Elastic proton scattering on tritium below the n-^{3}He threshold

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Elastic proton scattering on the ^{3}H nucleus is studied between p-^{3}H and n-^{3}He thresholds, in the energy region where the first excited state of the α particle is embedded in the continuum. Faddeev-Yakubovski equations are solved in configuration space by fully considering effects from isospin breaking and rigorously treating the Coulomb interaction. Different realistic nuclear Hamiltonians are tested, elucidating open problems in the description of the nuclear interaction.

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I. INTRODUCTION

During the past decade remarkable progress has been achieved in computational few-nucleon physics. First, Green function Monte Carlo [1] and no core shell model [2] techniques have been developed and perfected, enabling bound nuclei containing up to several nucleons to be described. Lately these techniques have also been adopted to treat the simplest case of elastic nucleon-nucleus scattering [3,4]. Great progress have been achieved also in treating the Coulomb interaction for three- and four-nucleon scattering [5,6], opening new frontiers to test nuclear interaction models and the three-nucleon force (3NF) in particular.

Despite the continuous effort to describe the strong nuclear interaction, it still remains the central issue in nuclear physics. If the on-shell part of the nucleon-nucleon (NN) interaction is well constrained by the two-nucleon scattering data, determination of the of-shell properties and three-nucleon interaction in particular, which can be tested only in $A \geqslant 3$ nuclei, is a highly nontrivial task. As a consequence, a major effort has been devoted to describing the three-nucleon system with special emphasis on the N-d scattering problem. However, owing to the large size of the deuteron, it turns to be difficult to find observables that reveal the strong sensitivity to the off-shell structure of the NN interaction. Low-energy nucleon-deuteron scattering—maybe apart from the $J^\pi = 1/2^+$ state, where strong correlations with triton binding energy are observed—is well described by theNN interaction alone and is quite insensitive to its of-shell structure. Only at large energies, well above deuteron breakup threshold, do off-shell effects become more pronounced [7]. Unfortunately, at large energies one faces increasing difficulty in controlling relativistic effects and the large number of partial waves involved in the scattering process [8]. Finding a relatively simple system at low energy that can be treated exactly and is sensitive to the off-shell behavior of the nuclear interaction is of great interest. A four-nucleon continuum containing a series of thresholds and resonances (see Fig. 1) seems to present an ideal laboratory. Already the simplest case of $n + {}^3$H scattering, which is free of complications from the Coulomb interaction, contains several broad resonances in the continuum and presents a serious test for nuclear interaction models: Most of the realistic Hamiltonians underestimate the elastic cross section at the resonance region by ∼10% [10–12].

The most interesting but also the most complex four-nucleon structure is a continuum of ^{4}He, which contains numerous resonances and thresholds (see Fig. 1). It has been suggested by Hofmann and Hale [13] that the ^{4}He system can be used as a database to fine-tune the three-nucleon interaction. Particularly delicate is the region between the p-^{3}H and n-^{3}He thresholds, owing to the existence of the $\mathcal{J}^\pi = 0^+$ resonance in the p-^{3}H threshold vicinity. Accurate treatment of the charge symmetry breaking effects is required to separate the two thresholds: For example, if one neglects the Coulomb repulsion between the two protons the resonance of the α particle moves below the p-^{3}H threshold [14], becoming a bound state and changing completely the near-threshold scattering dynamics. These facts reveal the particular relevance of this region to understanding the isospin structure of two- and three-nucleon interactions.

This work aims to attract attention to the p-^{3}H elastic scattering problem, which is studied for proton laboratory energies up to 1 MeV (i.e., below the n-^{3}He threshold). Six principally different realistic nuclear Hamiltonians will be used to study this system.

II. FADDEEV-YAKUBOVSKI EQUATIONS

The four-nucleon problem is solved using Faddeev-Yakubovski (FY) equations in configuration space [15,16]. In the FY formalism the four-particle wave function is written as a sum of 18 amplitudes. From those we distinguish amplitudes of type K, which incorporate $3 + 1$ particle channels; amplitudes of type H contain asymptotes of $2 + 2$ particle channels. By interchanging the order of the particles one can construct 12 different amplitudes of type K and 6 amplitudes of type H.

Further we use the isospin formalism; that is, we consider protons and neutrons as being degenerate states of the same particle—the nucleon, having a mass fixed to $\hbar^2/m = 41.47$ MeV fm^2. For the system of identical particles one has only two independent FY amplitudes, one of type K and the other of type H. The other 16 FY amplitudes can be obtained by applying particle permutation operators (i.e., interchanging the order of the particles in the system). Similarly, only two

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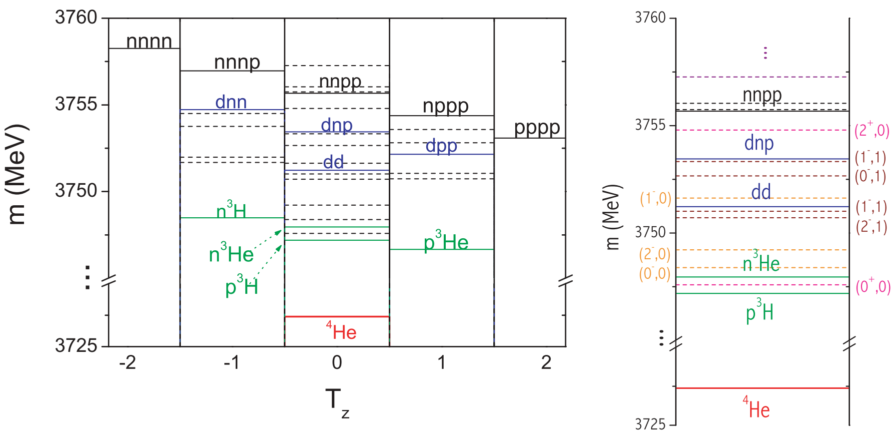

FIG. 1. (Color online) The experimental spectra of $4N$ bound and resonant states, as obtained using R-matrix analysis [9]. Resonances are indicated by the dashed lines. In the right pane of the figure the structure of ^{4}He is elucidated.

independent FY equations exist; by singling out $K \equiv K^4_{1,23}$ and $H \equiv H^{34}_{12}$ and including the three-nucleon force, the FY equations read [17,18]

$$\left(E - H_0 - V_{12} - \sum_{i<j} V^C_{ij}\right) K = V_{12}(P^+ + P^-) \times [(1+Q)K + H] + \frac{1}{2}\left[V^1_{23} + V^2_{31}\right]\Psi,$$
$$\left(E - H_0 - V_{12} - \sum_{i<j} V^C_{ij}\right) H = V_{12}\tilde{P}[(1+Q)K + H]. \quad (1)$$

Here V_{12} is the strong part of the NN interaction between nucleons (12), V^C_{ij} is the Coulomb interaction between the nucleons i and j, the three-nucleon force is represented by the terms V^1_{23} and V^2_{31}, which are symmetric for the cyclic particle permutations and contain a part of the 3NF acting in the particle cluster (123): $V_{123} = V^3_{12} + V^1_{23} + V^2_{31}$. The particle permutation operators P^+, P^-, $\tilde{P}$, and Q we use are simply

$$P^+ = (P^-)^{-1} = P_{23}P_{12},$$
$$Q = -P_{34},$$
$$\tilde{P} = P_{13}P_{24} = P_{24}P_{13}.$$

By employing operators defined here, the total wave function of the four-nucleon system is given by

$$\Psi = [1 + (1 + P^+ + P^-)Q](1 + P^+ + P^-)K + (1 + P^+ + P^-)(1 + \tilde{P})H. \quad (2)$$

The numerical implementation of these equations is described in detail in Ref. [17]. This formalism enables one to include the Coulomb interaction as well as to test different realistic nuclear interaction models, comprising nonlocal ones and models in conjunction with the three-nucleon interaction.

The ^{4}He system is predominantly of the total isospin $\mathcal{T} = 0$ state. However, the basis limited to $\mathcal{T} = 0$ does not allow unambiguous separation of the p-^{3}H and n-^{3}He channels. To separate these mirror channels we allow for full isospin-symmetry breaking by incorporating the total isospin $\mathcal{T} = 1$ and $\mathcal{T} = 2$ states in the partial wave basis. This operation allows us to account for the effects from both charge independence and charge symmetry breaking in the strong part of the nuclear interaction.

III. RESULTS

In this study proton scattering on ^{3}H nuclei for incident (laboratory) proton energies up to $E_p = 1$ MeV, that is, below n-^{3}He threshold, is considered. This energy region is very delicate owing to the presence of the first excitation of the α particle, which is physically situated just above p-^{3}H threshold (see Fig. 1). The subthreshold scattering cross section is very sensitive to the precise position of this resonance. Actually, the width of the resonance is strongly correlated with its position relative to the p-^{3}H threshold. If this state is slightly overbound the resonance peak in the excitation curve is naturally shifted to lower energies and becomes narrower. In case of underbinding one will have a much broader resonance, reflected in the flat excitation curve $\frac{d\sigma}{d\Omega}(E,\theta)$ as a function of energy. A very accurate description of the resonant scattering is therefore required. In particular, charge symmetry breaking term must be handled carefully to properly separate the p-^{3}H and n-^{3}He thresholds. Our scattering calculations, which do not restrain the total isospin $\mathcal{T}$, fully account for the isospin symmetry breaking and thus are perfectly suited.

Several very different realistic nuclear Hamiltonians have been tested in this work. These included the local configuration space potential of the Argonne group, AV18 [19], the nonlocal configuration space potentials INOY (referred to as IS-M model in Ref. [20] or INOY04′ in Refs. [12,21]) and ISUJ [22], as well as the chiral effective field theory based potential of the Idaho group derived up to next-to-next-to-next-to-leading order (I-N^{3}LO) [23]. The Urbana three-nucleon interaction was also used in conjunction with AV18 and I-N^{3}LO potentials. The parametrization, commonly called UIX [24], of this three-nucleon interaction was used together with the AV18 model for the NN interaction; however, the two-pion exchange parameter has been assigned a slightly different value of $A_{2\pi} = -0.03827$ MeV when using this force in conjunction with the I-N^{3}LO NN potential. The last adjustment allowed a fit of the triton binding energy to be made to its experimental value in a model we refer to as I-N^{3}LO+UIX* in what follows.

TABLE I. Convergence of p-^{3}H scattering length calculations for the INOY potential. Scattering lengths are presented as a function of the maximal value of the partial angular momenta ($j_{\max}$) allowed in the calculations.

$j_{\max}$	$B(^3\text{H})$	a_0	a_1
1	8.049	1.070	5.051
2	8.402	−30.65	5.583
3	8.481	−37.01	5.452
4	8.482	−36.87	5.377
5	8.483	−37.16	5.371
6	8.483	−37.35	

A partial wave expansion is one of the most common ingredients in few-nucleon calculations. This expansion converges rather fast for a low-energy system, since the effective centrifugal terms grow rapidly with the angular momentum. Nevertheless, because of the multitude of the degrees of freedom it represents, the partial wave basis for the four-nucleon system becomes considerable and requires thousands of amplitudes to achieve numerically converged results. Therefore partial wave convergence turns out to be a central issue for any calculation claiming numerical accuracy. In this study numerical convergence was studied as a function of $j_{\max} = \max(j_x, j_y, j_z)$ for amplitudes K and $j_{\max} = \max(j_x, j_y, l_z)$ for amplitudes H, where j_x, j_y, j_z, and l_z are the partial angular momenta as explained in Ref. [10]. In Table I the convergence for p-^{3}H scattering length calculations with INOY potential are presented. As one can see, the convergence pattern is not regular; however, final results seem to converge at better than the 0.5% level.

The first three columns of Table II give the calculated ground-state binding energies for three nucleons and the α particle. These values agree perfectly with the ones obtained using other *ab initio* methods. The obtained results for triplet and singlet p-^{3}H scattering lengths are also provided in Table II. These values are estimated with some small error, which arises from the numerical calculations having been performed for incident proton energies $E_p > 7$ keV and then extrapolated to get corresponding scattering lengths at $E_p = 0$. One can see an important spread of the model predicted values, in particular for the resonant singlet scattering length. As already explained this value is determined by the position of the α particle excitation relative to the p-^{3}H threshold, which are separated roughly by $\sim\hbar^2/(2\mu a_0^2) < 100$ keV (where μ is the reduced mass of the p-^{3}H system). Therefore only a few keV variation of the resonance position relative to the p-^{3}H threshold can shift the singlet scattering length by as much as 1 fm. Though it is difficult to establish correlation laws for an attractive four-nucleon system, one should stress how sensitive the scattering length is to the prediction of the ground-state binding energy of the α particle. As one can see, the 170-keV reduction in α particle binding energy obtained when passing from the INOY model to the ISUJ model permits a reduction in the resonant scattering length by almost 2 fm. This is not surprising however, once one shifts the α particle ground state relative to the p-^{3}H threshold—its excitation is also shifted in the same direction.

TABLE II. Different nuclear model predictions for bound state energies of the triton, ^{3}He, and ^{4}He in MeV together with p-^{3}H scattering lengths in femtometers.

Model	$B(^3\text{H})$	$B(^3\text{He})$	$B(^4\text{He})$	$a_0(p\text{-}^3\text{H})$	$a_1(p\text{-}^3\text{H})$
ISUJ	8.482	7.718	28.91	−35.5(2)	5.39(1)
INOY	8.483	7.720	29.08	−37.4(2)	5.37(1)
AV18	7.623	6.925	24.23	−15.5(1)	5.79(1)
AV18+UIX	8.483	7.753	28.47	−23.6(2)	5.47(1)
I-N^{3}LO	7.852	7.159	25.36	−19.6(2)	5.85(1)
I-N^{3}O+UIX*	8.482	7.737	28.12	−22.9(2)	5.59(1)
Exp.	8.482	7.718	28.30		

The p-^{3}H triplet channel ($J^\pi = 1^+$) is repulsive, whereas variation of the p-^{3}H triplet scattering length is rather moderate: This scattering length tends to decrease as the three-nucleon binding energy increases. The correlation is not perfect however; in particular, the results for the momentum space I-N^{3}LO potential tends to break the correlation. This is probably because for this potential we have used point-Coulomb repulsion, whereas the configuration space potentials employ a screened Coulomb interaction having the same parametrization as the one encoded in the electromagnetic part of the AV18 proton-proton potential.

In Fig. 2 the calculated p-^{3}H differential cross section at $\theta_{\text{c.m.}} = 120^\circ$ as a function of proton laboratory energy is compared to the experimental data. At very low energies, below 100 keV, the scattering cross section diverges owing to the effective Coulomb repulsion between the proton and triton. This behavior, which is easily described analytically, completely hides any strong interaction effects. Only beyond 100–150 keV are nuclear scattering amplitudes pronounced, revealing the clear resonant behavior of the singlet channel ($J^\pi = 0^+$), even though this wave is suppressed by the statistical factor of 3 compared to the triplet ($J^\pi = 1^+$) one. The resonant peak is particularly neat for INOY and ISUJ models, which place it too close to the p-^{3}H threshold,

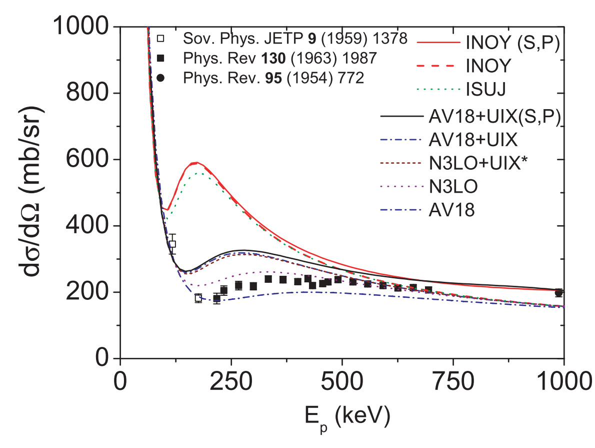

FIG. 2. (Color online) Various model calculations for the p-^{3}H excitation function $\frac{d\sigma}{d\Omega}(E)\big|_{\theta_{\text{c.m.}}=120^\circ}$ compared with experimental data from Refs. [25–27].

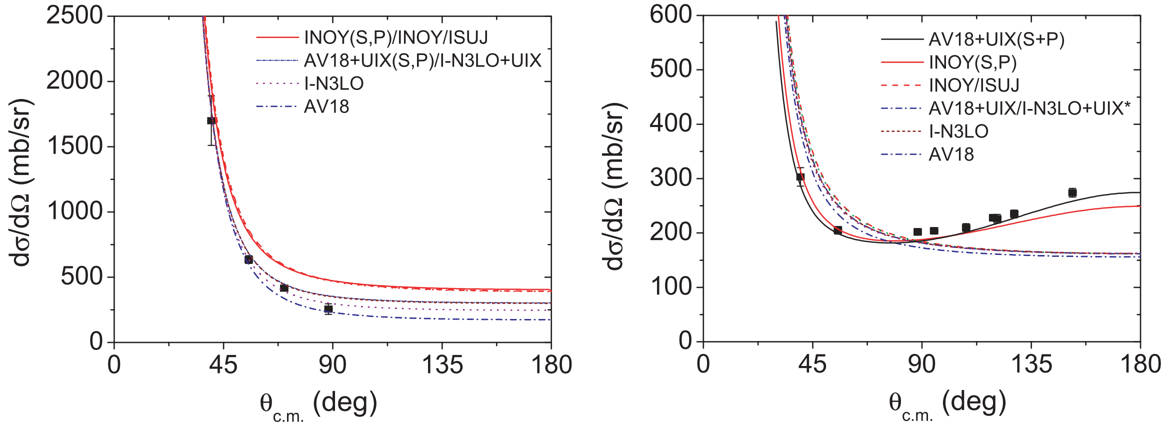

FIG. 3. (Color online) Calculated p-^{3}H differential cross sections compared with experimental data of Ref. [28] for incident proton energies of 0.3 MeV (left pane) and 0.9 MeV (right pane). If at 0.3 MeV the contribution of P waves is negligible, they are necessary at 0.9 MeV to reproduce bending in the experimental curve. However, if S-wave contributions to the elastic scattering cross section differ from model to model at 0.3 MeV, they merge into the same curve at 0.9 MeV.

strongly underestimating its width. The resonance width naturally increases as the resonance moves further into the continuum; therefore the other model predicted excitation functions $[\frac{d\sigma}{d\Omega}(E, \theta_{\text{c.m.}} = 120^\circ)]$ are much flatter than the INOY or ISUJ ones. Probably the best description of the experimental data in the resonance region ($E_p \sim 200$–600 keV) is achieved by the I-N3LO model. The AV18 Hamiltonian slightly underestimates the experimental cross section below 600 keV, but its excitation function is too flat. Inclusion of the Urbana-type three-nucleon force makes I-N3LO+UIX* and AV18+UIX model results almost indiscernible, whereas the elastic cross section is visibly overestimated in the resonance region. The agreement between the two model predictions is probably coincidental, because Hale and Hofmann [13] demonstrated a strong sensitivity of the p-^{3}H cross section to the 3NF parameters. Qualitatively the same behavior is observed, when analyzing the differential cross section as a function of the scattering angle. This is demonstrated in the left pane of Fig. 3 for incident protons of 300 keV.

Beyond the resonance region the S-wave cross sections for all models coincide. The AV18 prediction is the last one to integrate into the joint curve, and only for incident proton energies around 1 MeV. This is because AV18 places the resonance too far into the continuum, overestimating its width and extending its energy into the resonance region. The ^{4}He spectra also contain several P-wave resonances just above the n-^{3}He threshold (see Fig. 1). These resonances, and in particular a narrow $J^\pi = 0^-$ one, extend also into the region below the n-^{3}He threshold. Therefore description of the scattering cross section beyond $E_p \approx 400$ keV requires inclusion of the negative-parity states. This makes the calculations largely involved, explaining why calculations including P waves have been performed only for INOY and AV18+UIX models. P waves seems to improve the agreement with experimental data measured close to the n-^{3}He threshold. This region is best studied by analyzing the differential cross section as a function of the scattering angle, as is done in the right pane of Fig. 3 for incident protons of 900 keV, which is only ~100 keV below the n-^{3}He threshold. One can see that at this energy the S-wave contribution to the differential cross section is almost the same for all the models considered. P waves, having in particular a strong contribution from the $J^\pi = 0^-$ channel, are required to reproduce the bending of the experimental cross section curve at backward angles. The backward scattering cross section is slightly underestimated by the INOY potential, indicating that negative-parity resonances are displaced to higher energies by this interaction. A similar observation has been made by Deltuva *et al.* [29] for low-energy n-^{3}He scattering. It is obvious that NN P waves should have a strong impact on the negative-parity states of the nuclear system and therefore stronger NN P waves than those given by the INOY model are favored by the experimental data. However, the AV18+UIX model describes almost perfectly the differential cross section at 900 keV, which is mostly due to the much larger $J^\pi = 0^-$ phase shift than the one obtained using INOY. The interplay of different P waves can be best studied by using analyzing power data; unfortunately no such data exist for the p-^{3}H system below $E_p = 4$ MeV.

Before closing this section, the possibility of using the ^{4}He continuum as a laboratory to fine-tune two- and three-nucleon interactions should be emphasized. Indeed, p-^{3}H scattering at very low energies, being particularly sensitive to charge symmetry breaking terms in NN S waves, is important for understanding charge symmetry breaking of the nuclear interaction. At higher energies close to the n-^{3}He threshold the contribution of negative-parity states to the scattering process can be well separated from the contribution of the positive-parity ones. Negative-parity states demonstrate a stronger sensitivity to NN P waves. Furthermore, the study of the ^{4}He continuum undertaken in parallel with resonant scattering in n-^{3}H and p-^{3}He systems would open new frontiers for understanding isospin symmetry of NN P waves as well as isospin structure of the three-nucleon interaction. In view of such perspectives the importance of low-energy experiments in four-nucleon systems should be stressed. In particular, nucleon scattering on tritium data are very scarce, and tritium experiments have all but been completely abandoned for the past 30 years. The notable exception is the recently performed proton-tritium differential cross section measurement at Fudan University (Shanghai) [30] for proton incident energies from 1.4 to 3.4 MeV and detectors fixed at a backward laboratory angle of 165°. Hopefully this activity will be continued for lower energy protons as well as extended to use neutrons as projectiles.

IV. CONCLUSION

In this work the proton elastic scattering on tritons is studied for incident proton energies below 1 MeV using Faddeev-Yakubovski equations in configuration space and by fully accounting for isospin breaking effects in the nuclear interaction. Converged results are obtained for six qualitatively different realistic nuclear Hamiltonians. However, none of the tested Hamiltonians have been able to reproduce the shape of the near-threshold $J^\pi = 0^+$ resonance together with the ground-state binding energy of the α particle.

It has been demonstrated that the ^{4}He continuum is an ideal laboratory to fine-tune the two- and three-nucleon interaction models and the particular role this system can play in understanding charge symmetry breaking. Recent advances in computational techniques allow very precise calculations for four-nucleon scattering problem at low energies; therefore new four-nucleon scattering experiments and in particular ones concerning nucleon scattering on tritium are strongly anticipated.

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