Multiplicity fluctuations due to the temperature fluctuations in high-energy nuclear collisions

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We investigate the multiplicity fluctuations observed in high-energy nuclear collisions attributing them to intrinsic fluctuations of temperature of the hadronizing system formed in such processes. To account for these fluctuations, we replace the usual Boltzmann-Gibbs (BG) statistics by the nonextensive Tsallis statistics characterized by the nonextensivity parameter q, with $|q-1|$ being a direct measure of fluctuation. In the limit of vanishing fluctuations, $q \to 1$ and Tsallis statistics converge to the usual BG. We evaluate the nonextensivity parameter q and its dependence on the hadronizing system size from the experimentally observed collision centrality dependence of the mean multiplicity $\langle N\rangle$ and its variance $\mathrm{Var}(N)$. We attribute the observed system size dependence of q to the finiteness of the hadronizing source, with $q=1$ corresponding to an infinite, thermalized source with a fixed temperature, and with $q>1$ (which is observed) corresponding to a finite source in which both the temperature and energy fluctuate.

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I. INTRODUCTION

With the large number of particles produced in heavy ion collisions at the CERN Super Proton Synchrotron (SPS) and BNL Relativistic Heavy Ion Collider (RHIC), it is possible to study fluctuations in different physical observables on the event-by-event basis [1]. These fluctuations are potentially a very important source of information on the thermodynamic properties of strongly interacting systems formed in such collisions, such as its specific heat [2] (connected with fluctuations observed in particle multiplicities [3,4], in transverse momenta [5], and in other global observables), its chemical potential, or matter compressibility [6].

Fluctuations of multiplicity observed in heavy ion collisions exhibit spectacular and unexpected features as functions of the number of participants. Recent results on centrality dependence in Pb + Pb collisions at 158A GeV obtained by NA49 [3] and WA98 [4] experiments indicate that the scaled variance of the multiplicity distribution, $\mathrm{Var}(N)/\langle N\rangle$, increases when proceeding from the central toward peripheral collisions, i.e., when the number of participants decreases. Such behavior is confirmed by a comprehensive survey of multiplicity fluctuations of charged hadrons provided by the PHENIX experiment [7].

During the last decade, the statistical models of strong interactions were constantly used as an important tool for studying the fluctuation pattern observed in high-energy nuclear collisions experiments, which are quantified by the scaled variance mentioned above (see, for example, Refs. [8–10] and references therein). However, to minimize the effect of the possible participant number fluctuations, the analysis of particle number fluctuations has been restricted to the most central $A+A$ collisions only.

However, up to now, none of the models aimed at describing all the essential features of multiparticle production processes (such as multiplicities and distributions of particles in phase space and their composition) describe also the dependence of $\mathrm{Var}(N)/\langle N\rangle$ on N_P, the number of nucleons from the projectile nucleus participating in the collision (i.e., the projectile participant), observed experimentally [3]. They lead to multiplicity distributions of the (approximately) Poissonian form, independent of centrality, which highly underestimate the observed multiplicity fluctuations in noncentral collisions and thus are unable to reproduce, even qualitatively, the observed centrality dependence of the scaled variance. This remark applies not only to the Monte Carlo models such as HIJING [11], HSD [12], or UrQMD [13], which are based on string excitation and decay, but also to any statistical model that does not assume correlations among secondary particles (cf. discussion in Sec. III C). On the other hand, these results can be described by some specialized models addressing fluctuations directly, such as the percolation model [14], the model assuming interparticle correlations caused by the combination of strong and electromagnetic interactions [15], or the transparency, mixing, and reflection model [16]. Fluctuations in these models reflect some dynamical features of the production process (specific for the model considered).

In this paper, we address the problem of multiplicity fluctuations without resorting to any specific dynamical picture but, instead, by attributing them to some nonstatistical, intrinsic fluctuations existing in a hadronizing system produced in high-energy heavy ion collisions. To account for such fluctuations, we use a special version of statistical model based on nonextensive Tsallis statistics [17] in which fluctuations of the temperature are known to be directly connected with the nonextensivity parameter q [18,19], namely, $q = 1 + \mathrm{Var}(1/T)/\langle 1/T\rangle^2$. In what follows, they are assumed to be the true (if not the only) origin of the fluctuations observed in the experimental data. The resulting distributions are then power-like, $\exp(-E/T) \Longrightarrow \exp_q(-E/T) = [1-(1-q)E/T]^{1/(1-q)}$, and in the limit of $q \to 1$ they go smoothly to

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the usual Boltzmann distributions (in what follows, we address only the $q \geqslant 1$ case).[1] It is important to our further discussion that such nonexponential distributions of energy result in non-Poissonian multiplicity distributions of the produced secondaries [24].

However, to incorporate the new features of data reported in Ref. [25], we have to extend the notion of fluctuating temperature, replacing it by some q-dependent *effective temperature*, $T_{\rm eff}$, which accounts not only for the intrinsic fluctuations in the hadronizing source (as in Ref. [18]) but also for effects of the possible energy transfer taking place between the hadronizing source and its surroundings [26]. This will be done in Sec. II. Section III contains our results: the universal participant dependence (i.e., scaling in the variable $f = N_P/A$) is presented in Sec. III A; its explanation by the observation that in reality the hadronizing source is always of finite size is presented in Sec. III B; and the system size dependence of multiplicity and multiplicity fluctuations is presented in Sec. III C. Section IV summarizes our work. Some details are given in Appendixes A and B.

II. EFFECTIVE TEMPERATURE

In the proposed approach, we replace the standard Boltzmann-Gibbs exponential distribution,

$$g(E) = C \exp(-E/T), \tag{1}$$

with the Tsallis distribution (q-exponential) defined as

$$h_q(E) = C_q \left[1 - (1-q)\frac{E}{T_{\rm eff}}\right]^{\frac{1}{1-q}}, \tag{2}$$

where

$$q = 1 + \frac{{\rm Var}(T)}{\langle T\rangle^2}, \tag{3}$$

and

$$T_{\rm eff} = T_0 + (q-1)\frac{1}{Dc_p\rho}\phi, \tag{4}$$

with c_p, ρ, and D being, respectively, the specific heat under constant pressure, density, and the strength of the temperature fluctuations (cf. Appendix A for details). Effective temperature $T_{\rm eff}$ occurs when we experience both the fluctuations of the temperature T (around the value T_0) and some energy transfer taking place between the source and the surroundings given by ϕ.[2] Notice that in Eq. (4), energy transfer affects $T_{\rm eff}$ only in the presence of fluctuations, i.e., for $q \neq 1$. The predicted

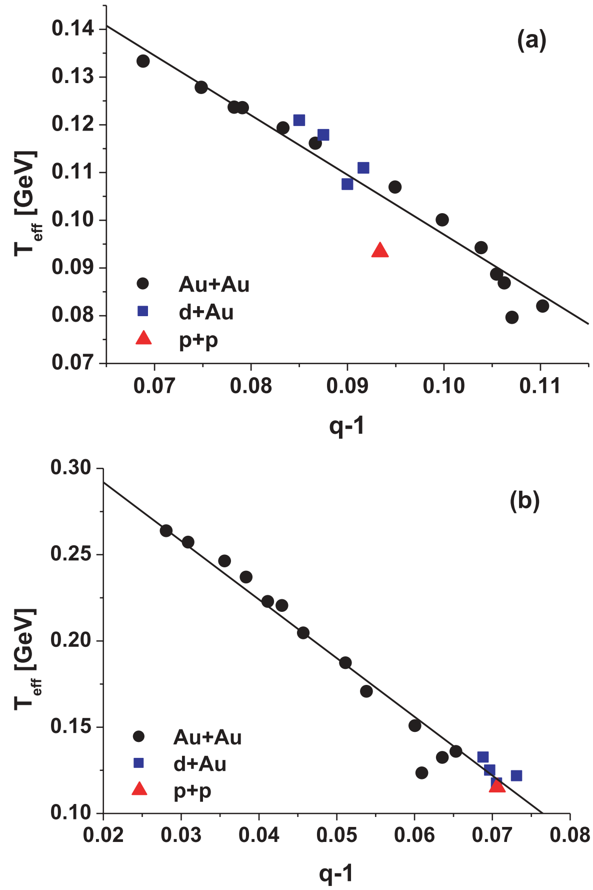

FIG. 1. (Color online) Dependence of the effective temperature $T_{\rm eff}$ (in GeV) on the parameter q for the production in different reactions of (a) negative pions and (b) antiprotons; all data points are from Ref. [25]. The solid lines show linear fits to the obtained results: (a) $T_{\rm eff} = 0.22 - 1.25(q-1)$; (b) $T_{\rm eff} = 0.36 - 3.4(q-1)$.

q dependence of $T_{\rm eff}$ is indeed observed experimentally, cf. Fig. 1. Namely, in Ref. [25] the transverse momentum spectra of pions and antiprotons produced in the interactions of $p+p$, $d+Au$ and Au + Au at $\sqrt{s_{NN}} = 200$ GeV at RHIC experiments [28] were analyzed using a nonextensive approach in which the slope of the p_T distribution determines the effective temperature $T_{\rm eff}$. Its shape gives the parameter of nonextensivity q.[3] They evaluated, among other things, the nonextensivity parameter q and the effective temperature $T_{\rm eff}$ for a different number of participants N_P. From their results, we can deduce the dependence of $T_{\rm eff}$ on the parameter q,

[1]In a sense, our work is a continuation of many previous works [20–23] showing that to adequately describe experimental data on energy and transverse momenta distributions, the usual statistical approach based on Boltzmann distributions should be modified and replaced by a generalized nonextensive formalism based on Tsallis statistics [17].

[2]Actually, in the work [26] accounting for the effect of viscosity, the source term in Eq. (4) takes the form $\phi = \eta f(u)$, where η denotes the coefficient of viscosity and function $f(u)$ contains terms dependent on the velocity u in the form of $\partial u_i/\partial x_k + \partial u_k/\partial x_i$ (as in Ref. [27]). Therefore the effective temperature there was given by $T_{\rm eff} = T_0 + (q-1)\eta f(u)/(Dc_p\rho)$.

[3]Note that in Ref. [25] the p_T spectra were analyzed in terms of the so-called escort probability distributions [29], i.e., $h_Q(E) = C_Q\,[1-(1-Q)E/\Lambda]^{Q/(1-Q)}$. However, this distribution function is, in fact, formally identical with $h_q(E) = C_q\,[1-(1-q)E/\lambda]^{1/(1-q)}$, which we are using in this work, provided that we identify: $q = 1(Q-1)/Q, \lambda = \Lambda/Q$ and $C_q = C_Q = (2-q)/\lambda = 1/\Lambda$. The mean value is now $\langle E\rangle = \lambda/(3-2q) = \Lambda/(2-Q)$ and is defined for $q \in (0, 1.5)$ [to be compared with the previous $Q \in (1/2, 2)$].

which is shown in Fig. 1. Notice that in all cases, we find that $\phi < 0$ and that $T_{\rm eff}$ seems to depend linearly on $(q-1)$. Negative values of ϕ mean that the energy is transferred from the interaction region to the surroundings (i.e., to the spectators of noninteracting nucleons).[4]

III. DIFFERENT FACETS OF MULTIPLICITY FLUCTUATIONS

A. Universal participant dependence

We start a discussion of multiplicity fluctuations by recollecting the result of Ref. [24] saying that if N particles are distributed in energy according to the N-particle Tsallis distribution described by the nonextensive parameter q, then their multiplicity distribution *has to be* of the negative-binomial type with $k^{-1} = q - 1$ (see Appendix B for details). It means therefore that we can expect that

$$\frac{\frac{\mathrm{Var}(N)}{\langle N\rangle} - 1}{\langle N\rangle} = q - 1 \tag{5}$$

(and, what is important, that $q-1$ determined this way does not depend on the acceptance of the detector).[5] On the other hand, if U is the accessible energy and $gT_{\rm eff}$ is the mean energy per particle detected in the acceptance region [with g being a parameter and $T_{\rm eff}$ effective temperature defined in Eq. (4)], then

$$\langle N\rangle = \frac{\langle U\rangle}{gT_{\rm eff}}. \tag{6}$$

Using Eq. (4), one can shown that

$$\frac{\langle N\rangle - n_0 N_P}{\langle N\rangle} = c(q-1), \tag{7}$$

where n_0 is the multiplicity in the single nucleon-nucleon collision measured in the region of acceptance; it is defined by the constraint that $\langle U\rangle = n_0 g T_0 N_P$, whereas $c = -\phi/\left(Dc_p\rho T_0\right)$ is a constant (notice that because $\phi < 0$ in the cases of interest here, cf. Fig. 1, c is positive). Comparing now Eqs. (5) and (7), one can expect that

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = 1 + c\left(\langle N\rangle - n_0 N_P\right). \tag{8}$$

Notice that Eqs. (7) and (8) do not depend on parameter g anymore; its role in the present work is to fit, if necessary, $\langle N\rangle$ in Eq. (6) to experimental data only.

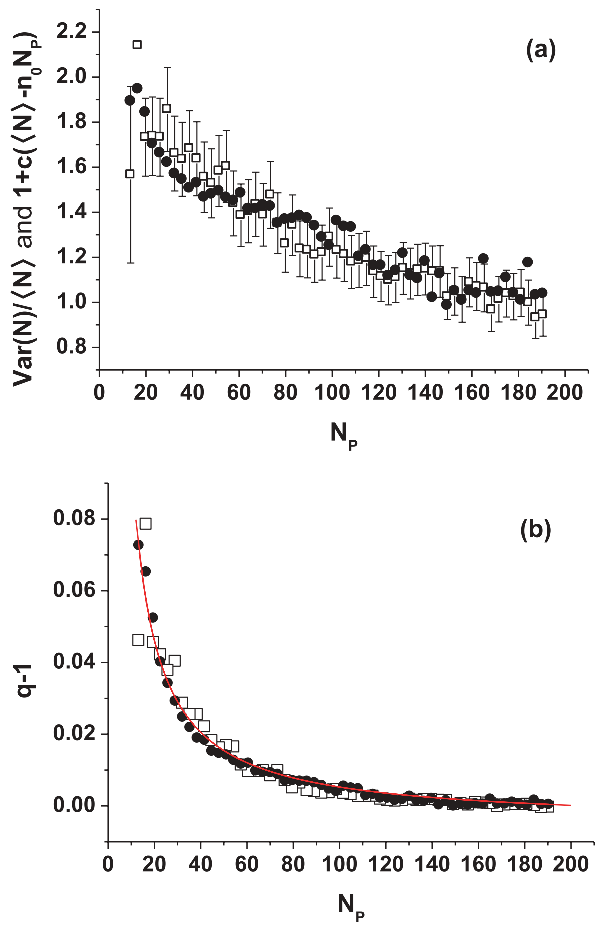

FIG. 2. (Color online) (a) Comparison of $\mathrm{Var}(N)/\langle N\rangle$ versus N_P (squares) with $1 + c(\langle N\rangle - n_0 N_P)$ versus N_P (circles) for $n_0 = 0.642$ and $c = 4.1$. Data are for negatively charged particles from Pb + Pb collisions as collected by NA49 experiment [3]. (b) The same, but translated to dependence of $q-1$ vs N_P. Squares were obtained from the $\mathrm{Var}(N)/\langle N\rangle$ vs N_P dependence; circles from the $\langle N\rangle$ vs N_P dependence. The values of n_0 and c are the same as before. The solid line shows the dependence in Eq. (16) for the nuclear mass number $A = 207$ and parameter $a = 0.98$.

This means that when using the notion of $T_{\rm eff}$, one should observe that $\mathrm{Var}(N)/\langle N\rangle$ and $\langle N\rangle$ are mutually connected. Figure 2 shows that this is indeed the case.[6] If $T_{\rm eff}$ depends on q (i.e., for $c \neq 0$), then the dependence $\mathrm{Var}(N)/\langle N\rangle$ on the number of participants N_P is connected to the dependence of $\langle N\rangle$ on N_P. Notice that if $\langle N\rangle$ is linear in N_P, then $\mathrm{Var}(N)/\langle N\rangle$ is constant. In particular, for $\langle N\rangle = n_0 N_P$ we have $\mathrm{Var}(N)/\langle N\rangle = 1$. Therefore the experimental fact that $\mathrm{Var}(N)/\langle N\rangle$ decreases with increasing N_P indicates *nonlinear* dependence of $\langle N\rangle$ on the number of participants N_P.

[4]Recently, Tsallis statistics was implemented in the so-called blast-wave model and applied to the transverse momentum spectra measured at RHIC [30]. As a result, the dependence of temperature and collective flow velocity on the $(q-1)$ parameter has been advocated. Similar dependence was also found in a recent analysis of the nonthermal equilibrium in heavy ion collisions performed using Tsallis distributions [31].

[5]This is because the mean accepted multiplicity for acceptance p is $\langle N\rangle = p\langle N_{p=1}\rangle$ and the scaled variance for the accepted particles is $\mathrm{Var}(N)/\langle N\rangle = (1-p) + p\,\mathrm{Var}\left(N_{p=1}\right)/\langle N_{p=1}\rangle$; therefore one gets from Eq. (5) that the nonextensivity parameter obtained this way does not depend on acceptance, i.e., $q = q_{p=1}$.

[6]In Fig. 2, the fitted value of $n_0 = 0.642$ is only a little greater than the multiplicity observed in $p+p$ collisions when calculated using the acceptance of the NA49 experiment. Notice also that the value of $c = 4.1$ obtained here for Pb + Pb collisions is not far from the value $1.25/0.22 = 5.7$ obtained for data from RHIC (i.e., for Au + Au collisions but at much higher energy), which we obtained in Fig. 1.

It should be stressed that data taken by the NA49 experiment [3] show that $\mathrm{Var}(N)/\langle N\rangle$ changes rather strongly with N_P. For peripheral collisions (i.e., for small N_P) one observes marked deviation of $\mathrm{Var}(N)/\langle N\rangle$ from unity. At the same time deviation of $\langle N\rangle$ from linearity in N_P is very weak. Our result based on the concept of effective temperature, $T_{\rm eff}$, and showing that $\mathrm{Var}(N)/\langle N\rangle$ and $\langle N\rangle$ are connected is therefore by no means trivial.

We close this section by reminding the reader that all previous attempts in this field addressed only the behavior of $\mathrm{Var}(N)/\langle N\rangle$ versus N_P, leaving aside the possible nonlinear dependence of $\langle N\rangle$ on N_P. Figure 2 shows that introducing $T_{\rm eff}$ results in a kind of scaling, namely, the same value of $q-1$ describes dependencies of both variance and mean multiplicity on N_P, following Eq. (8), which–we repeat–has its origin in the q dependence of $T_{\rm eff}$.[7]

B. Finite hadronizing source size and temperature fluctuations

We shall now derive the $q-1$ dependence on N_P seen in Fig. 2. Let us first observe that [2,32]

$$q-1=\frac{\mathrm{Var}(T)}{\langle T\rangle^2}=\frac{1}{C_V}, \tag{9}$$

i.e., the parameter q can be regarded as connected (via fluctuations of temperature) to the heat capacity under constant volume C_V. For a system with finite size remaining in contact with a heat bath, one has, following Lindhard's approach [33], that

$$\mathrm{Var}(U)+C_V^2\,\mathrm{Var}(T)=\langle T\rangle^2 C_V. \tag{10}$$

This is a kind of uncertainty relation (in the sense that it expresses the truth that in the case of conjugate variables one standard deviation in some measurement can only become small at the expense of the increase of some other standard deviation [34]). Relation (10) is supposed to be valid all the way from the canonical ensemble, where $\mathrm{Var}(T)=0$ and $\mathrm{Var}(U)=\langle T\rangle^2 C_V$, to the microcanonical ensemble, for which $\mathrm{Var}(T)=\langle T\rangle^2/C_V$ and $\mathrm{Var}(U)=0$. Equation (10) expresses the complementarity between both the temperature and energy and the canonical and microcanonical description of the system.[8]

To obtain realistic (intermediate) distributions, start from a system at a fixed temperature T. The standard deviation of its energy is

$$\mathrm{Var}(U)=\langle T\rangle^2\frac{\partial\langle U\rangle}{\partial T}=\langle T\rangle^2 C_V. \tag{11}$$

Inverting the canonical distribution $g_T(U)$, one can obtain

$$g_U(T)=-T^2\frac{\partial}{\partial T}\int_0^U g_T(U')\,dU' \tag{12}$$

and interpret it as a probability distribution of the temperature in the system [see Eq. (A10)]. The standard deviation of this distribution then yields[9]

$$\mathrm{Var}(T)=\frac{\langle T\rangle^2}{C_V}. \tag{13}$$

Because for a canonically distributed system the energy variance is $\mathrm{Var}(U)=\langle T\rangle^2 C_V$ and for an isolated system $\mathrm{Var}(U)=0$, the variance (expressing energy fluctuations in the system) for the intermediate case can be assumed to be equal to

$$\mathrm{Var}(U)=\langle T\rangle^2 C_V\xi,\quad \xi\in(0,1), \tag{14}$$

where the parameter ξ depends on the size of the hadronizing source. Inserting this into Eq. (10), one gets that q depends on ξ in the following way:

$$q-1=\frac{\mathrm{Var}(T)}{\langle T\rangle^2}=\frac{1-\xi}{C_V}. \tag{15}$$

Assuming now that the size of the thermal system produced in heavy ion collisions is proportional to the number of participating nucleons N_P, i.e., that $\xi\simeq f=N_P/A$, and taking into account that $C_V\cong aN_P$, we obtain that

$$q-1=\frac{1}{aN_P}(1-f). \tag{16}$$

This is the relation that nicely fits the data, see Fig. 2.

[7]One should keep in mind that both the q and $T_{\rm eff}$ evaluated from transverse characteristics of the reaction can differ from those obtained from its longitudinal characteristics. This means that these parameters in multiplicity distributions $P(N)$ (which are sensitive to $p=\sqrt{p_L^2+p_T^2}$) can differ substantially from those obtained by analyzing transverse momentum distribution. Actually, in Ref. [22] we advocate that $|1-q_T|\ll|1-q_L|$. Assuming now that $\mathrm{Var}(T)=\mathrm{Var}(T_T)+\mathrm{Var}(T_L)$, we obtain that $q\langle T\rangle^2=q_L\langle T_L\rangle^2+q_T\langle T_T\rangle^2+\langle T\rangle^2-\langle T_L\rangle^2-\langle T_T\rangle^2$, which for $\langle T_L\rangle>\langle T_T\rangle$, as observed in Ref. [21], leads to $q\simeq q_L$.

[8]In the same way as the (improper) eigenstates of position and momentum appear as extreme cases in the quantum mechanical uncertainty relations. It is worth knowing that in Ref. [35] the limiting cases of Tsallis statistics was investigated in which q was interpreted as a measure of thermal bath heat capacity: $q=1$, i.e., the canonical case, would correspond to an infinite bath (thermalized and with fixed temperature), whereas $q=-\infty$, i.e., microcanonical case, would correspond to a bath with null heat capacity (isolated and with fixed energy). All intermediate cases would then correspond to the finite heat capacity (both temperature and energy fluctuate).

[9]In our formalism presented here, we have [Eq. (B3)] that $g_{T,N}(U)=[\beta/\Gamma(N)](\beta U)^{N-1}\exp(-\beta U)$ with fluctuations given by $\mathrm{Var}(U)/\langle U\rangle^2=1/N$. One can now invert distribution $g_{T,N}(U)$ proceeding in analogous way as used in Appendix B to obtain multiplicity distributions $g_{T,U}(N)$ and obtain distribution of temperature, $g_{U,N}(T)=\frac{\partial}{\partial\beta}\int_0^U g_{T,N}(U')dU'=[U/\Gamma(N)](\beta U)^{N-1}\exp(-\beta U)$ with fluctuations $\mathrm{Var}(T)/\langle T\rangle^2\simeq\mathrm{Var}(\beta)/\langle\beta\rangle^2=1/N$. The fluctuations of temperature obtained this way have gamma distribution analogous to distribution (A10). Because $C_V\propto N$ one obtains Eqs. (11) and (13). This illustrates that we can deduce fluctuations (i.e., the corresponding probability distributions) of any quantity out of (T,U,N) provided only that the other two are constant. The implication of this fact will be discussed elsewhere.

To recapitulate: we know that if $U = \text{const}$ and $T = \text{const}$, then the multiplicity distribution $P(N)$ is Poissonian (cf. Appendix B1) and we also know how fluctuations of T change $P(N)$ from the Poissonian form to the negative-binomial distribution (NBD) one (see Appendix B2). We want now to see how big are fluctuations of T in our hadronizing systems formed in a collision process. It turns out that for $N = \text{const}$, we have either Eq. (11) or Eq. (13) depending on whether $T = \text{const}$ or $U = \text{const}$. We claim then that we can assume validity of Eq. (10) not only in the above limiting cases but also in the general case when both the energy U and the temperature T fluctuate at the same time; i.e., if fluctuations of the energy U are given by Eq. (14), then fluctuations of the temperature T are given by Eq. (15). Finally, knowing how big the fluctuations of T are, we can deduce the fluctuations of the multiplicity N. This will be performed in what follows.

C. System size dependence of mean multiplicity and multiplicity fluctuations

We shall now discuss the system size dependence of the mean multiplicity and multiplicity fluctuations. In our approach, the system size enters through C_V and ξ. We shall keep $\xi = N_P/A$ and connect C_V with the measured quantities considering two natural assumptions: either

$$C_V = aN_P \tag{17}$$

or

$$C_V = a'\langle N\rangle. \tag{18}$$

When used together with Eqs. (7) and (15), the second possibility leads to the very simple scaling relation

$$\langle N\rangle - \frac{N_P}{A}\langle N\rangle|_{N_P=A} = \frac{c}{a'}\left(1 - \frac{N_P}{A}\right), \tag{19}$$

whereas the first one results in a slightly more involved formula,

$$\left(\langle N\rangle - \frac{N_P}{A}\langle N\rangle|_{N_P=A}\right)\frac{N_P}{\langle N\rangle} = \frac{c}{a}\left(1 - \frac{N_P}{A}\right). \tag{20}$$

In both cases, $\langle N\rangle|_{N_P=A} = n_0 A$ is multiplicity extrapolated to $N_P = A$. As seen in Fig. 3, where comparison with experimental data [3] is presented, one observes different dependencies for the Pb + Pb collision and lighter nuclei for which in semicentral collisions such an effect is practically not observed. In addition, for peripheral collisions ($N_P < 0.15A$) one observes the deviation from the expected dependence of Eq. (19). Notice that deviation from the linear fit for Pb + Pb collisions concerns only the five most peripheral points, and the observed discrepancy means that the measured experimentally mean multiplicity is less than one particle higher than the expected value. The agreement with prediction given by Eq. (20) seems to be better, what means that the assumption $C_V = aN_P$ is more realistic. In what concerns multiplicity fluctuations, assumption (18) results [when combining Eqs. (16) and (5)] in the simple scaling relation

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = 1 + \langle N\rangle(q-1) = 1 + \frac{1}{a'}\left(1 - \frac{N_P}{A}\right), \tag{21}$$

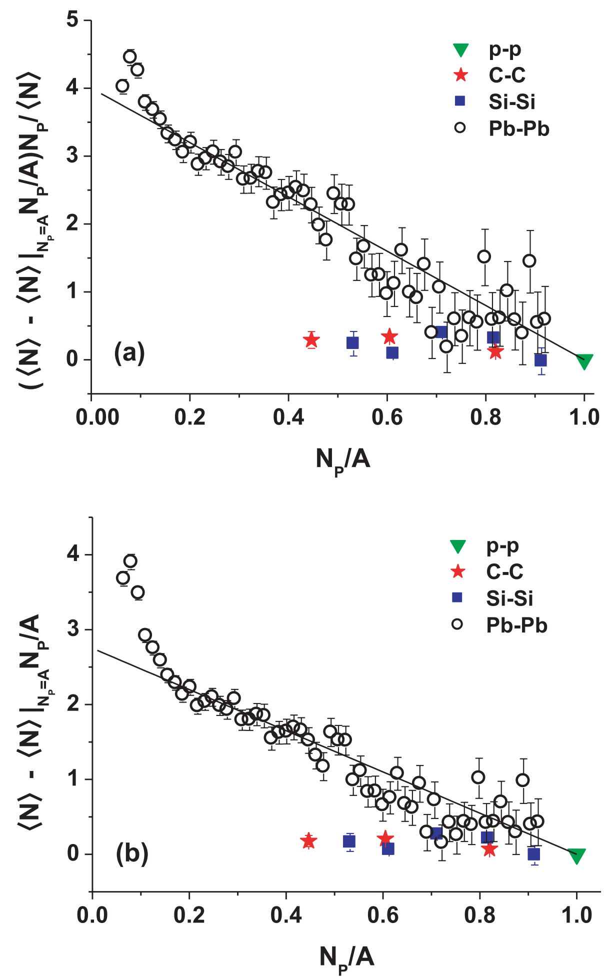

FIG. 3. (Color online) Mean multiplicities compared with predictions given by (a) Eq. (20) and (b) Eq. (19). We adopt $n_0 = 0.559, 0.575, 0.657$, and 0.642 for, respectively, $p + p$, C + C, Si + Si, and Pb + Pb collisions. Linear fit show the predictions of Eq. (19) with parameter $c/a' = 2.75$ and Eq. (20) with parameter $c/a = 1.8$. Only statistical errors are indicated.

i.e., in the multiplicity fluctuations dependent on the fraction of nucleons participating in the collision $f = N_P/A$. Such scaling was found recently by the NA49 Collaboration [3] when comparing collisions of C + C, Si + Si and Pb + Pb, see Fig. 4. If, instead, we use Eq. (17), then, taking into account the dependence of $\langle N\rangle$ on N_P given by Eq. (7), one obtains that, more exactly,

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = 1 + \langle N\rangle(q-1) = 1 + \frac{n_0\left(1 - \frac{N_P}{A}\right)}{a - \frac{c}{A}\left(\frac{A}{N_P} - 1\right)}. \tag{22}$$

We observe thus a weak dependence on the mass number A of colliding nuclei. For a small number of participants, $N_P \ll A$, we observe an additional increase of relative variance, $\mathrm{Var}(N)/\langle N\rangle$, in comparison with prediction (21). Notice that for $\phi > 0$, one expects just the opposite trend, i.e., the nonmonotonic behavior of $\mathrm{Var}(N)/\langle N\rangle$ with increasing number of participants N_P. The question about the sign of the

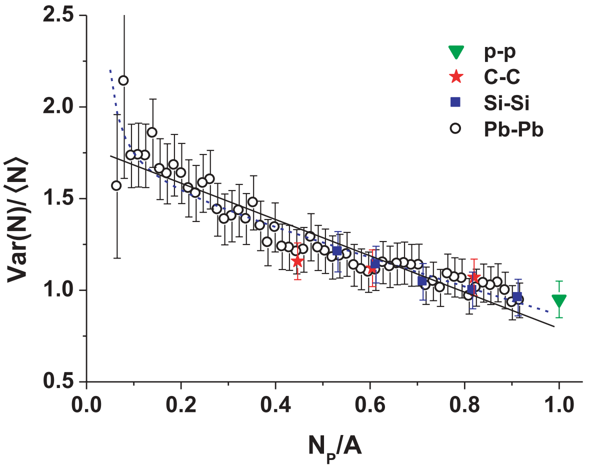

FIG. 4. (Color online) Scaled variance of the multiplicity distribution of negatively charged particles produced in $p+p$, semicentral $C+C$, semicentral $Si+Si$, and $Pb+Pb$ collisions as a function of the fraction of nucleons participating in the collision, N_P/A. The experimental data [3] are compared with prediction (23) with parameters $\omega_0 = 0.79$ and $a' = 1.0$ (solid line) and with prediction (22) with parameters $\omega_0 = 0.87$ and $a = 0.83$ (dashed line); the other parameters are the same as in Fig. 2 with $n_0 = 0.642$ and $c = 4.1$ for $A = 207$.

heat source term ϕ seems to still be open because of the lack of data in the region of small number of participants.

For central collisions (i.e., for $f = N_P/A \simeq 1$), multiplicity distributions are sub-Poissonian and $[\mathrm{Var}(N)/\langle N\rangle]|_{N_P=A} = \omega_0 < 1$. For example, within a statistical model with fixed volume, ω_0 varies in the range 0.5–1.0 [36]. In particular (see Refs. [8,10] and references therein), we note the following: (i) The global conservation laws imposed on each microscopic state of the statistical system lead to suppression of the particle number fluctuations. The final state scaled variance behavior in the canonical ensemble is characterized by $\omega_0 \cong 0.8$, whereas in the microcanonical ensemble, by $\omega_0 \cong 0.3$. (ii) In the primordial values of the scaled variance (i.e., calculated before decays of resonances), one can also observe the effect of quantum statistics. It turns out that the proper inclusion of Bose statistics leads to the additional enhancement of particle number fluctuations of the order of $\omega_0 \cong 1.05$–1.06, which is quite small at the chemical freeze-out. (iii) The particle number fluctuations can be also enhanced by including explicit production and decay of resonances (in either the grand canonical or canonical ensemble). They lead to $\omega_0 \cong 1.1$. The actual value of parameter ω_0 depends on the energy of collision and on the acceptance in which the multiplicity fluctuations are observed, and it varies in the range 0.8–1.0 [37]. For this reason, in Fig. 4 we compare experimental data with the formula

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = \omega_0 + \frac{1}{a'}\left(1 - \frac{N_P}{A}\right). \tag{23}$$

Finally, we note that the transverse momentum fluctuations measured in nuclear collisions at 158A GeV [5] and quantified by the measure[10] $\Phi(p_T)$ show a similar centrality dependence, namely, as seen in Fig. 5,

$$\Phi(p_T) = \Phi_{N_P=A} + b\left(1 - \frac{N_P}{A}\right). \tag{24}$$

This behavior of the transverse momentum fluctuations as a function of collision centrality was related in a superposition model to the centrality dependence of the multiplicity fluctuations [39] using the correlation of average transverse momentum and multiplicity observed in collisions [5]. To derive Eq. (24), notice that in general Φ is governed by the multiplicity fluctuations in the following way [40]:

$$\Phi(p_T) = \sqrt{\mathrm{Var}(p_T) + 2\langle p_T\rangle^2 \frac{\mathrm{Var}(N)}{\langle N\rangle}\left[1 - \rho\sqrt{\frac{\mathrm{Var}(p_T)}{\langle p_T\rangle^2}\frac{\langle N\rangle}{\mathrm{Var}(N)}} + 1\right]} - \sqrt{\mathrm{Var}(p_T)}, \tag{25}$$

where ρ is the correlation coefficient between N and $\sum_i^N p_{iT}$. As in Ref. [39], we can see that multiplicity fluctuations determine the behavior of $\Phi(p_T)$. In the first-order approximation and taking into account multiplicity fluctuations given by Eq. (23), we can write the transverse momentum fluctuations measure as

$$\Phi(p_T) \simeq \sqrt{\mathrm{Var}(p_T)}\left[\omega_0(1-\rho)\frac{\langle p_T\rangle^2}{\mathrm{Var}(p_T)} - \frac{1}{2}\rho\right] + (1-\rho)\sqrt{\mathrm{Var}(p_T)}\frac{\langle p_T\rangle^2}{\mathrm{Var}(p_T)}\frac{1}{a'}\left(1 - \frac{N_P}{A}\right), \tag{26}$$

which has the form of Eq. (24) in what concerns dependence on the variable N_P/A. The slope parameter b corresponds to the correlation coefficient $\rho \sim 0.99$ (for the previously estimated value of a' and for the transverse momentum fluctuations $\mathrm{Var}(p_T)/\langle p_T\rangle^2 \simeq 0.43$, as observed experimentally [41]). Once again we observe scaling behavior in the variable $f = N_P/A$.

[10] Measure $\Phi(p_T)$ has been introduced in Ref. [38] and is defined as $\Phi = \sqrt{\langle Z^2\rangle/\langle N\rangle} - \sqrt{\langle z^2\rangle}$, where $z = p_T - \langle p_T\rangle$ and $Z = \sum_i^N z_i$.

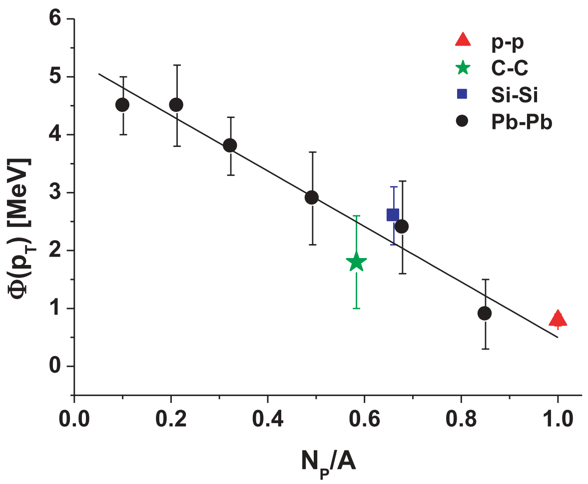

FIG. 5. (Color online) Transverse momentum fluctuations of negative particles defined by the measure Φ [38] as a function of number of the fraction of nucleons participating in the collision, N_P/A, for each of the four collisions. The experimental data are taken from Ref. [5] and compared with the prediction of Eq. (24) for parameters $\Phi_{N_P=A} = 0.48$ MeV and $b = 4.8$ MeV.

IV. SUMMARY

We have investigated some features of the multiplicity fluctuations observed in recent high-energy nuclear collisions by attributing them to the intrinsic, nonstatistical fluctuations of the temperature of the hadronizing system produced in such collisions. To this end, we used the Tsallis statistics approach in which such fluctuations are accounted for by the nonextensivity parameter q (more precisely by $q - 1$). When fluctuations are vanishing $q \to 1$, Tsallis statistics becomes the usual Boltzmann-Gibbs one, and the power-like Tsallis q exponents become the usual exponential distributions. We were considering a generalized q-exponential ensemble describing the system of N_P participating nucleons (assumed to be proportional to the size of the hadronizing system). It was found that in this case one can associate the nonextensivity parameter q with the number of participants N_P [see Eq. (16)]. It was found that data require a generalization of the usual notion of fluctuating temperature by adding the effects of the energy transfer between the hadronizing source and the surroundings composed of nucleons not participating directly in the reaction. This resulted in the introduction of a q-dependent effective temperature $T_{\rm eff}$. We have also allowed for fluctuations of the full accessible energy U (see Sec. III A); without these fluctuations, $\mathrm{Var}(N)/\langle N\rangle$ would be simply constant, independent of N_P/A.

The simplest possible explanation of the observed effect was the $q - 1$ dependence on the number of participants N_P provided by Eq. (16). We assumed that the parameter connected with the size of the collision region, ξ, is approximately given by $\xi = N_P/A$. Modifications of this assumption are possible and lead to better agreement with experimental data. Among others, the presence of nonlinear terms in the ξ variable in Eq. (15) would approve consistency with data [or, for example, assuming that $\xi = \zeta(N_P/A)$ with $\zeta(N_P/A)$ more complicated than used here, where $\zeta(N_P/A) = N_P/A$]. Nevertheless, we restricted our discussion to the simplest possible approximation to demonstrate the connection between temperature fluctuations and observables from relativistic ion collisions in a more transparent way.

Let us close with some remarks concerning the limitations of our analysis. First, let us first recall that the multiplicity fluctuations considered in this study were observed for a fixed number of projectile participants. Although in the collisions of identical nuclei, the average number of participants from the projectile equals approximately that from the target, the actual number of target participants fluctuates, whereas the number of projectile participants is kept fixed. However, one should notice that the multiplicity fluctuations were measured in the forward hemisphere ($1.1 < y_{\rm c.m.} < 2.6$) in which the influence from the target participants is marginal. The models which produce, independently of centrality, approximately Poissonian multiplicity distributions highly underestimate the observed multiplicity fluctuations in noncentral collisions. They are unable to reproduce, even qualitatively, the centrality dependence of the scaled variance (see,for example, Ref. [3] and references therein). The highest scaled variance is obtained from the hadron-string dynamics [12] and ultrarelativistic molecular dynamics [13]. However, both approaches (in their standard versions) show a flat scaled variance, $\omega \simeq 1.2$, and exhibit almost no dependence on N_p [42]. Finally, note that in comparison with experimental data (which have finite acceptance, equal to $p = 0.16$ in the case of NA49 data [3]), we have used multiplicity N given by this acceptance. Also, all parameters used to describe these data (such as n_0, a', ω_0, $\Phi_{N_p=A}$, and b) are acceptance dependent. However, the nonextensivity parameter q and its dependence on N_p obtained from comparison with experimental data do not depend on experimental acceptance (cf. footnote(5)).

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APPENDIX A: TEMPERATURE FLUCTUATIONS

We collect here the main points concerning the idea of temperature fluctuations [18,26]. Suppose that we have a nonhomogeneous thermodynamic system in different regions of which there are different temperatures T, which fluctuate around some mean temperature T_0. As a result of these fluctuations, the actual temperature T equals

$$T = T_0 - \xi(t)T, \qquad \text{(A1)}$$

where $\xi(t)$ describes the actual (not specified) stochastic process causing these fluctuations.

There must take place an exchange of energy (heat) between the regions mentioned above, in particular between any selected region and the rest of the system. This exchange eventually leads to equilibration of the temperature of the whole system. The corresponding process of heat conductance is described by [27]

$$c_p\rho\frac{\partial T}{\partial t}-\gamma(T'-T)=\phi, \tag{A2}$$

where c_p, ρ, and γ are, respectively, the specific heat under constant pressure, the density, and the coefficient of external conductance. The heat source term ϕ determines the amount of energy transfer per unit time and unit volume. Using Eqs. (A1) and (A2), one gets the following Langevin equation for the temperature T:

$$\frac{\partial T}{\partial t}+\left[\frac{1}{\tau}+\xi(t)\right]T=\frac{1}{\tau}\left[T_0+\frac{\tau}{c_p\rho}\phi\right], \tag{A3}$$

with coefficient $\tau = c_p\rho/\gamma$. As stated before, $\xi(t)$ describes stochastic changes of temperature in time. Let us assume that these changes are such that their mean value is zero, i.e.,

$$\langle\xi(t)\rangle=0, \tag{A4}$$

whereas, for sufficiently fast changes, its correlator is equal to

$$\langle\xi(y)\xi(t+\Delta t)\rangle=2D\delta(\Delta t). \tag{A5}$$

The constants τ and D define, respectively, the characteristic mean time for the temperature changes and their variance:

$$\langle T(t)\rangle=T_0+T(t=0)\exp\left(-\frac{t}{\tau}\right), \tag{A6}$$

$$\langle T^2(t=\infty)\rangle=\frac{1}{2}D\tau. \tag{A7}$$

Thermodynamic equilibrium in such a situation means that for $t \gg \tau$, the influence of the initial condition $T(t=0)$ vanishes and the mean-squared T has a value corresponding to the state of equilibrium.

Equation (A3) leads to the corresponding Fokker-Planck equation

$$\frac{df(T)}{dt}=-\frac{\partial}{\partial T}K_1f(T)+\frac{1}{2}\frac{\partial^2}{\partial T^2}K_2f(T), \tag{A8}$$

where the intensity coefficients are

$$K_1(T)=\frac{1}{\tau}\left(T_0+\frac{\tau}{c_p\rho}\phi\right)+\left(D-2\frac{1}{\tau}\right)T,$$
$$K_2(T)=2DT^2. \tag{A9}$$

Its solution is a gamma distribution in $1/T=\beta$:

$$f(T)=\frac{\mu}{\Gamma(\alpha)}\left(\frac{\mu}{T}\right)^{\alpha-1}\exp\left(-\frac{\mu}{T}\right), \tag{A10}$$

where

$$\alpha=\frac{1}{\tau D},\quad \mu=\frac{1}{\tau D}\left(T_0+\frac{\tau}{c_p\rho}\phi\right). \tag{A11}$$

The mean value and the relative variance of β are, respectively,[11]

$$\langle\beta\rangle=\left(T_0-\frac{\tau}{c_p\rho}\phi\right)^{-1}, \tag{A12}$$

$$\frac{\langle\beta^2\rangle-\langle\beta\rangle^2}{\langle\beta\rangle^2}=\frac{1}{\alpha}=\tau D. \tag{A13}$$

Temperature fluctuations in the form presented here when applied to the exponential Boltzmann-Gibbs formula, Eq. (1), lead to $h_q(E)=\int_0^\infty \exp(-E/T)f(T)\,d(1/T)$, i.e., to a Tsallis distribution, Eq. (2), with the nonextensivity parameter q equal to [18]

$$q=1+\frac{1}{\alpha}=1+\tau D, \tag{A14}$$

and with the effective temperature

$$T_{\text{eff}}=T_0+\frac{\tau}{c_p\rho}\phi=T_0+(q-1)\frac{1}{Dc_p\rho}\phi. \tag{A15}$$

In the case of no energy transfer, i.e., when $\phi=0$, one is left only with fluctuations, and then $T_{\text{eff}}=T_0$, as in Ref. [18]. Recently, temperature fluctuations due to the volume fluctuations in statistical models were discussed as well and introduced in the modeling of relativistic particle collision processes [43].

APPENDIX B: MULTIPLICITY DISTRIBUTIONS IN BOLTZMANN AND TSALLIS ENSEMBLES

We shall now recapitulate some basic ideas concerning multiplicity distributions that result from Boltzmann and Tsallis ensembles [24].

1. Poisson multiplicity distribution

This distribution arises in situations where, in some process, one has N independently produced secondaries with energies $\{E_{1,\ldots,N}\}$ distributed according to a Boltzmann distribution

$$g(E_i)=\frac{1}{\lambda}\exp\left(-\frac{E_i}{\lambda}\right),\quad \text{where}\quad \lambda=\langle E\rangle. \tag{B1}$$

The corresponding joint probability distribution is given by

$$g(\{E_{1,\ldots,N}\})=\frac{1}{\lambda^N}\exp\left(-\frac{1}{\lambda}\sum_{i=1}^N E_i\right). \tag{B2}$$

For independent energies $\{E_{1,\ldots,N}\}$, the sum $E=\sum_{i=1}^N E_i$ is distributed according to a gamma distribution

$$g_N(E)=\frac{1}{\lambda(N-1)!}\left(\frac{E}{\lambda}\right)^{N-1}\exp\left(-\frac{E}{\lambda}\right), \tag{B3}$$

the cumulative distribution function of which is

$$G_N(E)=1-\sum_{i=1}^{N-1}\frac{1}{(i-1)!}\left(\frac{E}{\lambda}\right)^{i-1}\exp\left(-\frac{E}{\lambda}\right). \tag{B4}$$

Equation (B3) follows immediately either by using characteristic functions or by sequentially performing integration of the joint distribution (B2) and noticing that

$$g_N(E)=g_{N-1}(E)\frac{E}{N-1}. \tag{B5}$$

[11]Notice that the relative variances of temperature T and its inverse β are roughly the same, $\frac{\langle\beta^2\rangle-\langle\beta\rangle^2}{\langle\beta\rangle^2}\simeq\frac{\langle T^2\rangle-\langle T\rangle^2}{\langle T\rangle^2}$.

For energies such that

$$\sum_{i=0}^{N} E_i \leqslant E \leqslant \sum_{i=0}^{N+1} E_i, \tag{B6}$$

the corresponding multiplicity distribution has a Poissonian form (with $\langle N\rangle = \frac{E}{\lambda}$)

$$P(N) = G_{N+1}(E) - G_N(E) = \frac{\langle N\rangle^N}{N!}\exp(-\langle N\rangle). \tag{B7}$$

Therefore, whenever N variables $\{E_{1,\ldots,N,N+1}\}$ follow an exponential distribution (B1) and satisfy condition (B6), then the corresponding multiplicity N has a Poissonian distribution (B7).

2. Negative-binomial multiplicity distribution

This distribution arises when in some process N independent particles with energies $\{E_{1,\ldots,N}\}$ are distributed according to a Tsallis distribution,

$$h(\{E_{1,\ldots,N}\}) = C_N\left[1-(1-q)\frac{\sum_{i=1}^{N} E_i}{\lambda}\right]^{\frac{1}{1-q}+1-N}. \tag{B8}$$

It means that there are some intrinsic (so far unspecified but summarily characterized by the parameter q) fluctuations present in the system under consideration. In this case, we do not know the characteristic function for the Tsallis distribution. However, because we are dealing here only with variables $\{E_{1,\ldots,N}\}$ occurring in the form of the sum, $E = \sum_{i=1}^{N} E_i$, one can still sequentially perform integrations of the joint probability distribution (B8) arriving at the formula corresponding to Eq. (B3):

$$h_N(E) = h_{N-1}(E)\frac{E}{N-1} = \frac{E^{N-1}}{(N-1)!\lambda^N} \times \prod_{i=1}^{N}[(i-2)q-(i-3)]\left[1-(1-q)\frac{E}{\lambda}\right]^{\frac{1}{1-q}+1-N}, \tag{B9}$$

with the cumulative distribution function given by

$$H_N(E) = 1 - \sum_{j=1}^{N-1}\tilde{H}_j(E), \tag{B10}$$

where

$$\tilde{H}_j(E) = \frac{E^{j-1}}{(j-1)!\lambda^j} \times \prod_{i=1}^{j}[(i-2)q-(i-3)]\left[1-(1-q)\frac{E}{\lambda}\right]^{\frac{1}{1-q}+1-j}. \tag{B11}$$

As before, for energies E satisfying the condition given by Eq. (B6), the corresponding multiplicity distribution is equal to the well-known negative-binomial distribution (NBD):

$$P(N) = H_{N+1}(E) - H_N(E) \tag{B12}$$

$$= \frac{(q-1)^N}{N!}\frac{q-1}{2-q}\frac{\Gamma\left(N+1+\frac{2-q}{q-1}\right)}{\Gamma\left(\frac{2-q}{q-1}\right)} \times \left(\frac{E}{\lambda}\right)^N\left[1-(1-q)\frac{E}{\lambda}\right]^{-N+\frac{1}{1-q}}$$

$$= \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\left(\frac{\langle N\rangle}{k}\right)^N}{\left(1+\frac{\langle N\rangle}{k}\right)^{N+k}}, \tag{B13}$$

where

$$k = \frac{1}{q-1}, \quad \langle N\rangle = \frac{E}{\lambda},$$

$$\mathrm{Var}(N) = \frac{E}{\lambda}\left[1-(1-q)\frac{E}{\lambda}\right] = \langle N\rangle + (q-1)\langle N\rangle^2. \tag{B14}$$

In the cases considered here, $q \in (1, 2)$. In the limiting cases of $q \to 1$, one has $k \to \infty$, and $P(N)$ becomes a Poisson distribution; whereas for $q \to 2$, one has $k \to 1$, and $P(N)$ becomes the so-called geometrical distribution.

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