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Critical temperature for α-particle condensation within a momentum-projected mean-field approach

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α-particle (quartet) condensation in homogeneous spin-isospin symmetric nuclear matter is investigated. The usual Thouless criterion for the critical temperature is extended to the quartet case. The in-medium four-body problem is strongly simplified by the use of a momentum-projected mean-field ansatz for the quartet. The self-consistent single-particle wave functions are shown and discussed for various values of the density at the critical temperature. Excellent agreement of the critical temperature with a numerical solution of the Faddeev-Yakubovsky equation is obtained.

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Introduction. The investigation of pairing in different Fermi systems is still on the forefront of active research. Examples are nuclear physics [1] and the physics of cold fermionic atoms [2]. The formation and condensation of heavier clusters in Fermi systems is much less studied.

In cold atom physics, the recent advent of trapping three different species of fermions [3] has opened up the possibility of creating gases of heavier clusters. For the time being, those may be trions (bound state of three different fermions), but in the future one also can think of quartets (bound state of four different fermions). The latter are specially interesting because of their bosonic nature and the possibility of Bose-Einstein condensation (BEC) of quartets. The description of quartet condensation has been attempted with an extension of the so-called Cooper problem to the four-body case in Ref. [4]. In Ref. [5], a variational procedure for the condensation of multicomponent fermion clusters has been proposed. A quartet phase has been found in a one-dimensional model with four different fermions [6].

On the other hand, in nuclear physics, quartet correlations are often very strong. This is rooted in the fact that it is a four-component fermion system (proton/neutron with spin up/down) with all fermions attracting one another, leading to the very strongly bound α particle. The formation of clusters has been an object of study almost since the beginning of nuclear physics [7]. Of course, pairing also exists in nuclei. Nuclei are very small quantum objects with only a (slowly) fluctuating phase (the conjugate variable to particle number N). Still, signatures of superfluidity are strong in nuclei, and one safely can extrapolate to the existence of superfluidity in neutron stars. On the other hand, as already mentioned, in nuclear physics the existence of quartets (α particles) as subclusters of nuclei is omnipresent. As is well known, many lighter nuclei with equal proton and neutron numbers ($Z = N$) show, for instance, in excited states, strong α clustering. The concept that these α particles may form a condensate in certain low-density states of nuclei and that this may, in analogy to the pairing case, be a precursor sign of α-particle condensation in infinite matter [8] has come up only recently [9]. Also heavy nuclei seem to have preformed α clusters in the surface because of their well-known spontaneous α decay properties.

Symmetric nuclear matter does not exist in nature because of the too strong Coulomb repulsion. However, in collapsing stars, so-called proto-neutron stars, the fraction of protons is still high [10], and the formation of α particles and, at sufficiently low temperature, their condensation may eventually be possible. At any rate, it seems evident that nuclear matter at various degrees of asymmetry is unstable with respect to cluster formation in the low-density regime. Several theoretical studies predict that α phases exist in certain temperature-density-asymmetry domains [11].

In view of the complexity of the task, the objective of the present work is quite modest. We want to study the critical temperature of α-particle condensation as a function of density and temperature in symmetric nuclear matter. Still, even this task will not be carried out down to the BEC limit. We will study the critical temperature T_c^α for the onset of formation of α particles in a thermal gas of nucleons. This will be done with a theory analogous to the famous Thouless criterion for the onset of formation of Cooper pairs in a superconductor. On the microscopic level, the problem is still very challenging, since it amounts to solving an in-medium four-body problem. In spite of that, solutions have already been worked out in the past, either solving approximately the Faddeev-Yakubovsky (FY) equations [12] or with an approximate ansatz [8].

In this work, we will continue along those lines. The final objective is to reach the BEC regime in a treatment similar to the one of Nozières and Schmitt-Rink (NSR) theory [13], but for quartets. Needless to say, this will only be possible if the whole formalism can radically be simplified. Actually, as we will show in this work, such a procedure may well exist. In any case, it is not conceivable that one treats condensation of bosonic clusters built out of N fermions on the level of nonlinear in-medium N-body equations for $N > 2$. On the

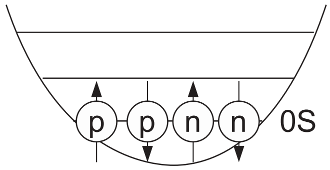

FIG. 1. Sketch of α-particle configuration, indicating that the two protons and two neutrons occupy the lowest $0S$ level in the mean-field potential of harmonic oscillator shape.

other hand, it is well known that nuclei can satisfactorily be described in mean-field approximation [14]. Projecting these mean-field (Hartree-Fock) type of solutions on zero total momentum ($\mathbf{K}=0$) will then allow these mean-field clusters to Bose condense. Actually it is well known among the nuclear physics community that even for such a small nucleus as the α particle a momentum-projected mean-field approach yields a very reasonable description [15]. The reason for this stems, as already mentioned, from the presence of four different fermions, all attracting one another with about the same force.

In Fig. 1, we sketch the situation, indicating that the two protons and two neutrons occupy the lowest $0S$ level of the mean-field potential. Actually, calculations show that the $0S$ orbital of the self-consistent mean field resembles very much an oscillator wave function of Gaussian shape. In this respect, the sketch in Fig. 1 is not so far from reality. We suspect that the situation is generic for all strongly bound quartets which may be produced in the future, and, therefore, our present study is of quite general interest. We will adopt this momentum-projected mean-field procedure in this work.

In-medium four-body equation. In-medium four-body equations have long been well documented in the literature [16]. In the present case of an in-medium quartet, the corresponding equation reads as follows [8]:

$$(E-\varepsilon_{1234})\Psi_{1234} = (1-f_1-f_2)\sum_{1'2'} v_{12,1'2'}\Psi_{1'2'34} + (1-f_1-f_3)\sum_{1'3'} v_{13,1'3'}\Psi_{1'23'4} + \text{permutations}, \quad (1)$$

where $\varepsilon_{1234}=\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4$ with $\varepsilon_i=\varepsilon(k_i)=k_i^2/(2m)+V^{\text{m.f.}}(k_i)$, where $V^{\text{m.f.}}$ is the Hartree-Fock-mean-field shift, and $f_i=f(\varepsilon_i)=[e^{(\varepsilon_i-\mu)/T}+1]^{-1}$ is the Fermi-Dirac distribution ($\hbar=c=k_B=1$). The matrix element of the interaction is $v_{12,1'2'}$ with the numbers $1,2,3,\ldots$ standing for all quantum numbers as momenta, spin, isospin, etc. This also applies to all other quantities in Eq. (1).

In Eq. (1), when $E=4\mu$, this signals quartet condensation in very much the same manner as in the two-body equation

$$(E-\varepsilon_1-\varepsilon_2)\Psi_{12} = (1-f_1-f_2)\sum_{1'2'} v_{12,1'2'}\Psi_{1'2'}, \quad (2)$$

where the approach of $T\to T_c$ so that $E\to 2\mu$ signals the transition to a superconducting or superfluid state (the well-known Thouless criterion [17]).

Of course, as already stated several times, the determination of T_c^α needs the heavy solution of the in-medium modified four-particle equation (1). Nonetheless, we here also will present for the first time an exact numerical solution of the in-medium four-body equation employing the FY method (see footnote 1).[1]

Now, following the discussion in the introduction, we make the following "projected" mean-field ansatz for the quartet wave function [4,5,18],

$$\Psi_{1234} = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\prod_{i=1}^{4}\varphi(\mathbf{k}_i)\chi^{ST}, \quad (3)$$

where χ^{ST} is the spin-isospin function which we suppose to be the one of a scalar ($S=T=0$). We will not further mention it. We work in momentum space, and $\varphi(\mathbf{k})$ is the as-yet-unknown single-particle $0S$ wave function. In position space, this leads to the usual formula [14] $\Psi_{1234}\to\int d^3R\prod_{i=1}^4\tilde{\varphi}(\mathbf{r}_i-\mathbf{R})$, where $\tilde{\varphi}(\mathbf{r}_i)$ is the Fourier transform of $\varphi(\mathbf{k}_i)$. If we take for $\varphi(\mathbf{k}_i)$ a Gaussian shape, this gives $\Psi_{1234}\to\exp[-c\sum_{1\leqslant i<k\leqslant 4}(\mathbf{r}_i-\mathbf{r}_k)^2]$, which is the translationally invariant ansatz often used to describe α clusters in nuclei. For instance, it is also employed in the α-particle condensate wave function of Tohsaki, Horiuchi, Schuck, Röpke (THSR) in Ref. [9].

Inserting the ansatz (3) into Eq. (1) and integrating over superfluous variables, or minimizing the energy, we arrive at the following nonlinear, Hartree-Fock type of equation for the single-particle $0S$ wave function $\varphi(k)=\varphi(|\mathbf{k}|)$:

$$A(k)\varphi(k)+3B(k)+3C(k)\varphi(k)=0, \quad (4)$$

where $A(k)$, $B(k)$, and $C(k)$ are given by

$$\mathcal{A}(k_1) = \int\prod_{i=2}^{4}\frac{d^3k_i}{(2\pi)^3}\left[\sum_{i=1}^{4}\frac{k_i^2}{2m}-4\mu\right] \times|\varphi(k_2)|^2|\varphi(k_3)|^2|\varphi(k_4)|^2(2\pi)^3\delta^{(3)}\left(\sum_{i=1}^{4}\mathbf{k}_i\right), \quad (5)$$

$$\mathcal{B}(k_1) = \int\prod_{i=2}^{4}\frac{d^3k_i}{(2\pi)^3}\frac{d^3k_1'}{(2\pi)^3}\frac{d^3k_2'}{(2\pi)^3}(1-f(\varepsilon_1)-f(\varepsilon_2)) \times v_{\mathbf{k}_1\mathbf{k}_2,\mathbf{k}_1'\mathbf{k}_2'}\varphi(k_1')\varphi(k_2')\varphi(k_2)|\varphi(k_3)|^2|\varphi(k_4)|^2 \times(2\pi)^3\delta^{(3)}\left(\sum_{i=1}^{4}\mathbf{k}_i\right), \quad (6)$$

$$\mathcal{C}(k_1) = \int\prod_{i=2}^{4}\frac{d^3k_i}{(2\pi)^3}\frac{d^3k_2'}{(2\pi)^3}\frac{d^3k_3'}{(2\pi)^3}(1-f(\varepsilon_2)-f(\varepsilon_3)) \times v_{\mathbf{k}_2\mathbf{k}_3,\mathbf{k}_2'\mathbf{k}_3'}\varphi(k_2)\varphi(k_2')\varphi(k_3)\varphi(k_3')|\varphi(k_4)|^2 \times(2\pi)^3\delta^{(3)}\left(\sum_{i=1}^{4}\mathbf{k}_i\right). \quad (7)$$

We want to point out here that we neglected in this pilot study the effects from the single-particle Hartree-Fock mean-field shifts. The direct term can be absorbed into the chemical potential, whereas the Fock term gives rise to an effective mass, usually of smaller influence, and it is taken care of here

[1] A solution of Eq. (1) is given in Ref. [12] which, however, contains some approximations (M. Beyer, private communication).

implicitly by adjusting the effective force, defined below in Eq. (9), to experimental data of the α particle.

From Eq. (4), we obtain the single-particle wave function in momentum space as

$$\varphi(k) = \frac{-3B(k)}{A(k) + 3C(k)}. \tag{8}$$

As seen in Eqs. (5)–(7), since $A(k)$, $B(k)$, and $C(k)$ depend on the wave function of $\varphi(k)$, Eq. (8) is strongly nonlinear. Its solution can be found by iteration.

For a general two-body force $v_{\mathbf{k}_1\mathbf{k}_2,\mathbf{k}_1'\mathbf{k}_2'}$, the equation to be solved is still rather complicated. We, therefore, proceed to the last simplification and replace the two-body force by a unique separable one, that is,

$$v_{\mathbf{k}_1\mathbf{k}_2,\mathbf{k}_1'\mathbf{k}_2'} = \lambda e^{-k^2/k_0^2} e^{-k'^2/k_0^2} (2\pi)^3 \delta^{(3)}(\mathbf{K} - \mathbf{K}'), \tag{9}$$

where $\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2$, $\mathbf{k}' = (\mathbf{k}_1' - \mathbf{k}_2')/2$, $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$, and $\mathbf{K}' = \mathbf{k}_1' + \mathbf{k}_2'$. This means that we take a spin-isospin averaged two-body interaction and disregard that in principle the force may be somewhat different in the $S, T = 0, 1$ or $1, 0$ channels. It is important to remark that for a mean-field solution, the interaction can only be an effective one, very different from a bare nucleon-nucleon force. This is contrary to the usual gap equation for pairs, to be considered below, where, at least in the nuclear context, a bare force can be used as a reasonable first approximation.

We are now ready to study the solution of Eq. (1) for the critical temperature T_c^α, defined by the point where the eigenvalue equals 4μ. For later comparison, the deuteron (pair) wave function at the critical temperature is also deduced from Eqs. (2) and (9) to be

$$\phi(k) = -\frac{1 - 2f(\varepsilon)}{k^2/m - 2\mu} \lambda e^{-k^2/k_0^2} \int \frac{d^3k'}{(2\pi)^3} e^{-k'^2/k_0^2} \phi(k'), \tag{10}$$

where $\phi(k)$ is the relative wave function of two particles given by $\Psi_{12} \to \phi(|\frac{\mathbf{k}_1 - \mathbf{k}_2}{2}|)\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)$, and $\varepsilon = k^2/(2m)$. We also neglected the momentum dependence of the Hartree-Fock mean-field shift in Eq. (10). With Eq. (10), the critical temperature of pair condensation is obtained from the following equation:

$$1 = -\lambda \int \frac{d^3k}{(2\pi)^3} \frac{1 - 2f(\varepsilon)}{k^2/m - 2\mu} e^{-2k^2/k_0^2}. \tag{11}$$

Results for the critical temperature T_c^α. To determine the critical temperature for α-particle condensation, we have to adjust the temperature so that the eigenvalue of Eq. (1) equals 4μ. The result is shown in Fig. 2(a). To get an idea of how this converts into a density dependence, we use for the moment the free gas relation between the density $n^{(0)}$ of uncorrelated nucleons and the chemical potential, that is,

$$n^{(0)} = 4 \int \frac{d^3k}{(2\pi)^3} f(\varepsilon). \tag{12}$$

We are well aware of the fact that this is a relatively gross simplification, for instance, at the lowest densities, and we intend to generalize our theory in the future so that correlations are included in the density. The two open constants λ and k_0 in Eq. (9) are determined so that binding energy (−28.3 MeV) and radius (1.71 fm) of the free ($f_i = 0$) α particle come out right. The adjusted parameter values are $\lambda = -992$ MeV fm^3,

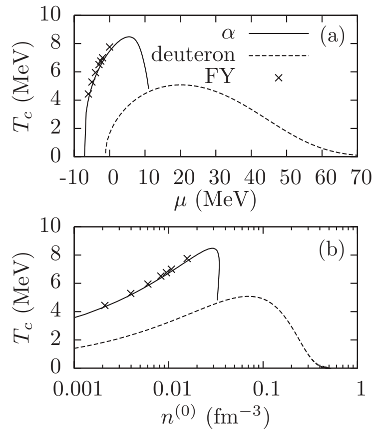

FIG. 2. Critical temperature of α and deuteron condensations as functions of (a) chemical potential and (b) density of free nucleon, derived from Eq. (4) for the α particle and Eq. (11) for the deuteron. Crosses (×) correspond to calculations of Eq. (1) with the Malfliet-Tjon interaction (MT I-III) using the FY method.

and $b = 1.43$ fm^{-1}. The results of the calculation are shown in Fig. 2.

In Fig. 2, the maximum of critical temperature $T_{c,\max}^\alpha$ is at $\mu = 5.5$ MeV, and the α condensation can exist up to $\mu_{\max} = 11$ MeV. It is very remarkable that the obtained results for T_c^α agree very well with the exact solution of Eq. (1) using the Malfliet-Tjon interaction (MT I-III) [19] with the FY method shown by crosses in Fig. 2 (the numerical solution only could be obtained for negative values of μ). This indicates that T_c^α is essentially determined by the Pauli blocking factors. These results for T_c^α are about 25% higher than the ones of our earlier publication [8]. We, however, checked that the underlying radius of the α particle in that work is larger than the experimental value and that T_c^α decreases with increasing radius of α particle. Furthermore, a different variational wave function was used in Ref. [8].

In Fig. 2 we also show the critical temperature for deuteron condensation derived from Eq. (11). In this case, the bare force is approximated with $\lambda = -1305$ MeV fm^3 and $k_0 = 1.46$ fm^{-1} to get the experimental energy (−2.2 MeV) and radius (1.95 fm) of the deuteron. It is seen that at higher densities, deuteron condensation wins over the one of α particle. The latter breaks down rather abruptly at a critical positive value of the chemical potential. Roughly speaking, this corresponds to the point where the α particles start to overlap. This behavior stems from the fact that Fermi-Dirac distributions in the four-body case, see Eq. (1), can never become step-like, as in the two-body case, even not at zero temperature, since the pairs in an α particle are always in motion. As a consequence, α condensation generally only exists as a BEC phase and the weak coupling regime is absent.

Figure 3 shows the normalized self-consistent solution of the wave function in momentum space derived from Eq. (8) and the wave function in position space defined by its Fourier transform $\tilde{\varphi}(r)$. Figures 3(a1) and 3(b1) are the wave functions of the free α particle. As discussed in the introduction, the wave function resembles a Gaussian, and this shape is approximately maintained as long as μ is negative, see Fig. 3(a2). On the

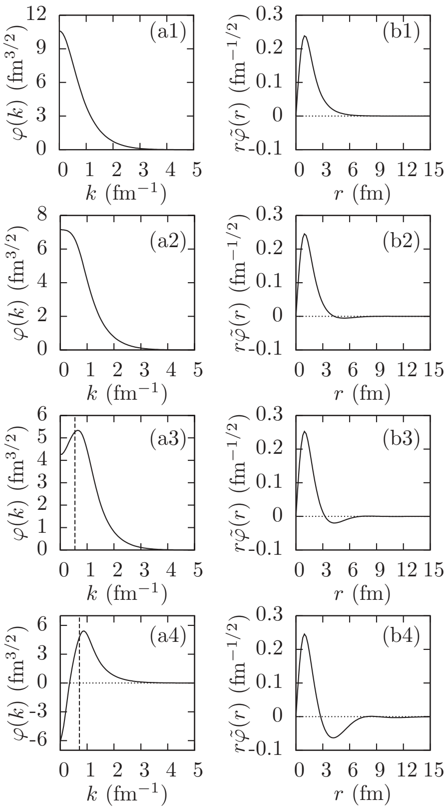

FIG. 3. Single-particle wave functions in (a) momentum space $\varphi(k)$ and (b) in position space $r\tilde{\varphi}(r)$ at critical temperature, Eq. (8). From top to bottom: (1) $\mu = -7.08$ MeV, $T_c = 0$ MeV, $n = 0$ fm^{-3}; (2) $\mu = -2.22$ MeV, $T_c = 6.61$ MeV, $n = 9.41 \times 10^{-3}$ fm^{-3}; (3) $\mu = 6.17$ MeV, $T_c = 8.45$ MeV, $n = 3.07 \times 10^{-2}$ fm^{-3}; and (4) $\mu = 10.6$ MeV, $T_c = 5.54$ MeV, $n = 3.34 \times 10^{-2}$ fm^{-3}. Figures (a1) and (b1) correspond to the wave functions for a free α particle. The vertical lines in (a3) and (a4) are at the Fermi wavelength $k_F = \sqrt{2m\mu}$.

contrary, the wave function of Fig. 3(a3), where the chemical potential is positive, has a dip around $k = 0$, which is due to the Pauli blocking effect. For the even larger positive chemical potential of Fig. 3(a4), the wave function develops a node. This stems from the structure of the wave function, derived in Eq. (4) from where one can realize that again this is a consequence of the Pauli blocking factor. The maximum of the wave function shifts to higher momenta and follows the increase of the Fermi momentum k_F, as indicated on Fig. 3. From a certain point on, the denominator in Eq. (8) develops a zero, and no self-consistent solution can be found any longer.

On the other hand, the wave functions in position space in Figs. 3(b2), 3(b3), and 3(b4) develop an oscillatory behavior, as the chemical potential increases. This is reminiscent to what happens in BCS theory for the pair wave function in position space [20].

Discussion and conclusions. In this work, we again studied the critical temperature of α-particle (quartet) condensation in homogeneous symmetric nuclear matter. We essentially confirmed the behavior of two previous studies [8,12]. The objective of the paper was to show that practically the same results as before can be obtained with a strongly simplifying ansatz for the four-particle wave function. Namely, this time, we used a momentum-projected mean-field variational wave function. This is based on the fact that the four different fermions of the quartet can occupy the same single-particle $0S$ wave function in the mean field. The latter is to be determined from a self-consistent nonlinear HF type of equation as a function of chemical potential or density. The relation between the chemical potential and density is taken from the free Fermi gas relation, Eq. (12). However, the total nucleon density of the system must be calculated from single nucleon occupation numbers including correlations, so that the contribution of bound states to the total nucleon density is taken into account, see Ref. [21]. To calculate the critical temperature not as function of the free nucleon density, see Fig. 2(b), but of the total nucleon density, a generalization *à la* NSR [13] must be performed; that is, we have at least to incorporate the contribution of the α-particle density including the condensate to the single-particle occupation numbers. This shall be investigated in future work.

Besides, in this work, we used the isospin-independent separable potential, Eq. (9), for the effective two-body interaction as a simplification.

The self-consistent wave function has been studied in momentum and position space. For negative chemical potential, the single-particle wave function behaves like a Gaussian. However, once the chemical potential turns positive, then the single-particle wave function in r space starts to oscillate. This is a well-known feature from ordinary pairing.

We, therefore, have demonstrated that a very simplifying momentum-projected mean-field ansatz suffices to account for the salient features of quartet condensation. This is very helpful for the next step, which is more complicated, i.e., the incorporation of quartet condensation self-consistently into the equation of state.

We should, however, be aware of the fact that our projected mean-field ansatz for the quartet wave function can only be a valid approximation as long as well-defined quartets exist. In the breakdown region seen in Fig. 2, this is certainly no longer the case. How the quartet phase evolves into a superfluid phase of pairs is an open question.

The success of our study in employing a very simplifying ansatz of the mean-field type for the quartet wave function may open wide perspectives. Besides pushing the description of quartet condensation much further, there exists the possibility that even for the case of a gas of trions, such a projected mean-field ansatz is a quite valid approach. In the case of three colors, such as quarks in the constituent quark model for nucleons, a harmonic confining potential is frequently assumed, and the three quarks can occupy the lowest $0S$ state, analogous to the case of quartets treated in the present paper. Of course, trions are composite fermions and cannot be treated in the same way as bosonic composites, since they form a new Fermi gas with their own new Fermi level. How this situation can eventually be treated has recently been outlined in Ref. [22].

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