

**Multipole mixing ratios of transitions in  $^{11}\text{B}$** G. Rusev,<sup>1,2</sup> A. P. Tonchev,<sup>1,2</sup> R. Schwengner,<sup>3</sup> C. Sun,<sup>1,4</sup> W. Tornow,<sup>1,2</sup> and Y. K. Wu<sup>1,4</sup><sup>1</sup>*Department of Physics, Duke University, Durham, North Carolina 27708, USA*<sup>2</sup>*Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708, USA*<sup>3</sup>*Institut für Strahlenphysik, Forschungszentrum Dresden-Rossendorf, D-01314 Dresden, Germany*<sup>4</sup>*Duke Free-Electron Laser Laboratory, Durham, North Carolina 27708, USA*

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The mixing ratios of  $M1$  and  $E2$  radiation for transitions in  $^{11}\text{B}$  have been determined by measuring the azimuthal asymmetry of the radiation emitted from levels populated by resonant absorption of polarized photons. The photon-scattering experiments were carried out at the Free-Electron Laser Laboratory at Duke University using nearly monoenergetic and linearly polarized photon beams. The mixing ratios were deduced from a comparison of the measured azimuthal asymmetries with calculations for the angular distributions of mixed transitions.

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Gamma-ray transitions in  $^{11}\text{B}$  are frequently used for flux calibration purposes in photon-scattering experiments with bremsstrahlung beams because of their large decay widths. The small number of strong deexcitations in  $^{11}\text{B}$ , distributed over a wide energy range, makes this nucleus a convenient calibration standard in experiments aimed at determining the dipole strength in nuclei up to the neutron separation energy. The three transitions in  $^{11}\text{B}$  at 4.444, 5.019, and 8.916 MeV, resulting from the deexcitation of states with  $J^\pi = 5/2^-, 3/2^-, 5/2^-$ , respectively, to the  $3/2^-$  ground state, are known to have mixed  $M1$  and  $E2$  radiations [1,2] from comparison of measurements and distorted-wave-Born approximation calculations. The angular distribution for these transitions depends on the contribution of dipole and quadrupole radiation defined by the mixing ratio  $\delta$ . Measurements of  $\delta$  are important for the correct determination of the photon flux if  $^{11}\text{B}$  is used as a calibration standard.

The multipole mixing ratio of the transitions at 4.444 and 5.019 MeV were measured by Bell *et al.* [1] using the  $^{12}\text{C}(t, \alpha)^{11}\text{B}$  reaction. The values for  $\delta$  obtained from the standard particle- $\gamma$ -ray correlation technique are  $\delta(4.444) = -0.19(3)$  and  $\delta(5.019) = +0.03(5)$ , respectively, applying the Rose-Brink phase convention [3]. The angular distribution of the transition at 8.916 MeV has been measured by Comsan *et al.* [2] using deuteron scattering on a  $^{12}\text{C}$  target. A value of  $\delta(8.916) = -0.11(4)$  has been deduced for the multipole mixing ratio without an explicit definition of the phase convention used.

In this Brief Report, we present the results of photon-scattering experiments on  $^{11}\text{B}$  with linearly polarized and nearly monoenergetic beams. The measured asymmetry of the scattered  $\gamma$  rays was used to derive the mixing ratio  $\delta$  in a model-independent way. This technique was outlined in Ref. [4] and also applied in Ref. [5] for branching transitions in the even-even nucleus  $^{164}\text{Dy}$ . The measurements were carried out at the High-Intensity  $\gamma$ -Ray Source (HI $\gamma$ S) facility at the Free-Electron Laser Laboratory, Duke University. Nearly monoenergetic photon beams were produced by Compton backscattering of a high-intensity Free-Electron Laser (FEL)

beam with an intense electron beam in the Duke storage ring. Presently, the energy of the backward-scattered photons can be tuned in a wide energy range, from about 1 to 100 MeV, by changing the energy of the electron beam and the FEL wavelength [6]. The polarization of the FEL photons, defined by the magnetic field of the undulators, is mostly preserved during the Compton backscattering because of a negligible recoil effect, leading to the production of intense photon beams with nearly 100% polarization. The HI $\gamma$ S facility is described in detail in Ref. [6].

The photon beam was collimated by a lead collimator with a length of 30.5 cm and a cylindrical hole with a diameter of 1.9 cm. The energy distribution of the incident photon beam was measured with a large volume high-purity germanium (HPGe) detector placed in the beam. The efficiency of the detector was 123% relative to the  $3 \times 3$  in. NaI scintillator detector. Three blocks of copper with thicknesses of 8 cm each were placed in the beam to reduce its intensity. A spectrum of the photon beam at 4.44 MeV is presented in Fig. 1(a). The spectrum was corrected for detector response, full-energy peak efficiency, measuring time, and attenuation in the copper blocks.

The beam irradiated the target positioned in an evacuated plastic tube made of polymethyl meta-acrylic. The average photon flux density was typically of  $2 \times 10^7 \text{ s}^{-1} \text{ cm}^{-2}$ . The scattered  $\gamma$  rays were detected with four HPGe detectors each with 60% efficiency. The HPGe detectors were positioned in a plane perpendicular to the beam ( $\theta = 90^\circ$ ) and at azimuthal angles,  $\phi = 0^\circ, 90^\circ, 180^\circ$ , and  $270^\circ$ , such that two of the detectors were in the horizontal plane and the other two were in the vertical plane. The distance from the center of the target to the front surface of the detectors was 10 cm. All detectors were equipped with passive shields made of 3 mm copper and 2 cm thick lead cylinders. Lead and copper absorbers with thicknesses of 5 and 3 mm, respectively, were placed in front of the detectors. In the experiments, we used a target made of metallic  $^{11}\text{B}$  with an enrichment of 99.50% and a thickness of  $392 \text{ mg/cm}^2$ . The boron powder was placed in a cylindrical plastic container.

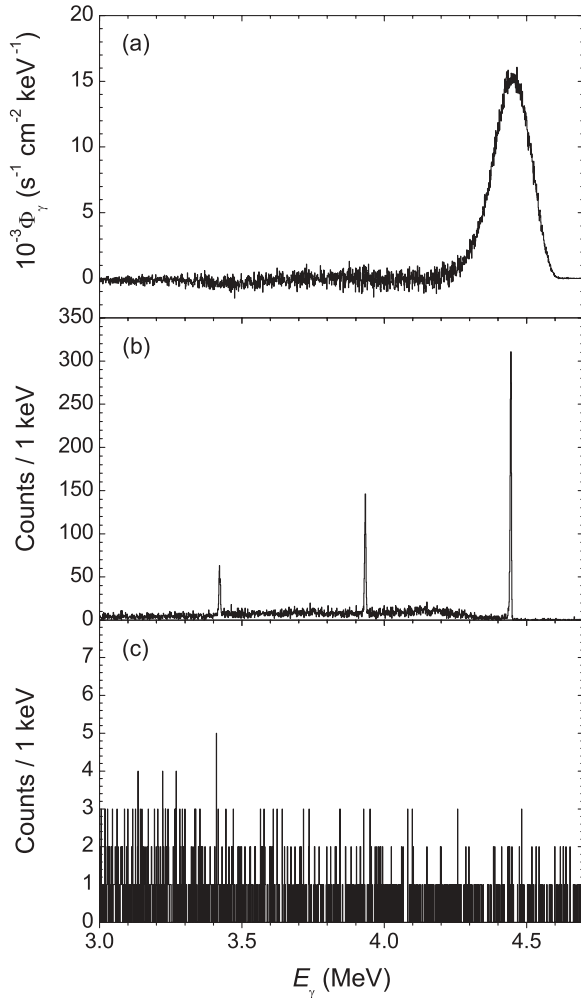


FIG. 1. Spectra from the measurement on  $^{11}\text{B}$  at 4.44 MeV photon-beam energy. (a) Energy distribution of the photon flux,  $\Phi_\gamma$ , of the incident beam, (b) scattered  $\gamma$  rays from  $^{11}\text{B}$  in the horizontal plane, and (c) background from the plastic container of the  $^{11}\text{B}$  powder with the same incident beam. The transition in  $^{12}\text{C}$  at  $E_\gamma = 4.437$  MeV, which can contaminate the peak at  $E_\gamma = 4.444$  MeV in  $^{11}\text{B}$ , is not seen in Panel (c). The spectra presented in Panels (b) and (c) were measured at similar photon fluxes.

The azimuthal asymmetry of the scattered  $\gamma$  rays was determined according to the relation

$$a = \frac{A_h/t_h - (\varepsilon_h/\varepsilon_v)A_v/t_v}{A_h/t_h + (\varepsilon_h/\varepsilon_v)A_v/t_v}, \quad (1)$$

where  $A_h$  and  $A_v$  are the registered  $\gamma$  rays for measuring times  $t_h$  and  $t_v$  in the horizontal plane and the vertical plane, respectively. The ratio of the full-energy peak efficiencies,  $\varepsilon_h/\varepsilon_v$ , of the horizontal and vertical detectors was deduced from the weighted mean of the values obtained for the transitions in  $^{56}\text{Co}$  with energies above 2 MeV to eliminate any dependence on the thickness of the lead and copper absorbers. For this purpose, a  $^{56}\text{Co}$  source was placed in the target position. This measurement resulted in a value of 1.011(10). The asymmetry is related to the analyzing power  $\Sigma(\theta, \phi)$  and

the degree of polarization of the photon beam  $P_\gamma$  by

$$\begin{aligned} a &= C(\theta)P_\gamma \Sigma(\theta, \phi) \\ &= C(\theta)P_\gamma \frac{W(90^\circ, 0^\circ) - W(90^\circ, 90^\circ)}{W(90^\circ, 0^\circ) + W(90^\circ, 90^\circ)}, \end{aligned} \quad (2)$$

where  $W(\theta, \phi)$  is the angular distribution of the emitted  $\gamma$  rays. It depends on the angular momenta of the excited state and the ground state, the type of transition, and the mixing ratio  $\delta$ . The correction factor  $C(\theta)$  is due to the finite opening angle of the detectors. We determined the value of  $C(\theta)$  by means of GEANT3 simulations for the detector setup. In the simulations the  $\gamma$  rays were emitted randomly from the target with an energy of 10 MeV and the direction was given by  $W(\theta, \phi)$ . We obtained values for  $C(\theta)$  of 0.950 and 0.941 for the spin sequences  $3/2 \rightarrow 5/2 \rightarrow 3/2$  and  $3/2 \rightarrow 3/2 \rightarrow 3/2$ , respectively. The degree of polarization of the photon beam for this particular experiment was determined to be nearly 100%. From the measured asymmetry of the scattered  $\gamma$  rays, we determined the mixing ratio by satisfying Eq. (2).

The angular distribution of a  $\gamma_2$  ray deexciting a level with angular momentum  $J_2$  to a level  $J_3$ , relative to an absorbed  $\gamma_1$  ray with a given direction of the vector of polarization exciting the level  $J_2$  from a level  $J_1$  with a transition of type  $L_1$ , can be calculated according to the relation [4,7,8]

$$\begin{aligned} W(\theta, \phi) &= \sum_{\lambda=0,2,4} B_\lambda(\vec{\gamma}_1)A_\lambda(\gamma_2)P_\lambda(\cos\theta) \\ &+ (\pm)_{L_1} \cos(2\phi) \sum_{\lambda=2,4} B'_\lambda(\vec{\gamma}_1)A_\lambda(\gamma_2)P_\lambda^{(2)}(\cos\theta), \end{aligned} \quad (3)$$

where  $P_\lambda(\cos\theta)$  and  $P_\lambda^{(2)}(\cos\theta)$  are Legendre polynomials and associated Legendre polynomials, respectively. The symbol  $(\pm)_{L_1}$  is +1 for an electric transition and -1 for a magnetic transition. The expansion coefficients  $A_\lambda$ ,  $B_\lambda$ , and  $B'_\lambda$  are given in the Krane-Steffen phase convention in Refs. [4] and [9]. The multipole mixing ratios for the transition rays  $\gamma_1$  and  $\gamma_2$  are  $\delta_1$  and  $\delta_2$ , respectively. For the case of deexcitation to the ground state these mixing ratios refer to the same transition in absorption ( $\delta_1$ ) and emission ( $\delta_2$ ), respectively. We used the relation  $\delta_1 = \delta_2$  in Eq. (3).

We performed measurements on  $^{11}\text{B}$  at photon-beam energies appropriate to populate the levels at 2.125, 4.445, 5.020, 7.286, and 8.920 MeV. The transition at 2.125 MeV has an isotropic angular and, hence, the analyzing power and the asymmetry are zero. A mixing ratio cannot be determined in this case. Spectra from the measurement at a beam energy of 4.44 MeV are shown in Fig. 1. The summed spectrum from  $^{11}\text{B}$  measured for 3251 s with the two detectors in the horizontal plane is shown in Fig. 1(b). The  $^{11}\text{B}$  powder was placed in a plastic container that contained  $^{12}\text{C}$  and therefore the spectrum from  $^{11}\text{B}$  may be contaminated by the ground-state transitions of the first  $2^+$  state in  $^{12}\text{C}$  at 4.439 MeV. Because the  $E2$  transitions in even-even nuclei had the largest intensity in the horizontal plane, we repeated only the measurement with the target container. The spectrum from the target container taken for 3273 s and the same beam intensity is presented in Fig. 1(c). It shows that an additional

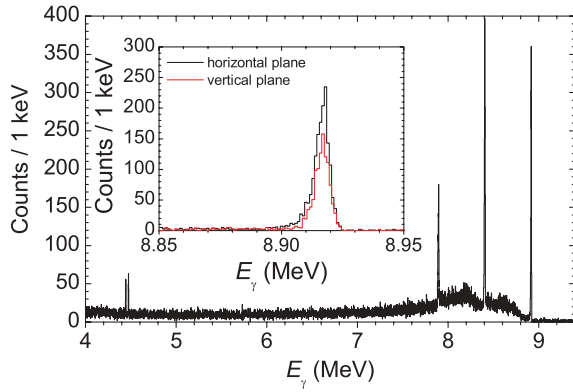


FIG. 2. (Color online) A spectrum from  $^{11}\text{B}$  measured for beam energy of 8.02 MeV with all four detectors. (Insert) A comparison of the spectra of  $\gamma$  rays at  $E_\gamma = 8.916$  MeV scattered in the horizontal plane (black) and in the vertical plane (red). The incident photons were linearly polarized in the horizontal plane.

correction for the intensity of the  $2^+$  ground-state transition in  $^{12}\text{C}$  is not required. All measurements were carried out long enough to obtain about 1000 counts in the peak area to reduce the statistical uncertainty. A comparison of spectra measured in the horizontal and vertical planes at a beam energy of 8.92 MeV is presented in Fig. 2, clearly showing the observed asymmetry.

The results of the measured asymmetry are presented in Table I. Using the predicted analyzing power and varying  $\delta$  to fulfill Eq. (2) numerically, the multipole mixing ratio was determined for the transitions at 4.444, 5.019, 7.283, and 8.916 MeV in  $^{11}\text{B}$  by comparing to the measured asymmetry.

TABLE I. Results for the observed asymmetries and the deduced mixing ratios for transitions in  $^{11}\text{B}$ .

$E_\gamma^a$ (keV)	$B^b$ (%)	$a^c$	$\delta^d$	$\delta^e$
Ground-state transitions				
2124.47	100	-0.008(27)		
4443.93	100	0.026(19)	$+0.158^{+0.025}_{-0.021}$	$+0.19(3)^f$
5019.08	85.8(4)	0.200(16)	$-0.036^{+0.013}_{-0.013}$	$-0.03(5)^f$
7282.92	88.4(3)	-0.213(30)	$+0.001^{+0.022}_{-0.021}$	
8916.3	97.3(1)	0.215(20)	$0.000^{+0.014}_{-0.014}$	$-0.11(4)^g$
Branching transitions				
2264.9	6.3(4)	-0.181(80)	$+0.028^{+0.073}_{-0.075}$	
2840.23	5.3(4)	0.177(66)	$-0.081^{+0.164}_{-0.126}$	
2895.21	14.2(4)	-0.37(5)	$-0.19^{+0.10}_{-0.17}$	$-0.05(20)^f$
4474.3	2.7(1)	-0.188(7)	$-0.061^{+0.025}_{-0.022}$	

<sup>a</sup>Transition energy taken from Ref. [10].

<sup>b</sup>Branching ratio.

<sup>c</sup>Asymmetry calculated according to Eq. (1).

<sup>d</sup>Deduced mixing ratio for emission in Krane-Steffen phase convention according to  $\delta_2$ .

<sup>e</sup>Mixing ratio known from literature given in the phase convention of Krane and Steffen.

<sup>f</sup>Taken from Ref. [1].

<sup>g</sup>Taken from Ref. [2].

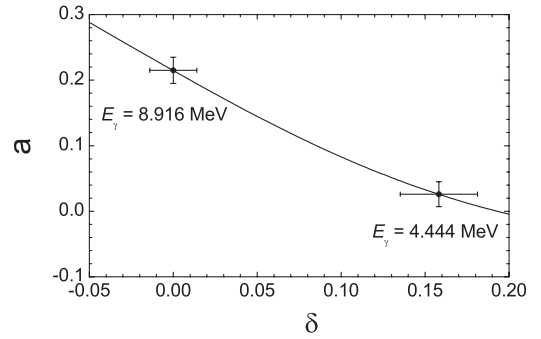


FIG. 3. Plot of the predicted asymmetry versus the mixing ratio for  $M1/E2$  radiation calculated for the  $3/2 \rightarrow 5/2 \rightarrow 3/2$  spin sequence (solid curve). The measured asymmetries for the transitions in  $^{11}\text{B}$  at 4.444 and 8.916 MeV are shown with the data points on the calculated line. The statistical uncertainty of the asymmetry is translated to the uncertainty of  $\delta$  presented with horizontal bars.

The vanishing mixing ratio of the 7.283 MeV transition proves the present method because even a very small  $M2$  admixture to this  $E1$  transition leads to an exceptionally large  $M2$  transition strength and is therefore unlikely (for example, an admixture of 0.04% results in  $B(M2) = 0.26$  W.u.). An example is shown in Fig. 3 for the spin sequence of  $3/2 \rightarrow 5/2 \rightarrow 3/2$  for the transitions at 4.444 and 8.916 MeV. The uncertainty of  $\delta$  was derived using the same approach for values of the asymmetry  $a + \Delta a$  and  $a - \Delta a$ , where  $\Delta a$  is the uncertainty of the asymmetry. The multipole mixing ratios obtained using the present approach are compared with those of the literature in Table I. We found good agreement of  $\delta$  with values given in previous work for the transitions at 4.444 and 5.019 MeV, while the transition at 8.916 MeV appears to have no or very small mixing within the uncertainties in contrast to the result obtained in Ref. [2]. The expected asymmetry as a function of  $\delta$  is shown in Fig. 4. There are two solutions of  $\delta$  for each of the observed asymmetries. We have chosen the  $\delta$  with the smaller absolute value because the second solution leads to  $E2$  admixtures that are unusually large compared with known values [11]. For example, the second solution for the

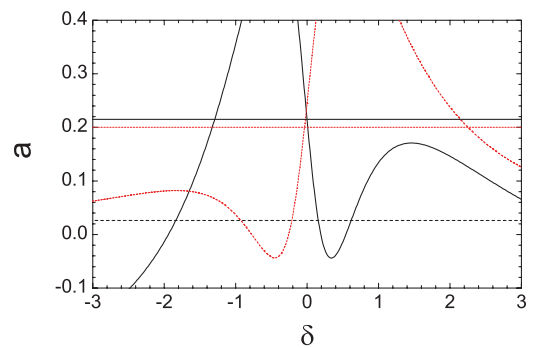


FIG. 4. (Color online) Plot of the predicted asymmetry versus the mixing ratio  $\delta$  calculated for the  $3/2 \rightarrow 5/2 \rightarrow 3/2$  spin sequence (solid black curve) and for the  $3/2 \rightarrow 3/2 \rightarrow 3/2$  spin sequence (dotted red curve). The measured asymmetries for the transitions in  $^{11}\text{B}$  at 4.444, 5.019, and 8.916 MeV are shown with dashed black, dotted red, and solid black lines, respectively.

8.916 MeV transition is  $\delta = -1.30$ , corresponding to an  $E2$  admixture of 63%.

Applying the same approach, we can deduce the multipole mixing ratio for the branching transitions from the levels at 5.020 and 8.920 MeV to the levels at 2.125 and 4.445 MeV, respectively, and from the level at 7.286 MeV to the levels at 4.445 and 5.020 MeV. For this purpose, we used the ratios  $\delta$  already obtained for the ground-state transitions given in Table I. Some of the branching transitions correspond to spin combinations of  $3/2 \rightarrow 3/2 \rightarrow 1/2$  and  $3/2 \rightarrow 5/2 \rightarrow 5/2$ . We obtained values of 0.969 and 0.982, respectively, for the correction factor  $C(\theta)$  in Eq. (2). The results for the multipole mixing ratios deduced for the branching transitions are given in Table I. The uncertainty  $\Delta\delta_1$  of the mixing ratios of the excitation transition was propagated in the uncertainty of  $\delta_2$  by calculating the predicted asymmetries for  $\delta_1 + \Delta\delta_1$  and  $\delta_1 - \Delta\delta_1$ .

In addition to the mixing ratio  $\delta$ , we can obtain the branching ratios for the observed transitions in  $^{11}\text{B}$  according to the relation

$$B_1 = \frac{A_1/(\varepsilon_1 W_1(90^\circ))}{A_1/(\varepsilon_1 W_1(90^\circ)) + A_2/(\varepsilon_2 W_2(90^\circ))}. \quad (4)$$

Here,  $A_1$  and  $A_2$  are the full-energy peak areas of the two transitions deexciting a given level. A sum spectrum of all

four detectors was used in the analysis to have higher statistics in the peaks (cf. Fig. 2). It corresponds to a spectrum measured with an unpolarized photon beam; therefore, the angular distribution  $W(\theta)$  is calculated from the first term of Eq. (3). The relative efficiencies  $\varepsilon_1$  and  $\varepsilon_2$  were deduced from the same GEANT3 simulations used to determine the coefficient  $C(\theta)$ . The deduced branching ratios are given in Table I.

Summarizing, we have presented an approach to determine multipole mixing ratios using a nuclear resonance fluorescence technique with linearly polarized photon beams. The measured asymmetries of the intensities of the scattered  $\gamma$  rays in the plane of polarization and perpendicular to the plane of polarization were compared with predictions for the analyzing power for various  $\delta$  leading to a direct measure of  $\delta$  without applying additional models. Measurements on  $^{11}\text{B}$  carried out at the HI $\gamma$ S facility revealed mixing ratios for the transitions at 4.444 and 5.019 that are consistent with literature values, whereas 8.916 MeV appears to be a pure  $M1$  transition. The method was proven by the results for the transitions at 2.125 and 7.283 MeV, which are isotropic and pure  $E1$  transitions, respectively.

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