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Quark gluon bags as reggeons

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The influence of the medium-dependent finite width of quark gluon plasma (QGP) bags on their equation of state is analyzed within an exactly solvable model. It is argued that the large width of the QGP bags not only explains the observed deficit in the number of hadronic resonances but also clarifies the reason why the heavy QGP bags cannot be directly observed as metastable states in a hadronic phase. The model allows us to estimate the minimal value of the width of QGP bags being heavier than 2 GeV from a variety of the lattice QCD data and get that the minimal resonance width at zero temperature is about 600 MeV, whereas the minimal resonance width at the Hagedorn temperature is about 2000 MeV. As shown, these estimates are almost insensitive to the number of the elementary degrees of freedom. The recent lattice QCD data are analyzed and it is found that in addition to the σT^4 term the lattice QCD pressure contains T-linear and $T^4 \ln T$ terms in the range of temperatures between 240 and 420 MeV. The presence of the last term in the pressure bears almost no effect on the width estimates. Our analysis shows that at high temperatures the average mass and width of the QGP bags behave in accordance with the upper bound of the Regge trajectory asymptotics (the linear asymptotics), whereas at low temperatures they obey the lower bound of the Regge trajectory asymptotics (the square root one). Since the model explicitly contains the Hagedorn mass spectrum, it allows us to remove an existing contradiction between the finite number of hadronic Regge families and the Hagedorn idea of the exponentially growing mass spectrum of hadronic bags.

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I. INTRODUCTION

The concept of strongly interacting QGP (sQGP) [1,2] has created a new framework for the quantum chromodynamic (QCD) phenomenology. However, the very strong argument that sQGP is not a weakly coupled gas but rather a strongly coupled liquid [1] does not clarify the question of what are the relevant degrees of freedom to formulate the proper statistical description of the sQGP. The idea that the dressed quarks and gluons should form the multiple binary colored bound states [1] and even multibody bound states [1,3] does not help much, because in the sQGP the colored objects must strongly interact with the surrounding media and form the colorless clusters whose interaction with each other is essentially reduced compared to their constituents. Such a behavior of interacting constituents is typical for the clusters formed by the molecules in the ordinary liquids [4,5] and by the nucleons in a nuclear liquid [5–7]. An existence of colorless clusters in the sQGP is indirectly supported by the huge quark-antiquark "energy" potentials found from the lattice QCD (LQCD) simulations [2,8], which indicate us that at energy densities right above the deconfinement transition there is no separation of valence quarks belonging to the same hadron. These huge values of binding energy indicate that the relevant degrees of freedom in the sQCD are heavy and large hadrons that nowadays are regarded as the QGP bags.

The idea that the relevant degrees of freedom in the QGP are the hadronic bags of any volumes and masses that contain quarks and gluons inside was first formulated in Ref. [9] and named gas of bags model (GBM). This work has unified several instructive results obtained earlier: it was shown [9] that the MIT bag model [10] leads to the Hagedorn mass spectrum of bags [11] and the phase transition to the QGP corresponds to a formation of the infinitely large bag. Further development in this direction led to many interesting findings [12,13]. The most promising of them is an inclusion of the quark gluon bags surface tension into statistical description [12] that allows one to simultaneously describe the first- and second-order deconfinement phase transition with the crossover.

The existence of the QGP bags with the Hagedorn mass spectrum near the transition temperature to hadronic phase is strongly supported by the fact that the Hagedorn resonances can naturally explain the extra baryon (antibaryon) [14] and kaon (antikaon) [15] production that was found in the 5% most central Au-Au collisions at the Relativistic Heavy Ion Collider (RHIC) [16]. Also it seems that the QGP bags with the Hagedorn mass spectrum can explain the fast chemical equilibration of hadrons in an expanding fireball [15]. Such an equilibration is a manifestation of the fact that the resonances with Hagedorn mass spectrum are the perfect particle reservoirs and perfect thermostats [17]. Hence our starting point is the quark gluon bags with surface tension model (QGBSTM) [12], which includes all the important features of the previously suggested statistical models discussed above.

The question of the relevant degrees of freedom in the sQGP is of principle importance not only to formulate the correct statistical description but also to explain the various phenomena that occur in the sQGP during its thermalization, expansion, and hadronization in the course of high-energy nuclear or elementary particle collisions. We mean not only the direct flow, conical flow, or the jet quenching but the mechanism of thermal and chemical equilibration of the sQGP whose investigation requires the development of nonperturbative methods suited to describe such processes. Unfortunately, the powerful methods suggested in the past were not developed further to exploit them for the present needs of heavy-ion phenomenology. For example, the well-known dual resonance model [18] is able to explain not only the Hagedorn mass spectrum but also the process of thermalization of decaying resonances [19]. Hence it could be a good starting point to study the process of thermalization of the QGP bags, but due to the lack of our knowledge on the relevant degrees of freedom of sQGP it is as yet unclear to what extent is it possible to use the dual resonance model for this purpose.

The problem of the relevant degrees of freedom in the sQGP has another important aspect. Indeed, the masses and bounds on charge or isospin of the sQGP constituents are intensively discussed in the LQCD community [20], but at the same time important characteristics such as their mean volume and lifetime have not yet caught the necessary attention. However, these quantities may put some new bounds on the spatial and temporal properties of the sQGP [21] created in high-energy collisions. If, for instance, the sQGP consists of droplets of finite (mean) size, then one could naturally resolve the HBT puzzles at RHIC energies [22]. However, the short life-time of heavy QGP bags found recently [21] may not only play an important role in all thermodynamic and hydrodynamic phenomena of the sQGP matter mentioned above but may also explain the absence of strangelets [23] or, more generally, why the finite QGP bags cannot be observed at energy densities typical for hadronic phase [21] (see below). Therefore, an investigation of the mass and volume distributions and the lifetime of the QGP bags and their consequences for both the experimental observables and theoretical studies is very pertinent.

The work is organized as follows. First, we thoroughly discuss the two conceptual problems of the GBM and its generalizations that are typical for finite systems created in high-energy collisions. Then in Sec. III we present the finite width model (FWM) of QGP bags [21] and in Sec. IV we show how both conceptual problems can be naturally resolved within the FWM that is a principally new kind of model compared to the GBM generalizations and early attempts to derive the mass-volume spectrum of the QGP bags [13,24]. Section V is devoted to the analysis of the LQCD pressure and trace anomaly. These results allow us to estimate the width of the QGP bags from the LQCD thermodynamics in Sec. VI. In Sec. VII we compare two regimes of the FWM and show that in the high-pressure regime the FWM QGP bags behave in accordance with the upper bound [25] of the asymptotic behavior of the Regge trajectories for the mass and width of hadronic resonances, whereas in the low-pressure limit they obey the lower limit of the asymptotic behavior of the Regge trajectories [25]. Thus, we explicitly demonstrate that the large and/or heavy QGP bags can be regarded as the objects belonging to the Regge trajectories. Furthermore, we establish the close relations between the Hagedorn idea of exponentially increasing hadronic mass spectrum and finite number of Regge trajectories of QGP bags within the Regge poles method. Our conclusions are formulated in Sec. VII.

II. CONCEPTUAL PROBLEMS OF GBM

Despite the positive features of the GBM [9,26] and its generalizations [12,13,27], all of them face two conceptual problems. The first one can be formulated by asking a very simple question: Why are the QGP bags never directly observed in the experiments? The routine argument applied to both high-energy heavy-ion and hadron collisions is that there exists a phase transition and, hence, the huge energy gap separating the QGP bags from the ordinary (light) hadrons prevents the QGP coexistence at the hadron densities below the phase transition. The same line of arguments is also valid, if the strong crossover exists. The problem, however, arises from the fact that in the laboratory experiments we are dealing with finite systems. From the finite volume exact analytical solutions of the constrained statistical multifragmentation model (SMM) [6,7] and the GBM [9,26], found in Ref. [28] and [29,30], respectively, it is known that in thermally equilibrated finite system there is a non-negligible probability of finding the small and not too heavy QGP bags, say with the mass of 10–15 GeV, even in the hadronic phase. Therefore, for finite volume systems created in high-energy nuclear or elementary particle collisions such QGP bags could appear as any other metastable states in statistical mechanics, because in this case the statistical suppression is just a few orders of magnitude and not of the order of the Avogadro number.

Moreover, the finite volume solution of the GBM [29], in which the mean mass of the QGP bag is proportional to its volume, predicts the decay time $\tau_n \approx \frac{V}{\pi n V_0 T}$ for the collective state $n (n = 1, 2, 3, \ldots, V_0 \approx 1\ \mathrm{fm}^3$ [21,29]) of the mixed phase having the finite volume V and temperature T. Therefore, if a single statistical state with $n \geqslant 1$ had a pressure close to zero, its lifetime would be determined by the temperature and the volume. If, in addition, such a state could emit the photons or dileptons to reduce its temperature without an essential reduction of its volume, it could live for a very long time compared to the typical lifetime of heavy hadronic resonances. In particular, one could think of the strangelets [23], as a possible example for such a state.

Consequently, if such QGP bags can be created in high-energy nuclear and in elementary particle collisions or in some astrophysical phenomena there must be a reason that prevents their direct experimental detection. As we will show in the following there is an inherent property of the strongly interacting matter equation of state (EOS) that prevents the appearance of such QGP bags inside of the hadronic phase even in finite systems and that is also responsible for the instability of large or heavy strangelets.

The second conceptual problem is rooted in a huge deficit in the number of observed hadronic resonances [31] with masses above 2.5 GeV predicted by the Hagedorn model [11] and used, so far, by all other subsequent models discussed above. Thus, there is a paradox situation with the Hagedorn mass spectrum: it was predicted for heavy hadrons that nowadays must be regarded as QGP bags, but it can be experimentally established up to hadronic masses of about 2.3 GeV [31]. Of course, one could argue that heavy hadronic resonances cannot be established experimentally because both their large width and very large number of decay channels lead to great difficulties in their identification. But the point is that, despite the recent efforts of Ref. [32], the influence of large width of heavy resonances on their EOS properties and the corresponding experimental consequences were not studied in full.

The recent step in this direction was made in Ref. [21]. We introduced the finite and medium-dependent width into statistical description and studied its influence on the system's pressure at vanishing baryonic chemical potential there. We argued that the FWM requires the inclusion of the width of the QGP bags that, on one hand, explains the experimentally observed deficit of heavy hadronic resonances compared to any of the previous GBM generalizations, and, on the other hand, we demonstrated that the new physical effect, *the subthreshold suppression of the QGP bags* of the FWM, naturally resolves the first conceptual problem formulated above.

III. BASIC INGREDIENTS OF THE FWM

The most convenient way to study the phase structure of any statistical model similar to the GBM or QGBSTM is to use the isobaric partition [12,26,28] and find its rightmost singularities. Hence, we assume that after the Laplace transform the FWM grand canonical partition $Z(V,T)$ generates the following isobaric partition:

$$\hat{Z}(s,T) \equiv \int_0^\infty dV \exp(-sV)\, Z(V,T) = \frac{1}{[s - F(s,T)]}, \tag{1}$$

where the function $F(s,T)$ contains the discrete F_H and continuous F_Q mass-volume spectrum of the bags

$$F(s,T) \equiv F_H(s,T) + F_Q(s,T) = \sum_{j=1}^{n} g_j e^{-v_j s} \phi(T, m_j) + \int_{V_0}^\infty dv \int_{M_0}^\infty dm\, \rho(m,v) \exp(-sv) \phi(T,m). \tag{2}$$

The density of bags of mass m_k, eigenvolume v_k, and degeneracy g_k is given by $\phi_k(T) \equiv g_k \phi(T, m_k)$ with

$$\phi_k(T) \equiv \frac{g_k}{2\pi^2} \int_0^\infty p^2 dp \exp\left[-\frac{\left(p^2 + m_k^2\right)^{1/2}}{T}\right] = g_k \frac{m_k^2 T}{2\pi^2} K_2\left(\frac{m_k}{T}\right). \tag{3}$$

The mass-volume spectrum $\rho(m,v)$ is the generalization of the exponential mass spectrum introduced by Hagedorn [11]. Similar to the GBM and QGBSTM, the FWM bags are assumed to have the hard core repulsion of the Van der Waals type that generates the suppression factor proportional to the exponential of bag eigenvolume $\exp(-sv)$. Because the mass-volume spectrum $\rho(m,v)$ can be written in a form containing the discrete part F_H, hereafter we will not distinguish the discrete bags from the bags of continuous spectrum, if their properties are similar. However, we will keep the sum and integrals in Eq. (2) explicitly, because they correspond to different phases of the model.

The first term of Eq. (2), F_H, represents the contribution of a finite number of low-lying hadron states up to mass $M_0 \approx 2$ GeV [21] which correspond to different flavors. This function has no s singularities at any temperature T and can generate a simple pole of the isobaric partition, whereas the mass-volume spectrum of the bags $F_Q(s,T)$ can be chosen to generate an essential singularity $s_Q(T) \equiv p_Q(T)/T$ that defines the QGP pressure $p_Q(T)$ at zero baryonic densities [12,24,26].

It is known from the definition of pressure in the grand canonical ensemble that in the thermodynamic limit its partition behaves as $Z(V,T) \simeq \exp[pV/T]$. An exponentially increasing $Z(V,T)$ generates the rightmost singularity $s^* = p/T$ of the function $\hat{Z}(s,T)$ in variable s. This is because the integral over V in Eq. (1) diverges at its upper limit for $s < p/T$. Therefore, the rightmost singularity s^* of $\hat{Z}(s,T)$ gives us the system pressure:

$$p(T) = T \lim_{V\to\infty} \frac{\ln Z(V,T)}{V} = T\, s^*(T). \tag{4}$$

The singularity s^* of $\hat{Z}(s,T)$ (1) can be calculated from the transcendental equation [12,26] $s^*(T) = F(s^*, T)$.

As long as the number of sorts of bags, n, is finite, the only possible singularities of $\hat{Z}(s,T)$ (1) are simple poles. For example, for the ideal gas [$n = 1; v_1 = 0; F_Q \equiv 0$ in Eq. (2)] $s^* = g_1 \phi(T, m_1)$ and thus from Eq. (4) one gets $p = T g_1 \phi(T, m_1)$ that corresponds to the grand canonical ensemble ideal gas EOS for the particles of mass m_1 and degeneracy g_1. However, for an infinite number of sorts of bags, i.e., for $F_Q \neq 0$, there may appear an essential singularity of $\hat{Z}(s,T)$ that corresponds to a different phase. This property is used in the FWM.

Here we use the parametrization of the spectrum $\rho(m,v)$ introduced in [21]. It assumes that

$$\rho(m,v) = \frac{\rho_1(v)\, N_\Gamma}{\Gamma(v)\, m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H} - \frac{(m - Bv)^2}{2\Gamma^2(v)}\right], \tag{5}$$

$$\rho_1(v) = f(T) v^{-b} \exp\left[-\frac{\sigma(T)}{T} v^{\varkappa}\right], \tag{6}$$

where we drop the unimportant dependences. As one can see from (5) the mass spectrum has a Hagedorn like parametrization and the Gaussian attenuation around the bag mass Bv (B is the mass density of a bag of a vanishing width) with the volume-dependent Gaussian width $\Gamma(v)$ or width hereafter. We will distinguish it from the true width defined as $\Gamma_R = \alpha \Gamma(v) (\alpha \equiv 2\sqrt{2\ln 2})$. We stress that the Breit-Wigner attenuation of a resonance mass cannot be used in the spectrum (5) because in case of finite width it would lead to a divergency of the mass integral in Eq. (2) above T_H.

The normalization factor obeys the condition

$$N_\Gamma^{-1} = \int_{M_0}^\infty \frac{dm}{\Gamma(v)} \exp\left[-\frac{(m - Bv)^2}{2\Gamma^2(v)}\right]. \tag{7}$$

The constants $a > 0$ and $b > 0$ will be specified later.

The present choice of mass-volume spectrum (5) is a natural extension of early attempts [24] to explore the bag volume as a statistically independent degree of freedom to derive an internal pressure of large bags. As it will be shown such a simple parametrization not only allows us to resolve both of the conceptual problems discussed above, but also it gives us an exactly solvable model. Our further motivation for the mass-volume spectrum (5) is based on two facts: first, for all known hadronic resonances the width and mass are independent characteristics, and, second, in a dense medium the reaction rates may change and may lead to the medium dependence of the resonance width. To take both of them into account we introduced the volume dependence into the Gaussian width and we came to conclusion that the proper characteristic to indicate the impact of a medium is not the resonance width, but the average resonance width. The latter allows us to get simultaneously the mass and temperature dependences of the resonance width averaged with respect to the resonance volume [see Eq. (8)]. Moreover, as will be shown later such a parametrization of the mass-volume spectrum leads not only to a single choice for the Gaussian width volume dependence but also allows us to introduce the concept of Regge trajectories for the averaged quantities of the QGP bags.

The volume spectrum in Eq. (6) contains the surface free energy ($\varkappa = 2/3$) with the T-dependent surface tension that is parameterized as $\sigma(T) = \sigma_0 \cdot [\frac{T_c - T}{T_c}]^{2k+1} (k = 0, 1, 2, \ldots)$ [12,33], where $\sigma_0 > 0$ can be a smooth function of temperature. For T being not larger than the tricritical temperature T_c such a parameterization is justified by the usual cluster models like the FDM [4,5] and SMM [6,7,34], whereas the general consideration for any T can be driven by the surface partitions of the Hills and Dales model [33]. In Ref. [12] it was argued that at low baryonic densities the first-order deconfinement phase transition degenerates into a crossover just because of negative surface tension coefficient for $T > T_c$. The other consequences of the present surface tension parametrization and the discussion of the absence of the curvature free energy in Eq. (6) can be found in Refs. [12,35].

The spectrum (5) has a simple form but is rather general because both the width $\Gamma(v)$ and the bag mass density B can be medium dependent. It clearly reflects the fact that the QGP bags are similar to the ordinary quasiparticles with the medium-dependent characteristics (life-time, most probable values of mass and volume). Now we are ready to derive the infinite bag pressure for two choices of the width: the volume-independent width $\Gamma(v) \equiv \Gamma_0$ and the volume-dependent width $\Gamma(v) \equiv \Gamma_1 = \gamma v^{\frac{1}{2}}$. As will be seen below the latter resolves both of the conceptual problems discussed earlier, whereas the former parametrization is used for a comparison.

IV. ANALYSIS OF THE FWM SPECTRUM

First we note that for large bag volumes ($v \gg M_0/B > 0$) the factor (7) can be found as $N_\Gamma \approx 1/\sqrt{2\pi}$. Similarly, one can show that for heavy free bags ($m \gg BV_0$, $V_0 \approx 1$ fm^3 [21], ignoring the hard core repulsion and thermostate)

$$\rho(m) \equiv \int_{V_0}^{\infty} dv\, \rho(m, v) \approx \frac{\rho_1\left(\frac{m}{B}\right)}{B\, m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H}\right]. \quad (8)$$

It originates in the fact that for heavy bags the Gaussian in Eq. (5) acts like a Dirac δ function for either choice of Γ_0 or Γ_1. Thus, the Hagedorn form of Eq. (8) has a clear physical meaning and, hence, it gives an additional argument in favor of the FWM. Also it gives an upper bound for the volume dependence of $\Gamma(v)$: the Hagedorn-like mass spectrum (8) can be derived if for large v the width Γ increases slower than $v^{(1-\varkappa/2)} = v^{2/3}$.

Similarly to Eq. (8), one can estimate the width of heavy free bags averaged over bag volumes and get $\overline{\Gamma(v)} \approx \Gamma(m/B)$. Thus, for $\Gamma_1(v)$ the mass spectrum of heavy free QGP bags must be the Hagedorn-like one with the property that heavy resonances have to develop the large mean width $\Gamma_1(m/B) = \gamma\sqrt{m/B}$ and, hence, they are hardly observable. Applying these arguments to the strangelets, we conclude that, if their mean volume is a few cubic fermis or larger, they should survive a very short time, which is similar to the results of Ref. [23] predicting an instability of such strangelets.

Note also that such a mean width is essentially different from both the linear mass dependence of string models [36] and from an exponential form of the nonlocal field theoretical models [37]. Nevertheless, as we demonstrate while discussing the Regge trajectories (30) and (33) the mean width $\Gamma_1(m/B)$ leads to the linear Regge trajectory of heavy free QGP bags for large values of the invariant mass squared.

Next we calculate $F_Q(s, T)$ (2) for the spectrum (5) performing the mass integration. There are, however, two distinct possibilities, depending on the sign of the most probable mass:

$$\langle m \rangle \equiv Bv + \Gamma^2(v)\beta, \quad \text{with} \quad \beta \equiv T_H^{-1} - T^{-1}. \quad (9)$$

If $\boldsymbol{\langle m \rangle > 0}$ for $v \gg V_0$, one can use the saddle point method for mass integration to find the function $F_Q(s, T)$

$$F_Q^+(s, T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \frac{\rho_1(v)}{\langle m \rangle^a} \exp\left[\frac{(p^+ - sT)v}{T}\right] \quad (10)$$

and the pressure of large bags

$$p^+ \equiv T\left[\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right]. \quad (11)$$

To get Eq. (10) one has to use in Eq. (2) an asymptotic form of the K_2 function $\phi(T, m) \simeq (mT/2\pi)^{3/2} \exp(-m/T)$ for $m \gg T$, collect all terms with m in the exponential, get a full square for $(m - \langle m \rangle)$ and make the Gaussian integration. The resulting mass attenuation of the obtained spectrum $\frac{N_\Gamma M_0^a}{\Gamma(v) m^a} \exp[-\frac{(m-\langle m \rangle)^2}{2\Gamma^2(v)}]$ at the fixed bag volume is shown in Fig. 1 as the solid curve for the typical range of parameters ($a = 2$).

Because for $s < s_Q^*(T) \equiv p^+(v \to \infty)/T$ the integral (10) diverges at its upper limit, the partition (1) has an essential singularity that corresponds to the QGP pressure of an infinite large bag. One concludes that the width Γ cannot grow faster than $v^{1/2}$ for $v \to \infty$, otherwise $p^+(v \to \infty) \to \infty$ and $F_Q^+(s, T)$ diverges for any s. Thus, for $\boldsymbol{\langle m \rangle > 0}$ the phase structure of the FWM with $\Gamma(v) = \Gamma_1(v)$ is similar to the QGBSTM [12].

The volume spectrum of bags $F_Q^+(s, T)$ (10) is of general nature and, in contrast with the one suggested in Ref. [24], has a clear physical meaning. One can also see that two general origins of the bulk part of bag's free energy

$$-p^+ v = -T\left[\beta\langle m \rangle - \tfrac{1}{2}\Gamma^2(v)\beta^2\right] \quad (12)$$

are the bag's most probable mass and its width. Choosing different T-dependent functions $\langle m \rangle$ and $\Gamma^2(v)$, one obtains different equations of state. Comparing the v power of the exponential prefactor in Eq. (10) to the continuous volume spectrum of bags of the QGBSTM [12], we find that $a + b \equiv \tau \leqslant 2$.

It is possible to use the spectrum (10) not only for infinite system volumes but also for finite volumes $V \gg V_0$. In this case the upper limit of integration should be replaced by finite V (see Refs. [29,30] for details). This will change the singularities of partition (1) to a set of simple poles $s_n^*(T)$ in the complex s plane that are defined by the same equation as for $V \to \infty$. Similarly to the finite V solution of the GBM [29,30], it can be shown that for finite T the FWM simple poles may have small positive or even negative real part that would lead to a non-negligible contribution of the QGP bags into the total spectrum $F(s, T)$ (2). In other words, if the spectrum (10), was the only volume spectrum of the QGP bags, then there would exist a finite (non-negligible) probability to find heavy QGP bags ($m \gg M_0$) in finite systems at low temperatures $T \ll T_c$. Therefore, using the results of the finite volume GBM and SMM, we conclude that the spectrum (10) itself cannot explain the absence of the QGP bags at $T \ll T_c$ and, hence, an alternative explanation of this fact is required.

Such an explanation corresponds to the case $\langle \boldsymbol{m} \rangle \leqslant \mathbf{0}$ for $v \gg V_0$. From Eq. (9) one can see that for the volume-dependent width $\Gamma(v) = \Gamma_1(v)$ the most probable mass $\langle m \rangle$ inevitably becomes negative at low T, if $0 < B < \infty$. In this case the maximum of the Gaussian mass distribution is located at resonance masses $m = \langle m \rangle \leqslant 0$. This is true for any argument of the K_2 function in $F_Q(s, T)$ (2). Because the lower limit of mass integration M_0 lies above $\langle m \rangle$, only the tail of the Gaussian mass distribution may contribute into $F_Q(s, T)$. A thorough inspection of the integrand in $F_Q(s, T)$ shows (see the dashed curve in Fig. 1 for $a = 2$) that above M_0 it is strongly decreasing function of resonance mass and, hence, only the vicinity of the lower limit of mass integration M_0 sizably contributes into $F_Q(s, T)$. Applying the steepest descent method and the K_2-asymptotic form for $M_0 T^{-1} \gg 1$

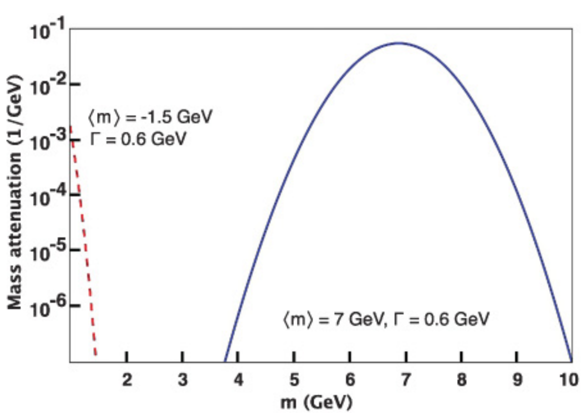

FIG. 1. (Color online) The resulting mass attenuation $\frac{N_\Gamma M_0^a}{\Gamma(v) m^a} \exp[-\frac{(m-\langle m \rangle)^2}{2\Gamma^2(v)}]$ as the function of bag mass at the fixed bag volume for positive and negative values of the most probable bag mass. The solid (dashed) curve corresponds to $\langle m \rangle = 7$ GeV($\langle m \rangle = -1.5$ GeV). Both curves are shown for the same width $\Gamma(v) = 600$ MeV and $a = 2$.

one obtains

$$F_Q^-(s, T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \frac{\rho_1(v) N_\Gamma \Gamma(v) \exp\left[\frac{(p^- - sT)v}{T}\right]}{M_0^a [M_0 - \langle m \rangle + a\Gamma^2(v)/M_0]} \tag{13}$$

with the formal expression for the pressure of QGP bag

$$p^-\big|_{v \gg V_0} = \frac{T}{v}\left[\beta M_0 - \frac{(M_0 - Bv)^2}{2\Gamma^2(v)}\right]. \tag{14}$$

We would like to stress that the last result requires $B > 0$ and it cannot be obtained for a weaker v growth than $\Gamma(v) = \Gamma_1(v)$. Indeed, if $B < 0$, then the normalization factor (7) would not be $1/\sqrt{2\pi}$ but would become $N_\Gamma \approx [M_0 - \langle m \rangle]\Gamma^{-1}(v) \exp[\frac{(M_0 - Bv)^2}{2\Gamma^2(v)}]$ and, thus, it would cancel the leading term in pressure (14). Note, however, that the inequality $\langle \boldsymbol{m} \rangle \leqslant \mathbf{0}$ for all $v \gg V_0$ with positive B and finite $p^-(v \to \infty)$ is possible for $\Gamma(v) = \Gamma_1(v)$ only. In this case the pressure of an infinite bag is

$$p^-(v \to \infty) = -T\frac{B^2}{2\gamma^2}. \tag{15}$$

Also it is necessary to point out that the only width $\Gamma(v) = \Gamma_1(v)$ does not lead to any divergency in the bag pressure in thermodynamic limit. This is clearly seen from Eqs. (11) and (14) because the multiplier $\Gamma^2(v)$ stands in the numerator of the pressure (11), whereas in the pressure (14) it appears in the denominator. Thus, if one chooses the different v dependence for the width, then either p^+ or p^- would diverge for the bag of infinite size.

The new outcome of this case with $B > 0$ is that for $T < T_H$ the spectrum (13) contains the lightest QGP bags having the smallest volume because every term in the pressure (14) is negative. The finite volume of the system is no longer important because only the smallest bags survive in Eq. (13). Moreover, if such bags are created, they would have mass about M_0 and the width about $\Gamma_1(V_0)$, and, hence, they would not be distinguishable from the usual low-mass hadrons. Thus, the case $\langle \boldsymbol{m} \rangle \leqslant \mathbf{0}$ with $B > 0$ leads to the *subthreshold suppression of the QGP bags* at low temperatures, because their most probable mass is below the mass threshold M_0 of the spectrum $F_Q(s, T)$. Note that such an effect cannot be derived within any of the GBM-kind models proposed earlier. The negative values of $\langle m \rangle$ that appeared in the expressions above serve as an indicator of a different physical situation comparing to $\langle m \rangle > 0$ but have no physical meaning because $\langle m \rangle \leqslant 0$ does not enter the main physical observable p^-.

V. COMPARISON WITH LQCD RESULTS

The obtained results give us an instructive opportunity to make a bridge between the particle phenomenology, some experimental facts and the LQCD. For instance, if the most probable mass of the QGP bags is known along with the QGP pressure, one can estimate the width of these bags directly from Eqs. (12) and (14). To demonstrate the new possibilities let us now consider several examples of the QGP EOS and relate them to the above results. First, we study the possibility of getting the MIT bag model pressure $p_{\rm bag} \equiv \sigma T^4 - B_{\rm bag}$ [10] by the stable QGP bags, i.e., $\Gamma(v) \equiv 0$. Equating the pressures p^+ and $p_{\rm bag}$, one finds that the Hagedorn temperature is related to a bag constant $B_{\rm bag} \equiv \sigma T_H^4$. Then the mass density of such bags $\frac{\langle m \rangle}{v}$ is identical to

$$B = \sigma T_H (T + T_H)\left(T^2 + T_H^2\right), \tag{16}$$

and, hence, it is always positive. Thus, the MIT bag model EOS can be easily obtained within the FWM, but, as was discussed earlier, such bags should be observable.

Second, we consider the stable bags, $\Gamma(v) \equiv 0$, but without the Hagedorn spectrum, i.e., $T_H \to \infty$. Matching $p^+ = -B$ and $p_{\rm bag}$, we find that at low temperatures the bag mass density $\frac{\langle m \rangle}{v} = B$ is positive, whereas for high T the mass density cannot be positive and, hence, one cannot reproduce $p_{\rm bag}$ because in this case $B \leqslant 0$ and the resulting pressure is not p^- (14), but rather a zero, as seen from Eqs. (13) and (14) and the N_Γ expression for the limit $\Gamma(v) \to 0$.

One can try to reproduce $p_{\rm bag}$ with the finite T-dependent width $\Gamma(v) = 2\sigma T^5 v$ for $T_H \to \infty$. Then one can get $p_{\rm bag}$ from p^+, but only for low temperatures obeying the inequality $\frac{\langle m \rangle}{v} = B_{\rm bag} - 2\sigma T^4 > 0$. Thus, the last two examples show us that without the Hagedorn mass spectrum one can not get the MIT bag model pressure.

The FWM is a phenomenological model including two independent functions, B and γ, which parametrize the QGP bag pressure and require additional information as an input. However, the FWM provides us with some general results. Equating p^+ and $p^-(v \to \infty)$, one can find the transition width coefficient and pressure as

$$\gamma_\pm^2 = -\frac{B}{\beta}, \quad p^\pm = \frac{BT\beta}{2}, \tag{17}$$

and one can easily get that this transition, indeed, corresponds to $\langle m \rangle = 0$. Thus, although both expressions for pressure were obtained by different methods they match at the correct value of the most probable mass. Because $B > 0$ and $\gamma_\pm^2 > 0$ it follows that such a transition must occur at some temperature $T_\pm = c_\pm T_H$ that is below T_H, i.e., $0 < c_\pm < 1$.

Another general conclusion concerns the temperature dependence of the QGP pressure in the limit $T \to 0$. For nonvanishing $\gamma_0 \equiv \gamma(T = 0) > 0$ there are, however, two possibilities. The first one corresponds to finite $B_0 \equiv B(T = 0) > 0$ values. Then from Eq. (15) one concludes that in the limit $T \to 0$ the QGP pressure linearly depends on temperature $p^-(v \to \infty) \to -T\frac{B_0^2}{2\gamma_0^2}$. The second possibility corresponds to the divergent behavior of $B \to \frac{g_0}{T^D}$ (with $D > 0$) provided that $\frac{\langle m \rangle}{v} < 0$ for $v \to \infty$. The latter requires that $D \leqslant 1$ for finite γ_0. In this case at $T \to 0$ the QGP pressure should behave as $p^-(v \to \infty) \to -\frac{g_0^2}{2\gamma_0^2}T^{1-2D}$. Note that either of these possibilities is a manifestation of the nonperturbative effect because in the limit $\gamma = 0$ they cannot be obtained.

In Refs. [38–40] it was reported that the LQCD data exhibit the first of these possibilities. The corresponding EOS of the QGP has an additional linear temperature dependence

$$p_{\rm lin} = \sigma T^4 - A_1 T + A_0 \tag{18}$$

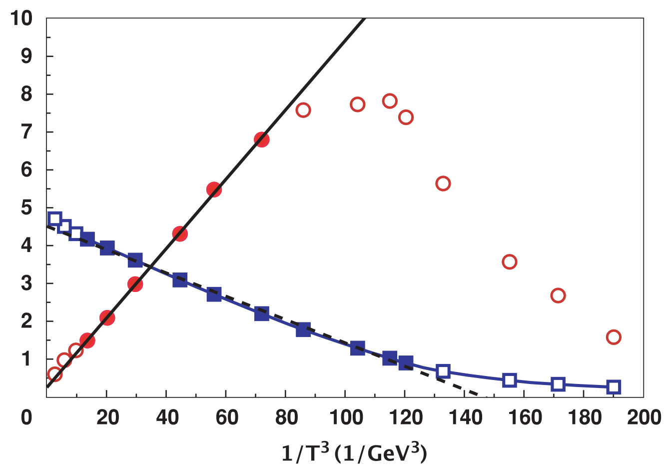

FIG. 2. (Color online) LQCD data for trace anomaly (circles) and pressure per T^4 (squares) as the functions of T^{-3}. Straight lines represent the fit of the filled symbols. See details in the text. The curve connecting the squares is to guide the eyes.

with $A_1 > 0$, $A_0 \geqslant 0$. However, the recent analysis [41] of new LQCD data [42] demonstrated not a linear, but the quadratic, T dependence of the trace anomaly and pressure in the range of temperatures between about $1.1T_c$ and $4T_c$. Therefore, to clarify the question of an additional T dependence of the LQCD pressure we analyzed the old LQCD data [43,44], in which the finite size effects are accounted for, and also we fitted the newest LQCD data that are found for the almost physical quark masses [45]. The fit of pT^{-4} from Ref. [45] as function of T^{-3} shown in Fig. 2 by dashed line clearly demonstrates the linear T^{-3} dependence $pT^{-4} = a_0 + a_1 T^{-3}$ with $a_0 \approx 4.5094$ and $a_1 \approx -0.0304$ GeV^3 for 10 data points in the range $T \in [202.5; 419.09]$ MeV.

The linear T dependence of pressure is rooted in the behavior of the trace anomaly $\delta = (\varepsilon - 3\,p)T^{-4}$ (here ε denotes the energy density). Indeed, plotting δ as the function of T^{-3} (see circles in Fig. 2) we found three different types of behavior. As one can see from Fig. 2 up to $T^{-3} \approx 72.056$ GeV^{-3} (for $T \geqslant 240.31$ MeV) the function δ grows nearly linearly, and for $T^{-3} \geqslant 120.43$ GeV^{-3} (or $T \leqslant 202.5$ MeV) it decreases nearly linearly, whereas between these values of T^{-3} the function δ remains almost constant. The analysis shows that six LQCD data points of the function δ that belong to the range $T^{-3} \in [13.585; 72.056]$ GeV^{-3} are, indeed, described by $\delta = \tilde{a}_0 + \tilde{a}_1 T^{-3}$ with $\tilde{a}_0 \approx 0.2514$ and $\tilde{a}_1 \approx 0.0916$ GeV^3 and $\chi^2/\mathrm{DOF} \approx 0.063$, i.e., with extremely high accuracy. The linear T^{-3} dependence of pT^{-4} is observed in a slightly wider range of T^{-3} because of the approximately constant behavior of the δ function at the moderate values of T^{-3}, but with lower quality fit that, however, is comparable with that one of Ref. [39].

The reason for lower quality of the pressure fit can be seen from its relation to the lattice trace anomaly

$$\frac{p_{\rm fit}}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} d\,T \frac{\delta}{T} = \tilde{a}_0 \ln\left[\frac{T}{T_0}\right] - \frac{\tilde{a}_1}{3}\left[T^{-3} - T_0^{-3}\right], \tag{19}$$

and, hence, one gets $a_1 = -\frac{\tilde{a}_1}{3}$, which is well supported by the LQCD data. The last equality in (19) is obtained from the linear fit of δ and, hence, T_0 and $p_0 \equiv p_{\rm fit}(T_0)$ are the constants of integration.

Equation (19) shows that for the temperatures between 240.31 and 419.09 MeV the LQCD pressure [45] does not have a constant term, i.e., $A_0 = 0$ for $p_{\rm lin}$, but there exist higher order corrections (T^5 and higher) to a pressure. They are small in this range of temperatures because $\tilde{a}_0 \ll 1$, but, in principle, can be taken into account to improve the quality of the linear fit of pT^{-4} function found in Ref. [39]. However, our main point is that either rough or refined analysis of the modern LQCD data strongly suggests an existence of the linear T-dependent term in the LQCD pressure for $T \in [240.31; 419.09]$ MeV.

Because neither the nonrelativistic hadron gas with the hard core repulsion represented by $F_H(s, T)$ in Eq. (2) nor its relativistic analog analyzed in Ref. [46] can generate the linear T dependence of pressure, it is possible that such a dependence is an inherent property of the LQCD data. Assuming this fact, one obtains that at low T the LQCD pressure of the QGP phase should behave as $p_{\rm LQCD}(T \to 0) \to -|a_1|T$. Comparing the linear T dependence of the LQCD pressure with the FWM pressure at low temperatures (15), we conclude that the present model with nonzero width coefficient correctly grasps the nonperturbative features of the QGP EOS and we consider it as one of the strongest arguments in favor of the FWM.

VI. WIDTH ESTIMATE

Such a behavior of the LQCD pressure allows us to roughly estimate the width $\Gamma_1(V_0)$ and to study the possible restrictions on thermodynamic functions. Because the FWM pressure depends on two functions, to find them it is necessary to know the form of the QGP pressure in the hadronic phase. Unfortunately the present LQCD data do not provide us with such detailed information and, hence, some additional assumptions are inevitable. Consider first the pressure (18) with $A_0 = 0$ [39] for nonvanishing B_0 and γ_0. T_H is uniquely fixed to be a positive solution of equation $A_0 = A_1 T_H - \sigma T_H^4$. Matching $p_{\rm lin}$ with $p^-(v \to \infty) = -T\frac{B^2}{2\gamma^2}$ for $T \leqslant c_\pm T_H$ we can determine B/γ ratio in this region of temperatures. However, equating $p_{\rm lin}$ and p^+, one obtains the width coefficient for $T \geqslant c_\pm T_H$

$$\gamma^2 = 2\beta^{-1}\left[\sigma T_H T\left(T^2 + T T_H + T_H^2\right) - B(T)\right]. \qquad (20)$$

To have a positive finite width for all $T \geqslant c_\pm T_H$, it is necessary that $(T - T_H)$ is a divisor of the difference staying in the square brackets of Eq. (20). Then the simplest possibility is to suppose that

$$B(T) = \sigma T_H^2\left(T^2 + T T_H + T_H^2\right) \qquad (21)$$

for any T. Evidently, $B(T)$ in Eq. (21) is positive and does not vanish at $T = 0$. In addition to a simplicity another advantage of such a choice is that Eq. (21) does not require any new constant or any new function that is not involved in Eq. (20). Moreover, comparing ansatz (21) with the mass density (16) obtained for the pure MIT bag model pressure, one can see that they differ only by a term $\sigma T_H T^3$ that at low $T \leqslant 0.5 T_H$ is a negligible correction to Eq. (21). Therefore, for low temperatures the ansatz (21) looks quite reasonable because in this region it corresponds to the mass density of the most popular EOS of modern QCD phenomenology.

TABLE I. The values of the resonance width for different models. Model A corresponds to the pure gluodynamics for the SU(2)$_C$ color group [43]. Model B describes the SU(3)$_C$ color group LQCD data with two quark flavors [44] and Model C corresponds to the LQCD of SU(3)$_C$ color group with three quark flavors [45].

Model Ref.	T_c (MeV)	$\Gamma_R(V_0, 0)$ (MeV)	$\Gamma_R(V_0, T_H)$ (MeV)
A	170	410	1420
A	200	616	2133
B	170	391	1355
B	200	587	2034
C	196	596	2066

It follows from Eq. (21) that $\gamma_0^2 = B_0^2/(2A_1) = T_H B_0/2 = \sigma T_H^5/2$ for $T = 0$ and that $\gamma^2 = 2TB(T)$ for $T \geqslant c_\pm T_H$. As an example, let us consider the true width for the SU(3) color group with two flavors analyzed in Ref. [39] (model B in Table I) for $T_c = 200$ MeV. Because for $A_0 = 0$ it is found $A_1 \approx (1.5T_c)^3$ and on the other hand the FWM requires $A_1 = \sigma T_H^3$, then one obtains $T_H \approx 0.94 T_c$ for $\sigma = \frac{37}{90}\pi^2$. Thus, the true width for the SU(3) color group with two flavors is $\Gamma_R(V_0, T = 0) \approx 1.22 V_0^{\frac{1}{2}} T_c^{\frac{5}{2}} \alpha \approx 587$ MeV and $\Gamma_R(V_0, T = T_H) = \sqrt{12}\,\Gamma_R(V_0, T = 0) \approx 2034$ MeV. These estimates clearly demonstrate that there is no way to detect the decays of such short living QGP bags, even if they are allowed by the subthreshold suppression. The sensitivity of these results to T_c value for models A and B is given in Table I, which supports our main conclusion for the short lifetime of the QGP bags.

The model C in Table I corresponds to $p_{\rm lin}$ pressure, but with $A_0 = 0$ and $A_1 = \frac{\tilde{a}_1}{3}$ obtained from the fit of the function δ. As one can see from Table I the minimal width of the QGP bags found for the same value of the transition temperature T_c practically does not depend on the number of the QGP degrees of freedom. Such a property is an additional argument in favor of the ansatz (21).

Now we would like to study the sensitivity of the width estimates to the choice of the LQCD pressure. For this purpose we will study the model described by Eq. (19). Let us assume that the pressure of the QGP below $T = 419.09$ MeV is given by Eq. (19) that we found for temperatures between 240.31 and 419.09 MeV. Choosing T_0 to be T_H, we obtain

$$p_{\rm QGP} = \tilde{a}_0 T^4 \ln\left[\frac{T}{T_H}\right] - \frac{\tilde{a}_1}{3}T + \frac{\tilde{a}_1 T^4}{3T_H^3}, \qquad (22)$$

because the FWM pressure p^+ (11) must vanish at $T = T_H$. Because the coefficient $\tilde{a}_0 \approx 0.2514 \ll 1$ is much smaller than the Stefan-Boltzmann constant over three $\sigma = \frac{95}{180}\pi^2 \approx 5.2 \gg 1$ for the SU(3)$_C$ LQCD with three flavors, then the logarithmic term in Eq. (22) remains a small correction for all temperatures below a few tens of T_H value. Furthermore, the logarithmic term in Eq. (22) cannot describe an asymptotic behavior of the LQCD pressure at large T. Thus, there is only a single possibility to match Eq. (22) with the LQCD data, namely to identify the last term in the right-hand side of Eq. (22) with the Boltzmann limit of the LQCD pressure at $T \gg T_H$. This condition fixes the value of T_H:

$$\frac{\tilde{a}_1}{3\,T_H^3} = \sigma \equiv \frac{95}{180}\pi^2 \Rightarrow T_H = \left[\frac{\tilde{a}_1}{3\sigma}\right]^{\frac{1}{3}} \approx 180 \text{ MeV}. \quad (23)$$

Matching p^+ and $p_{\rm QGP}$ (22) and expanding the logarithmic function at $T = T_H$, one can find the width coefficient for $T \geqslant c_\pm T_H$ as

$$\gamma_+^2 = 2\beta^{-1}\left[\sigma T_H T\left(T^2 + T T_H + T_H^2\right) - B(T) + \tilde{a}_0 T^4 \sum_{k=0} \frac{(-1)^k}{k+1}\left(\frac{T-T_H}{T_H}\right)^k\right]. \quad (24)$$

Note that in evaluating the pressure (22) the coefficient $\tilde{a}_1$ was written to take into account Eq. (23).

As in a previous case, it is necessary that $(T - T_H)$ is a divisor of the difference staying in the square brackets in Eq. (24). Again here we consider the simplest generalization of Eq. (21) that satisfies the necessary condition for $T \geqslant c_\pm T_H$:

$$B(T) = \sigma T_H^2\left(T^2 + T T_H + T_H^2\right) + \tilde{a}_0 T^{4-l} T_H^l, \quad (25)$$

where power l can be 0, 1, 2, 3, or 4. Note that for $l = 1$ one obtains exactly the same T dependence as for the mass density (16) of the MIT bag model pressure and, hence, $l = 1$ is of a special interest.

Substituting Eq. (25) into Eq. (24) one finds

$$\gamma_+^2 = 2T T_H\left[\sigma T_H\left(T^2 + T T_H + T_H^2\right) + \tilde{a}_0 T^{4-l}\frac{\left[T^l - T_H^l\right]}{T - T_H} - \frac{\tilde{a}_0 T^4}{T_H}\sum_{k=0}\frac{(-1)^k}{k+2}\left(\frac{T-T_H}{T_H}\right)^k\right]. \quad (26)$$

Assume now that the expressions for the pressure (22) and the mass density of bags (25) are valid for $T < c_\pm T_H$ as well. These assumptions allow us to find the width coefficient in the region $T < c_\pm T_H$

$$\gamma_-^2 = \frac{B(T)^2}{2\left[\sigma\left(T_H^3 - T^3\right) + \tilde{a}_0 T^3 \ln[T_H/T]\right]}. \quad (27)$$

Taking the limit $T \to 0$ in Eq. (27) one finds the width coefficient at zero temperature as

$$\gamma_-^2(T = 0) = \frac{[\sigma + \tilde{a}_0\delta_{l,4}]^2}{2\sigma} T_H^5, \quad (28)$$

where $\delta_{l,k}$ denotes the Kronecker symbol. The last result shows that the logarithmic term in the pressure (22) modifies our previous estimates for the width coefficient at $T = 0$ by about 10% for $l = 4$ only, whereas for $l \leqslant 3$ and, hence, for $l = 1$, the resonance width coefficient at $T = 0$ remains unchanged. The corrections of the same order of magnitude are generated by $B(T)$ (25) at $T = T_H$:

$$\gamma_+^2(T = T_H) = 2\,[3\sigma - l\tilde{a}_0]\,T_H^5. \quad (29)$$

Thus, the resonance width values given in Table I remain almost the same for the pressure behavior as in Eq. (22).

VII. ASYMPTOTIC BEHAVIOR OF THE REGGE TRAJECTORIES

The behavior of the width of hadronic resonances was extensively studied almost 40 years ago in the Regge poles method, dictated by an intensive analysis of the strongly interaction dynamics in high-energy hadronic collisions. A lot of effort was put forward [25,47] to elucidate the asymptotical behavior of the resonance trajectories $\alpha(S)$ for $|S| \to \infty$ (S is an invariant mass square in the reaction). Because the Regge trajectory determines not only the mass of resonances but also their width, it would be interesting to compare these results with the FWM predictions. Note that nowadays there is great interest in the behavior of the Regge trajectories of higher resonances in the context of the five-dimensional string theory holographically dual to QCD [48] that is known as anti-de-Sitter space/conformal field theory (AdS/CFT).

In our research we follow Ref. [25] that is based on the following most general assumptions: (I) $\alpha(S)$ is an analytical function, having only the physical cut from $S = S_0$ to $S = \infty$; (II) $\alpha(S)$ is polynomially restricted at the whole physical sheet; (III) there exists a finite limit of the phase trajectory at $S \to \infty$. Using these assumptions, it was possible to prove [25] that for $S \to \infty$ the upper bound of the Regge trajectory asymptotics at the whole physical sheet is

$$\alpha_u(S) = -g_u^2\,[-S]^\nu\,, \quad \text{with} \quad \nu \leqslant 1, \quad (30)$$

where the function $g_u^2 > 0$ should increase slower than any power in this limit and its phase must vanish at $|S| \to \infty$.

However, in Ref. [25] it was also shown that, if in addition to (I)–(III) one requires that the transition amplitude $T(s, t)$ is a polynomially restricted function of S for all nonphysical $t > t_0 > 0$, then the real part of the Regge tragectory does not increase at $|S| \to \infty$ and the trajectory behaves as

$$\alpha_l(S) = g_l^2[-[-S]^{\frac{1}{2}} + C_l], \quad (31)$$

where $g_l^2 > 0$ and C_l are some constants. Moreover, Eq. (31) defines the lower bound for the asymptotic behavior of the Regge trajectory [25]. The expression (31) is a generalization of a well-known Khuri result [49]. It means that for each family of hadronic resonances the Regge poles do not go beyond some vertical line in the complex spin plane. In other words, it means that in asymptotics $S \to +\infty$ the resonances become infinitely wide, i.e., they are moving out of the real axis of the proper angular momentum J and, therefore, there are only a finite number of resonances in the corresponding transition amplitude. At first glance it seems that the huge deficit of heavy hadronic resonances compared to the Hagedorn mass spectrum (the second conceptual problem of Sec. II) supports such a conclusion. Because there are a finite number of resonance families [50] it is impossible to generate from them an exponential mass spectrum and, hence, the Hagedorn mass spectrum cannot exist for large resonance masses. Consequently, the GBM and its followers run into deep trouble. We, however, believe that the FWM with negative value of the most probable bag mass $\langle m\rangle \leqslant 0$ can help to resolve this problem as well.

First we note that the direct comparison of the FWM predictions with the Regge poles asymptotics is impossible because the resonance mass and its width $\Gamma(v)$ are independent variables in the FWM. Nevertheless, we can relate their average values and compare them to the results of Ref. [25].

To illustrate this statement, we recall our result on the mean Gaussian width of the free bags averaged with respect to their volume by the spectrum (8) [see two paragraphs after Eq. (8) for details]

$$\overline{\Gamma_1(v)} \approx \Gamma_1(m/B) = \gamma\sqrt{\frac{m}{B}}. \quad (32)$$

Using the formalism of Ref. [25], it can be shown that at zero temperature the free QGP bags of mass m and mean resonance width $\alpha\overline{\Gamma_1(v)}|_{T=0} \approx \alpha\,\gamma_0\sqrt{\frac{m}{B_0}}$ precisely correspond to the following Regge trajectory

$$\alpha_r(S) = g_r^2[S + a_r(-S)^{\frac{3}{4}}] \quad \text{with} \quad a_r = \text{const} < 0. \quad (33)$$

Indeed, substituting $S = |S|e^{i\phi_r}$ into Eq. (33), then expanding the second term on the right hand side of Eq. (33) and requiring $\text{Im}[\alpha_r(S)] = 0$, one finds the phase of physical trajectory [one of four roots of one fourth power in Eq. (33)]

$$\phi_r(S) \to \frac{a_r \sin\frac{3}{4}\pi}{|S|^{\frac{1}{4}}} \to 0^-, \quad (34)$$

which is vanishing in the correct quadrant of the complex S plane. Considering the complex energy plane $E = \sqrt{S} \equiv M_r - i\frac{\Gamma_r}{2}$, one can determine the mass M_r and the width Γ_r

$$M_r \approx |S|^{\frac{1}{2}} \quad \text{and}$$
$$\Gamma_r \approx -|S|^{\frac{1}{2}}\phi_r(S) = \frac{|a_r||S|^{\frac{1}{4}}}{\sqrt{2}} = \frac{|a_r|M_r^{\frac{1}{2}}}{\sqrt{2}}, \quad (35)$$

of a resonance belonging to the trajectory (33).

Comparing the mass dependence of the width in Eq. (35) with the mean width of free QGP bags (32) taken at $T = 0$, it is natural to identify them,

$$a_r^{\text{free}} \approx -\alpha\,\gamma_0\sqrt{\frac{2}{B_0}} = -4\gamma_0\sqrt{\frac{\ln 2}{B_0}}, \quad (36)$$

and to deduce that the free QGP bags belong to the Regge trajectory (33). Such a conclusion is in line both with the well-established results on the linear Regge trajectories of hadronic resonances [50] and with theoretical expectations of the dual resonance model [18], the open string model [1,36], the closed string model [1], and the AdS/CFT [48]. Such a property of the FWM also gives a very strong argument in favor of both the volume-dependent width $\Gamma(v) = \Gamma_1(v)$ and the corresponding mass-volume spectrum of heavy bags (5).

Next we consider the second way of averaging the mass-volume spectrum with respect to the resonance mass

$$\overline{m}(v) \equiv \frac{\int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\,\rho(m,v)\,m\,e^{-\frac{\sqrt{k^2+m^2}}{T}}}{\int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\,\rho(m,v)\,e^{-\frac{\sqrt{k^2+m^2}}{T}}}, \quad (37)$$

which is technically simpler than averaging with respect to the resonance volume, but we will make the necessary comments on the other way of averaging in the appropriate places.

Using the results of Sec. IV one can find the mean mass (37) for $T \geqslant c_{\pm}T_H$ (or for $\langle m\rangle \geqslant 0$) to be equal to the most probable mass of bag from which one determines the resonance width:

$$\overline{m}(v) \approx \langle m\rangle \quad (38)$$

and

$$\Gamma_R(v) \approx 2\sqrt{2\ln 2}\,\Gamma_1\left[\frac{\langle m\rangle}{B+\gamma^2\beta}\right] = 2\,\gamma\sqrt{\frac{2\ln 2\,\langle m\rangle}{B+\gamma^2\beta}}. \quad (39)$$

The last two equations lead to a vanishing ratio $\frac{\Gamma_R}{\langle m\rangle} \sim \langle m\rangle^{-\frac{1}{2}}$ in the limit $\langle m\rangle \to \infty$. Comparing Eqs. (38) and (39) with the mass and width (35) of the Regge trajectory (33) and applying absolutely the same logic we used for the free QGP bags, we conclude that the location of the FWM heavy bags in the complex energy plane is identical to that of resonances belonging to the trajectory (33) with

$$\langle m\rangle \approx |S|^{\frac{1}{2}} \quad \text{and} \quad a_r \approx -4\gamma\sqrt{\frac{\ln 2}{B+\gamma^2\beta}}. \quad (40)$$

The most remarkable output of such a conclusion is that the medium-dependent FWM mass and width of the extended QGP bags obey the upper bound for the Regge trajectory asymptotic behavior obtained for pointlike hadrons [25]!

It is also interesting that the resonance width formula (39) is generated by the most probable volume

$$v_E(m) \approx \frac{m}{\sqrt{B^2 + 2\gamma^2 s^*}} = \frac{m}{B+\gamma^2\beta} \quad (41)$$

of heavy resonances of mass $m \gg M_0$ that are described by the continuous spectrum $F_Q(s,T)$ (2). This result can be easily found by maximizing the exponential in $F_Q(s,T)$ with respect to resonance volume v at fixed mass m and by recalling that at high temperatures the rightmost singularity of the isobaric partition (1) is defined by the pressure (11) as $s^* = \frac{p^+}{T}$.

The extracted values of the resonance width coefficient along with the relation (21) for $B(T)$ allow us to estimate a_r as

$$a_r \approx -4\sqrt{\frac{2TT_H}{2T - T_H}\ln 2}. \quad (42)$$

This expression shows that for $T \to T_H/2 + 0$ the asymptotic behavior (33) breaks down because the resonance width diverges at fixed $|S|$. However, from Eq. (42) it follows that $a_r^2(T = T_H) \approx 22.18T_H$ and $a_r^2(T \gg T_H) \approx 11.09T_H$. In other words, for a typical value of the Hagedorn temperature $T_H \approx 190$ MeV [see a discussion after Eq. (19)] Eq. (42) gives a reasonable range of the invariant mass $|S|^{\frac{1}{2}} \gg a_r^2(T = T_H) \approx 4.21$ GeV and $|S|^{\frac{1}{2}} \gg a_r^2(T \gg T_H) \approx 2.1$ GeV for which Eq. (33) is true.

Now we can find the spin of the FWM resonances

$$J = \text{Re}\,\alpha_r(\langle m\rangle^2) \approx g_r^2\,\langle m\rangle\left[\langle m\rangle - \frac{a_r^2}{4}\right], \quad (43)$$

which has a typical Regge behavior up to a small correction. Such a property can also be obtained within the dual resonance model [18], within the models of open [1] and closed string

[1,36], and within AdS/CFT [48]. These models support our result (43) and justify it. Note, however, that in addition to the spin value the FWM determines the width of hadronic resonances. The latter allows us to predict the ratio of widths of two resonances having spins J_2 and J_1 and appearing at the same temperature T to be as follows

$$\frac{\Gamma_R\left[\frac{\langle m\rangle|_{J_2}}{(B+\gamma^2\beta)}\right]}{\Gamma_R\left[\frac{\langle m\rangle|_{J_1}}{(B+\gamma^2\beta)}\right]} \approx \frac{\sqrt{v|_{J_2}}}{\sqrt{v|_{J_1}}} \approx \frac{\sqrt{\langle m\rangle|_{J_2}}}{\sqrt{\langle m\rangle|_{J_1}}} \approx \left[\frac{J_2}{J_1}\right]^{\frac{1}{4}}, \quad (44)$$

which, perhaps, can be tested at the Large Hadron Collider (LHC).

Now we turn to the analysis of the low temperature regime, i.e., to $T \leqslant c_{\pm}T_H$. Using previously obtained results from (37) one finds

$$\overline{m}(v) \approx M_0, \quad (45)$$

i.e., the mean mass is volume independent. Taking the limit $v \to \infty$, we get the ratio $\frac{\Gamma(v)}{\overline{m}(v)} \to \infty$ that closely resembles the case of the lower bound of the Regge trajectory asymptotics (31). Similarly to the analysis of high temperature regime, from Eq. (31) one can find the trajectory phase and then the resonance mass M_r and its width Γ_r

$$\phi_r(S) \to -\pi + \frac{2|C_l||\sin(\arg C_l)|}{|S|^{\frac{1}{2}}}, \quad (46)$$

$$M_r \approx |C_l||\sin(\arg C_l)| \quad \text{and} \quad \Gamma_r \approx 2|S|^{\frac{1}{2}}. \quad (47)$$

Again comparing the averaged masses and width of FWM resonances with their counterparts in Eq. (47), we find similar behavior in the limit of large width of resonances. Therefore, we conclude that at low temperatures the FWM obeys the lower bound of the Regge trajectory asymptotics of Ref. [25].

The other way of averaging, i.e., with respect to the resonance volume, in the leading order gives the most probable resonance volume of the continuous spectrum $F_Q(s,T)$ (2) defined by the left equation (41) again. Substituting in it the corresponding rightmost singularity $s^* = \frac{p^-}{T}$ and using (15) for p^-, one finds $v_E(m) \to \infty$ that leads to an infinite value of the most probable resonance width defined in this way. Note that such a result is supported by the high temperature mean width behavior if $T \to T_H/2 + 0$. As one can see from Eqs. (42) and (35), in the latter case the trajectory (33) also demonstrates a very large width compared to a finite resonance mass.

A more refined analysis of the most probable volume (41) shows that the second derivative of the exponential in $F_Q(s,T)$ with respect to the resonance volume vanishes at large resonance masses m and, hence, one needs to account for even weaker dependences on resonance volume v and to inspect higher-order derivatives with respect to v, but this task is out of the scope of present work and we leave it for future investigation.

Thus, these estimates demonstrate that at any temperature the FWM QGP bags can be regarded as the medium induced Reggeons that at $T \leqslant c_{\pm}T_H$ (i.e., for $\langle m\rangle \leqslant 0$) belong to the Regge trajectory (31) and otherwise they are described by the trajectory (33). Of course, both of the trajectories (31) and (33) are valid in the asymptotic $|S| \to \infty$, but the most remarkable fact is that, to our knowledge, the FWM gives us the first example of a model that reproduces both of these trajectories and, thus, obeys both bounds of the Regge asymptotics. Moreover, because the FWM contains the Hagedorn-like mass spectrum at any temperature, the subthreshold suppression of QGP bags removes the contradiction between the Hagedorn ideas on the exponential mass spectrum of hadrons and the Regge poles method in the low-temperature domain! Furthermore, the FWM opens the possibility of applying the Regge poles method to a variety of processes in a strongly interacting matter and account, at least partly, for some of the medium effects.

VIII. CONCLUSIONS AND PERSPECTIVES

Here we present the novel statistical approach to study the QGP bags with medium dependent width. We argue that the volume-dependent width of the QGP bags $\Gamma(v) = \gamma v^{\frac{1}{2}}$ leads to the Hagedorn mass spectrum of heavy bags. Such behavior of a width allows us to explain a huge deficit of heavy hadronic resonances in the experimental mass spectrum. The key point of our treatment is the presence of Gaussian attenuation of bag mass. Perhaps the nonlocal field theoretical models may shed light on the origin of the Gaussian mass attenuation.

Under plausible assumptions we derive the general expression for the bag pressure p^+ that accounts for the effect of finite width in the EOS. We argue that the obtained spectrum itself cannot explain the absence of directly observable QGP bags and strangelets in the high-energy nuclear and elementary particle collisions. Then we demonstrate the possibility to "hide" the heavy QGP bags for $T \leqslant c_{\pm}T_H$ by their *subthreshold suppression*. The latter occurs due to the fact that at low temperatures the most probable mass of heavy bags $\langle m\rangle$ becomes negative and, hence, is below the lower cut-off M_0 of the continuous mass spectrum. Consequently, only the lightest bags of mass about M_0 and of smallest volume V_0 may contribute into the resulting spectrum, but such QGP bags will be indistinguishable from the low-lying hadronic resonances with the short lifetime. We show how the FWD can reproduce a few EOS of the QGP and discuss the corresponding restrictions.

We analyze the recent LQCD data for the trace anomaly and extract the linear T-dependent term for the LQCD pressure and higher-order corrections to it. This linear T-dependent term in the LQCD pressure is naturally associated with the FWM pressure at low temperatures p^-. Using such a dependence we estimate the volume-dependent width under plausible assumptions and find it almost insensitive to the number of color and flavor degrees of freedom of the LQCD data. These estimates clearly demonstrate that such short living QGP bags cannot be established experimentally. We believe that our finding introduces the new time scale into the high-energy nuclear and elementary particles collisions and requires some modifications of the present picture of the collision process and its subsequent stages.

With the help of formalism of Ref. [25] we show that the average mass and width of heavy or large free QGP bags belong to the linear Regge trajectory (33). Similarly, we find that at hight temperatures the average mass and width of the QGP bags behave in accordance with the upper bound of the Regge trajectory asymptotics (30) (linear trajectory), whereas at low temperatures they obey the lower bound of the Regge trajectory asymptotics (31) (square root trajectory). Because the model explicitly contains the Hagedorn spectrum, it removes an existing contradiction between the finite number of hadronic Regge families and the Hagedorn idea of the exponentially growing mass spectrum of hadronic bags. Such a result creates a new look onto the large and/or heavy QGP bags as the medium-induced Reggeons and opens a principal possibility to apply all the strength of the Regge poles method to a variety of processes in a strongly interacting media and to account, at least partly, for some of the medium effects.

In addition to these general results the FWM allows us to make some predictions that can be soon tested experimentally. Thus, the relation between the maximal spin of the bag and the most probable mass (43) or the dependence between the maximal spin and the mean volume (44) can be, perhaps, tested at LHC CERN during the hadronic collision runs. It is also probable that the switch between the Regge trajectories (31) and (33) can be verified at the Facility for Antiproton and Ion Research (at GSI) and the Nuclotron-based Ion Collider Facility (at JINR) energy range. It is clear that the lower bound trajectory (31) cannot be established in a laboratory, but it seems reasonable to expect that the experimenters will be able to find how the parameters of the upper bound trajectory (33) are changing with the colliding energy. The found parameters of bag trajectories in combination with the Hanbury Brown-Twiss analysis of their volume may help us to determine the resonance width coefficient γ or even the volume dependence of the resonance width itself.

The generalization of all of the above results to nonzero baryonic densities can be made straightforwardly by assuming the dependence of the model functions B and $\Gamma_1(v)$ on the baryonic chemical potential. The more detailed quantitative experimental consequences of the FWM will be published elsewhere [51].

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