**, 068501 (2007).

[19] L. Alvarez-Ruso, L. S. Geng, S. Hirenzaki, and M. J. Vicente Vacas, Phys. Rev. C **75**, 055501 (2007).

[20] E. Hernandez, J. Nieves, and M. Valverde, Phys. Rev. D **76**, 033005 (2007).

[21] S. Winkelmann, Diploma thesis, Universitaet Giessen, 2008, available online at http://theorie.physik.uni-giessen.de/dipl.