Bulk viscosity in quasiparticle models

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We discuss transport properties of dynamical fluid composed of quasiparticles whose masses depend on temperature and charge chemical potentials. Based on the relativistic kinetic theory formulated under the relaxation time approximation, we derive a general expression for the bulk viscosity in the quasiparticle medium. We show that dynamically generated particle masses imply an essential modification of the fluid compressibility. As an application of our results we consider a class of quasiparticle models with the chiral phase transition belonging to $O(4)$ and $Z(2)$ universality class. Based on the Ginzburg-Landau and the scaling theory we study the critical properties of the bulk viscosity ζ near the phase transition. We show that extrapolating the results of kinetic theory under the relaxation time approximation near the critical region the ζ does not show singular behavior near the $O(4)$ and $Z(2)$ critical point through static critical exponents.

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I. INTRODUCTION

In the hydrodynamical evolution of fluid leading dissipative processes can be quantified by the transport coefficients, the shear η and bulk ζ viscosities. Their values and properties not only carry information on how far the system appears from an ideal hydrodynamics but can also provide relevant insight into the fluid dynamics and its critical phenomena [1–8]. For certain materials, e.g., helium, nitrogen or water, the shear viscosity to entropy ratio η/s is known experimentally to show a minimum at the phase transition [3]. However, the bulk viscosity ζ/s was argued to be large or even divergent at the critical point [5–8]. The recent lattice gauge theory calculations seem to be consistent with the expectation of decreasing η/s and increasing ζ/s toward the quantum chromodynamics (QCD) phase transition from above [9,10]. Thus, transport coefficients are of particular interest to quantify the properties of strongly interacting relativistic fluid and its phase transition [3].

In modeling strongly interacting media near equilibrium, the interactions usually lead to a quasiparticle description with its mass depending on thermal parameters [11]. The thermodynamics of perturbative QCD is also to large extent quantified by thermal quark and gluon masses that are T and μ dependent [12]. In the QCD-like chiral models where the phase transition is governed by an effective mass generated through the dynamics, the quasiparticle mass plays a role of an order parameter and thus is sensitive to change in thermal parameters.

In this section we discuss the bulk and shear viscosities of the dynamical fluid composed of quasiparticles with T- and μ-dependent masses. Our calculations are based on the kinetic theory in the relaxation time approximation. We derive a general expression for the viscosities in the quasiparticle medium that has a broad spectrum of applications. At vanishing chemical potential or for fixed particle masses our results are consistent with that formulated in Ref. [13] and Ref. [14], respectively.

Extrapolating the calculated bulk viscosity to the phase transition region we discuss its critical properties. Based on the Ginzburg-Landau and the scaling theory we show that under the relaxation time approximation the ζ is not expected to show a singular behavior near the $O(4)$ and $Z(2)$ critical point through the static critical exponents.

II. SHEAR AND BULK VISCOSITIES FROM TRANSPORT THEORY

The transport parameters, the shear η and the bulk ζ viscosities, are defined as coefficients of the space-space component of a deviation of the energy-momentum tensor from equilibrium. In a medium composed of bosons and/or fermions with the momentum distribution function $f(p,x)$ for a particle and $\bar{f}(p,x)$ for an antiparticle the energy-momentum tensor is defined as

$$T^{\mu\nu} = \int d\Gamma \frac{p^\mu p^\nu}{E}[f + \bar{f}], \tag{2.1}$$

where $d\Gamma = g d^3p/(2\pi)^3$ is the integration measure in the momentum space with the degeneracy factor g associated with the particle quantum numbers. The four-momentum $p^\mu = (E, \mathbf{p})$ with $E(\mathbf{p}) = \sqrt{\tilde{\mathbf{p}}^2 + \mathbf{M}^2}$ and M being a particle mass.[1]

Assuming that the medium appears near equilibrium we introduce the particle-momentum distribution $f(p,x)$ in the following form

$$f = [e^{(E - \mathbf{p}\cdot\mathbf{u} \mp \mu)/T} \pm 1]^{-1}, \tag{2.2}$$

where $\mathbf{u}$ is the flow velocity and μ is the chemical potential related with any conserved charges. The ± 1 corresponds to fermion and boson statistics, whereas $\mp\mu$ to particle and antiparticle contributions, respectively.

In our further discussion we assume that M is not necessarily a bare particle mass but rather a dynamical quasiparticle

[1]In general, it is not necessary to specify this dispersion relation. The following derivation is valid for any $E(\mathbf{p})$.

mass that depends on temperature and chemical potentials, thus $M = M(T,\mu)$ in Eq. (2.2).

A deviation of the system from equilibrium is quantified by the corresponding change of distribution functions $\delta f = f - f_0$ with f_0 being the equilibrium particle-momentum distribution. In the relaxation time approximation [15], the δf is obtained from

$$p^\mu \partial_\mu f_0 = -\frac{p\cdot u}{\tau}\delta f, \tag{2.3}$$

where the collision time

$$\tau^{-1} = n_f(T,\mu)\langle v\sigma(T,\mu)\rangle\,, \tag{2.4}$$

is determined by the thermal-averaged total scattering cross section $\langle v\sigma\rangle$, with the relative velocity of two colliding particles v and the particle density n_f in equilibrium. The δf results in the corresponding change in the energy-momentum tensor

$$\delta T^{\mu\nu} = -\int d\Gamma \frac{p^\mu p^\nu}{E^2} p^\alpha \partial_\alpha[\tau f_0 + \bar\tau \bar f_0]. \tag{2.5}$$

The thermodynamic quantities are time dependent through thermal parameters. The time dependence of T and μ are obtained from the charge number density $\partial_0 j^0 = 0$ and the energy density $\partial_0 T^{00} = 0$ conservations. The charged current $j^\mu = (j^0, \mathbf{j})$ is defined by

$$j^\mu = \int d\Gamma \frac{p^\mu}{E}[f - \bar f], \tag{2.6}$$

The energy and charge density conservations can be expressed in terms of the thermodynamic quantities as

$$\begin{aligned}\frac{\partial\epsilon}{\partial t} &= -(\epsilon + P)\nabla\cdot\mathbf{u}\\ &= -\left(T\frac{\partial P}{\partial T} + \mu\frac{\partial P}{\partial\mu}\right)\nabla\cdot\mathbf{u}\end{aligned} \tag{2.7}$$

$$\frac{\partial n}{\partial t} = -n\nabla\cdot\mathbf{u} = -\frac{\partial P}{\partial\mu}\nabla\cdot\mathbf{u}\,, \tag{2.8}$$

where the energy ϵ and charge n densities are related with the pressure P through the thermodynamic relations: $\epsilon = T\partial P/\partial T - P + \mu\partial P/\partial\mu$. From the chain rules

$$\frac{\partial P}{\partial t} = \frac{\partial P}{\partial\epsilon}\frac{\partial\epsilon}{\partial t} + \frac{\partial P}{\partial n}\frac{\partial n}{\partial t}\,, \tag{2.9}$$

and from Eq. (2.8) one gets

$$\frac{\partial P}{\partial t} = -\left[\frac{\partial P}{\partial\epsilon}\left(T\frac{\partial P}{\partial T} + \mu\frac{\partial P}{\partial\mu}\right) + \frac{\partial P}{\partial n}\frac{\partial P}{\partial\mu}\right]\nabla\cdot\mathbf{u}. \tag{2.10}$$

The pressure P is a function of T and μ, thus

$$\frac{\partial P}{\partial t} = \frac{\partial P}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial P}{\partial\mu}\frac{\partial\mu}{\partial t}\,, \tag{2.11}$$

therefore one arrives at the following equations for the time dependence of T and μ;

$$\begin{aligned}\frac{\partial T}{\partial t} &= -T\left(\frac{\partial P}{\partial\epsilon}\right)_n \nabla\cdot\mathbf{u}\,,\\ \frac{\partial\mu}{\partial t} &= -\left[\mu\left(\frac{\partial P}{\partial\epsilon}\right)_n + \left(\frac{\partial P}{\partial n}\right)_\epsilon\right]\nabla\cdot\mathbf{u}\,.\end{aligned} \tag{2.12}$$

Consequently, the change of the energy-momentum tensor (2.5) becomes

$$\begin{aligned}\delta T^{\mu\nu} &= \delta T_f^{\mu\nu} + \delta T_{\bar f}^{\mu\nu},\\ \delta T_f^{\mu\nu} &= \int d\Gamma\tau\frac{p^\mu p^\nu}{TE} f_0(1\pm f_0) q_f(\mathbf{p};T,\mu),\end{aligned} \tag{2.13}$$

where we have introduced

$$\begin{aligned}q_{f,\bar f}(\mathbf{p};\mathbf{T},\mu) = \Bigg[&-\frac{\tilde{\mathbf{p}}^2}{3\mathbf{E}} + \left(\frac{\partial\mathbf{P}}{\partial\epsilon}\right)_{\mathbf{n}}\left(\mathbf{E} - \mathbf{T}\frac{\partial\mathbf{E}}{\partial\mathbf{T}} - \mu\frac{\partial\mathbf{E}}{\partial\mu}\right)\\ &- \left(\frac{\partial P}{\partial n}\right)_\epsilon\left(\frac{\partial E}{\partial\mu} \mp 1\right)\Bigg]\partial_k u_l\delta^{kl} - \frac{p_k p_l}{2E}W^{kl},\end{aligned} \tag{2.14}$$

with the following tensor decomposition

$$\begin{aligned}\partial_k u^l &= \frac{1}{2}\left(\partial_k u^l + \partial_l u^k - \frac{2}{3}\delta_{kl}\partial_i u^i\right) + \frac{1}{3}\delta_{kl}\partial_i u^i\\ &\equiv \frac{1}{2}W_{kl} + \frac{1}{3}\delta_{kl}\partial_i u^i.\end{aligned} \tag{2.15}$$

With the above decomposition, the change of the tensor T^{ij} can be written as a sum of traceless W_{ij} and scalar part

$$\delta T^{ij} = -\zeta\delta_{ij}\partial_k u^k - \eta W_{ij}, \tag{2.16}$$

which defines the transport coefficients, the bulk ζ and the shear η viscosities, respectively. Implementing in Eq. (2.13) the energy conservation

$$\int d\Gamma E[\tau f_0(1\pm f_0)q_f + \bar\tau\bar f_0(1\pm\bar f_0)q_{\bar f}] = 0 \tag{2.17}$$

leads to the final expression for the transport coefficients obtained under the relaxation time approximation in the quasiparticle model with T- and μ-dependent masses.

The bulk $\zeta = \zeta_f + \zeta_{\bar f}$ and the shear $\eta = \eta_f + \eta_{\bar f}$ viscosities in a medium composed of one type of particle/antiparticle is obtained as

$$\eta = \frac{1}{15T}\int\frac{d^3p}{(2\pi)^3}\frac{\vec p^4}{E^2}[g\tau f_0(1\pm f_0) + \bar g\bar\tau\bar f_0(1\pm\bar f_0)], \tag{2.18}$$

and

$$\begin{aligned}\zeta = -\frac{1}{3T}\int\frac{d^3p}{(2\pi)^3}\Bigg\{&\frac{M^2}{E}[g\tau f_0(1\pm f_0) + \bar g\bar\tau\bar f_0(1\pm\bar f_0)]\\ &\times\left[\frac{\vec p^2}{3E} - \left(\frac{\partial P}{\partial\epsilon}\right)_n\left(E - T\frac{\partial E}{\partial T} - \mu\frac{\partial E}{\partial\mu}\right)\right.\\ &\left.+\left(\frac{\partial P}{\partial n}\right)_\epsilon\frac{\partial E}{\partial\mu}\right] - \frac{M^2}{E}[g\tau f_0(1\pm f_0) - \bar g\bar\tau\bar f_0(1\pm\bar f_0)]\\ &\times\left(\frac{\partial P}{\partial n}\right)_\epsilon\Bigg\},\end{aligned} \tag{2.19}$$

respectively. The above results can be obviously generalized to any systems composed of different particle species by summing up their contributions in Eqs. (2.18) and (2.19).

The derivatives of pressure $\partial P/\partial\epsilon|_n$ and $\partial P/\partial n|_\epsilon$ entering in Eq. (2.19) can be expressed in terms of the net particle number density n, the entropy density s, and different susceptibilities χ_{xy} that are defined by

$$n = \frac{\partial P}{\partial \mu}, \quad s = \frac{\partial P}{\partial T}, \quad \chi_{xy} = \frac{\partial^2 P}{\partial x \partial y}. \tag{2.20}$$

Applying the Jacobian methods to the above derivatives of P one finds

$$\left(\frac{\partial P}{\partial \epsilon}\right)_n = \frac{s\chi_{\mu\mu} - n\chi_{\mu T}}{C_V \chi_{\mu\mu}}, \tag{2.21}$$

$$\left(\frac{\partial P}{\partial n}\right)_\epsilon = \frac{nT\chi_{TT} + (n\mu - sT)\chi_{\mu T} - s\mu\chi_{\mu\mu}}{C_V \chi_{\mu\mu}}, \tag{2.22}$$

with C_V being the specific heat at constant volume and at constant s/n. The C_V can be expressed through different susceptibilities as

$$C_V = T\left(\frac{\partial s}{\partial T}\right)_V = T\left[\chi_{TT} - \frac{\chi_{\mu T}^2}{\chi_{\mu\mu}}\right]. \tag{2.23}$$

The shear viscosity (2.18) has the same form as previously obtained in Refs. [14,16] where the thermal modification of particle dispersion relations was not included. Thus, the shear viscosity is not directly affected by the quasiparticle dynamics. However, the bulk viscosity is essentially modified by the terms $\partial E/\partial T$ and $\partial E/\partial \mu$ that appear only when there is an explicit (T, μ) dependence of the particle mass. For $M(T, \mu) = \text{const.}$, Eq. (2.19) is reduced to the result obtained in Ref. [14]. At vanishing chemical potential Eq. (2.19) coincides with the expression recently formulated in Ref. [13] for an interacting quark-gluon plasma.

III. TRANSPORT COEFFICIENTS NEAR THE PHASE TRANSITION

To quantify the transport properties of thermodynamic systems characterized by viscosity coefficients one would need to formulate a specific model for particle interactions. However, there are some generic properties of ζ that can be discussed in a model-independent way through the universality arguments. In this context, of particular interest is the specific behavior of ζ near the phase transition. For the bare particle mass, the dynamics is entering only through the collision time. Thus, information on phase transition can be contained only in the change of the thermal averaged cross section. However, in the quasiparticle picture due to dynamical particle masses, further information on the phase transition appears in ζ through an explicit contribution of different observables that are sensitive to critical phenomena. From Eq. (2.19) it is clear that such observables are the specific heat C_V and different susceptibilities. In addition, in the model where the particle mass $M(T, \mu)$ is related with an order parameter, the thermal derivatives of M can be as well singular at the phase transition. This is the case, for example, when considering effective models related with the chiral phase transition [5,17].

To study the sensitivity of the bulk viscosity to the phase transition we separate from Eq. (2.19) the term $\zeta^{(\mathrm{der})}$ that can be singular near the critical point,

$$\begin{aligned}\zeta^{(\mathrm{der})} = &-\frac{1}{3T}\int \frac{d^3p}{(2\pi)^3}\frac{M^2}{E}\\ &\times\Bigg\{[g\tau f_0(1\pm f_0) + \bar{g}\bar{\tau}\bar{f}_0(1\pm \bar{f}_0)]\\ &\times\left[-\left(\frac{\partial P}{\partial \epsilon}\right)_n\left(E - T\frac{\partial E}{\partial T} - \mu\frac{\partial E}{\partial \mu}\right)\right.\\ &\left.+\left(\frac{\partial P}{\partial n}\right)_\epsilon \frac{\partial E}{\partial \mu}\right]\\ &-[g\tau f_0(1\pm f_0) - \bar{g}\bar{\tau}\bar{f}_0(1\pm \bar{f}_0)]\left(\frac{\partial P}{\partial n}\right)_\epsilon\Bigg\}. \end{aligned} \tag{3.1}$$

Hereafter, assuming a validity of the relaxation time approximation we discuss the critical behavior of $\zeta^{(\mathrm{der})}$ in different models with a phase transition. In particular, based on the Ginzburg-Landau as well as on the scaling theory we analyze the scaling of ζ in the $O(4)$ and $Z(2)$ universality classes that are expected in the QCD chiral phase transition.

A. Mean-field scaling of the bulk viscosity in the Ginzburg-Landau model

According to the Ginzburg-Landau theory, close to the phase boundary the thermodynamic potential may be expanded in a power series of the order parameter M, which plays the role of the dynamical particle mass

$$\begin{aligned}\Omega(T, \mu) \sim\ &\Omega_0(T, \mu; M = 0)\\ &+ \frac{a(T, \mu)}{2}M^2 + \frac{b(T, \mu)}{4}M^4 + \frac{c}{6}M^6 - mM,\end{aligned} \tag{3.2}$$

with $c > 0$. The order parameter is determined in such a way that the free energy is minimized. Thus, M is obtained as the solution of the gap equation for $m = 0$

$$M^2 = \frac{-b}{2c} \pm \frac{1}{2c}\sqrt{b^2 - 4ac}\,. \tag{3.3}$$

The fluctuations of M are defined through the susceptibility

$$\chi = \left.\frac{\partial M}{\partial m}\right|_{m=0} = \frac{1}{a + 3bM^2 + 5cM^4}\,. \tag{3.4}$$

With the particular choice of the parameters, $a = 0$ for $b > 0$ the thermodynamic potential describes the second-order phase transition. However, for $a = b = 0$ the system exhibits the tricritical point (TCP). Thus, the Ginzburg-Landau model has generic critical properties expected in the QCD chiral phase transition.

In the vicinity of the phase transition the temperature and chemical potential dependence of the coefficients $a(T, \mu)$ and $b(T, \mu)$ can be parameterized as

$$a(T, \mu) = \alpha|T - T_c| + \beta|\mu - \mu_c|, \tag{3.5}$$

with constant α and β.

From the thermodynamic potential (3.2) one gets all relevant thermodynamic quantities near the phase transition.

In particular, near the TCP the singular part of susceptibilities

$$\chi_{\mu\mu} \sim \frac{\beta^2}{b}, \quad \chi_{\mu T} \sim \frac{\alpha\beta}{b}, \quad \chi_{TT} \sim \frac{\alpha^2}{b}. \tag{3.6}$$

and derivatives of the dynamical mass

$$\frac{\partial M}{\partial T} \sim -\frac{1}{M}\frac{\alpha}{b}, \quad \frac{\partial M}{\partial \mu} \sim -\frac{1}{M}\frac{\beta}{b}. \tag{3.7}$$

With the above scaling relations and from Eqs. (2.21) and (2.23) one finds that near the TCP the pressure derivatives and the specific heat are finite and that the potentially singular part $\zeta^{(\mathrm{der})}$ behaves as

$$\zeta^{(\mathrm{der})} \sim -\frac{M^3}{C_V}\left(s - n\frac{\alpha}{\beta}\right)\left(\frac{\partial M}{\partial T} - \frac{\alpha}{\beta}\frac{\partial M}{\partial \mu}\right). \tag{3.8}$$

Consequently, due to the vanishing coefficient of the second bracket in Eq. (3.8) together with the scaling [17]

$$\frac{M^2}{b} \sim M^4\chi \sim t^0, \tag{3.9}$$

the singularities of the susceptibilities do not show up in ζ near TCP.

The singular part of $\zeta^{(\mathrm{der})}$ also vanishes at the second-order transition because there $\zeta^{(\mathrm{der})} \sim M^2 \to 0$. In addition, if the explicit chiral symmetry breaking is absent then also the regular part of ζ vanishes. Thus the ζ is precisely zero at the critical point.

The above example shows that within the mean-field dynamics the bulk viscosity is nonsingular at the second-order phase transition and at the TCP.

B. Scaling of the bulk viscosity in the $O(4)$ and $Z(2)$ universality class

To verify the singular behavior of ζ along the second-order line and at the critical end point (CEP) that belongs to the $O(4)$ and $Z(2)$ universality class, respectively, one needs to go beyond the mean-field approximation. The scaling behavior of different observables in the vicinity of phase transition emerges from the scaling function of the singular part of the free energy.

Along the $O(4)$ transition line and for $\mu/T \ll 1$, the singular part of the free energy can be parameterized as [18]

$$F_s(T,\mu) \simeq t^{2-\alpha} f_s(1, t^{-\beta\delta}h), \tag{3.10}$$

where

$$t = \bar{t} + A\bar{\mu}^2, \tag{3.11}$$

with $\bar{t} = |T - T_c|/T_c$, $\bar{\mu} = \mu/T_c$ and h being an external field. The α, β, and δ are the critical exponents in the $O(4)$ universality class.

From the scaling function (3.10), by taking derivatives, one finds the properties of all relevant quantities required in Eq. (2.19) to find the behavior of the bulk viscosity near the $O(4)$ line. In the following we consider only the transition point at $\mu = 0$. From Eq. (3.10) one gets

$$\chi_{\mu\mu} \sim t^{1-\alpha}, \quad \chi_{TT} \sim t^{-\alpha}, \quad C_V \sim t^{-\alpha},$$
$$M \sim t^{\beta}, \quad \partial M/\partial T \sim t^{\beta-1}. \tag{3.12}$$

Substituting the above scaling to Eq. (2.19) one finds that near the $O(4)$ transition point the singular part of the bulk viscosity $\zeta^{(\mathrm{der})}$ scales as

$$\zeta^{(\mathrm{der})} \sim t^{\alpha+4\beta-1}. \tag{3.13}$$

Thus, with the $O(4)$ critical exponents: $\alpha \simeq -0.24$ and $\beta \simeq 0.38$, the singular part of ζ vanishes at the critical point. Consequently, in this approach, there is no singularity in the bulk viscosity along the $O(4)$ line.

The above scaling of ζ can be different near the CEP, because it belongs to the $Z(2)$ universality class of the three-dimensional Ising model. To match the spin system parameters, the reduced temperature t and the external field h, to those in the QCD chiral models we follow the discussion of Ref. [19] and replace: $t \to a_t\bar{t} + b_t\bar{\mu}$ and $h \to a_h\bar{t} + b_h\bar{\mu}$. The singular part of the free energy in the $Z(2)$ universality class is parameterized as:

$$F_s(t,\mu) \sim h^{\frac{1+\delta}{\delta}} f_s(h^{-1/\beta\delta}t, 1)\,. \tag{3.14}$$

This scaling form of the free energy leads to the following critical behavior of the thermodynamic quantities near CEP at $t = 0$:

$$\chi_{\mu\mu,TT,\mu T} \sim h^{-\gamma/\beta\delta}, \quad C_V \sim h^{-\gamma/\beta\delta},$$
$$M \sim h^{1/\delta}, \quad \partial M/\partial T \sim h^{-\gamma/\beta\delta}. \tag{3.15}$$

Consequently, the singular part of the bulk viscosity (2.19) in the $Z(2)$ universality class scales as

$$\zeta^{(\mathrm{der})} \sim h^{\gamma/\beta\delta+4/\delta-1}. \tag{3.16}$$

Substituting into Eq. (3.16) the $Z(2)$ exponents: $\beta \simeq 0.31$, $\delta \simeq 5.2$, and $\gamma \simeq 1.25$ one finds $\zeta \sim h^{0.54}$. Thus, similarly as in the $O(4)$ universality class the bulk viscosity is nonsingular near the CEP.

The above scalings of ζ, obtained under relaxation time approximation, different from that recently found in Ref. [8]. There, it was argued that the critical behavior of the bulk viscosity is governed by the critical exponent of the specific heat because $\zeta \sim C_V$.[2] Consequently, the bulk viscosity was shown to diverge at the CEP in the $Z(2)$ universality class. In our approach, under the relaxation time approximation, the scaling of ζ is determined by the product

$$\zeta^{(\mathrm{der})} \sim \frac{M^3}{C_V}\left(\frac{\partial M}{\partial T} + C\frac{\partial M}{\partial \mu}\right). \tag{3.17}$$

Consequently, ζ is proportional to C_V^{-1} rather than to C_V. The same, negative power of C_V is also reported in Ref. [20]. The bulk viscosity contains both the regular and derivative terms: the former is always positive and the latter can change its sign dependently on the model. However, the overall sign of ζ should be always positive for any T and μ. This condition could be used as a constraint and a limitation on the thermal parameter dependence of dynamical quasiparticle masses to satisfy the H theorem.

[2]For the recent discussion of a possible problem with such scaling behavior of the bulk viscosity see, e.g., Ref. [21].

From the above discussion, valid under relaxation time approximation, one sees that ζ is not expected to show divergent behavior near the critical end point through the static critical exponents. However, an extension of the relaxation time approximation, that allows proper incorporation of a long-range fluctuations of soft modes, can result in divergence of ζ at the CEP as well as at the second-order $O(4)$ transition [20]. In this case, divergence of the bulk and shear viscosities is governed by the dynamic rather than static critical exponents [22,23].[3]

In the presence of the dynamical critical phenomena the bulk viscosity was shown to scale as $\zeta \sim t^{-z\nu+\alpha}$ at the second-order phase transition, with the dynamical critical exponent z, determining the critical slowing down of the system relaxation time $\tau \sim t^{-z\nu}$ and with ν and α being the static critical exponents of the correlation length and the specific heat, respectively [23]. Consequently, when approaching toward the $Z(2)$ critical point the bulk viscosity diverges due to the large value of $z \simeq 3$ expected in the dynamical universality class of the three-dimensional Ising model [22,24]. The ratio, $\zeta/\tau \sim C_V^{-1}$, however, is proportional to the inverse of the specific heat and thus vanishes at the critical point. This scaling property of the ratio is consistent with our finding in Eq. (3.17) obtained within the relaxation time approximation. However, due to the fact that in the relaxation time approximation the τ stays finite even at the second-order critical point the bulk viscosity obtained in our approach does not include a dynamical critical behavior that is quantified by the dynamical critical exponent.

IV. CONCLUSIONS

We have studied the nonequilibrium properties of a quasiparticle medium at finite temperature and density using the kinetic theory in the relaxation time approximation. Assuming that the quasiparticle masses are temperature and chemical potential dependent, we have derived a consistent expression for the bulk viscosity coefficient ζ. We have shown that in the presence of dynamical mass M the fluid compressibility is essentially modified. Our result for ζ is valid in different physical systems where interactions yield some modification in particle dispersion relations through explicit variation of M with thermal parameters.

We have applied our results to a class of effective chiral models where the dynamical mass is identified as the order parameter for the chiral phase transition. We extrapolated the bulk viscosity obtained in the relaxation time approximation to the phase transition and studied the influence of critical fluctuations on ζ. We have shown that under the mean-field dynamics as well as in the presence of quantum fluctuations implemented through the scaling functions, the bulk viscosity is not sensitive to the chiral phase transition. This is because the singularities generated from the susceptibilities are totally canceled in ζ at the $O(4)$ as well as at the $Z(2)$ critical point. A possible modification of our conclusions due to a proper treatment of soft modes at the phase transition is not excluded. Nevertheless, the scaling properties given in this article indicate a tendency of the transport coefficients when approaching the critical point. The phenomenological relevance of soft modes at the phase transition strongly depends on how large the critical region is. If the critical region is very narrow in thermal parameters then it may be rather hard to observe the singularity of bulk viscosity around the critical point.

[3]The dynamic universality class of the QCD critical point is that of the model H [24].

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