Pion cross section parametrizations for intermediate energy, nucleus-nucleus collisions

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Space radiation and cosmic ray transport codes require simple and accurate models for hadron production in intermediate energy, nucleus-nucleus collisions. Several arithmetic parametrization models for pion production are compared to laboratory frame data. It is found that models based on high energy parametrizations are unable to describe intermediate energy, differential cross section data. However, simple thermal model parametrizations, when appropriately transformed from the center of momentum to the laboratory frame, are able to account for the data. Heavy ion transport codes that require algebraic cross section formulas can therefore use arithmetic parametrizations at high energy, but should use thermal model parametrizations at intermediate energy.

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There is considerable interest in providing arithmetic parametrizations of cross sections for pion production in high energy proton-proton [1–8] and nucleus-nucleus collisions [1–3,9–11]. Such formulas are very suitable for input into particle transport codes, which are able to run quickly and efficiently, if the cross sections are provided in a simple, algebraic form. These are used in cosmic ray physics [12–14], astrophysics [15–20], nuclear physics [21,22], simulations of particle physics experiments [23,24], and space radiation problems [25–27]. Consider the following issues. First, the peak of the cosmic ray spectrum occurs, accidentally, near the pion production threshold [28] at around 300 MeV. We shall refer to the intermediate energy region as being the energy above the pion threshold extending up to the GeV region. Second, transport codes that are used for cosmic ray [12–14] and space radiation applications [25–27] therefore require cross sections that are accurate in the intermediate energy region. Third, transport codes also require cross sections calculated in the laboratory frame, because the Boltzmann transport equation or Monte-Carlo simulation is usually set up in such a way that projectiles are transported through a stationary target. Cross sections are usually calculated in the nucleon-nucleon center of momentum frame. Although the techniques for Lorentz transforming to the laboratory frame are well known, one must deal with extra complications such as double valued functions of angles [29] and the fact that cross sections in the laboratory frame are much more complicated than their relatively simple counterparts in the center of momentum (cm) frame. Fourth, space radiation, heavy ion experiment simulations, heavy ion therapy, and some cosmic ray applications require the transport of heavy nuclei in the intermediate energy region. In summary, it is useful to provide cross section parametrizations for intermediate energy, nucleus-nucleus collisions evaluated in the laboratory frame. Such cross sections for pion production are the subject of the present work.

The main purpose of this Brief Report is to discuss the applicability of arithmetic cross section parametrizations for use in transport codes, where one is interested in heavy ion transport through a finite medium, such as an atmosphere or dense material. The cross section parametrizations described below work well for high energy nucleon-nucleon collisions, and it might be tempting to multiply them by nuclear mass number factors [1–3,9,10] for use in lower energy nucleus-nucleus collisions, as input for heavy ion transport codes. As the article reports, the parametrizations, in fact, do not work well when extended down to intermediate energy nucleus-nucleus collisions. Thermal models work much better. Therefore, it is recommended that heavy ion pion production cross section inputs to transport codes use arithmetic parametrizations at high energy, but thermal models at intermediate energy. We now discuss the details.

Several high energy parametrizations have been studied previously [1–8]. For example, Badhwar, Stephens, and Golden [3] have parametrized the Lorentz-invariant differential cross section for pion production in proton-proton collisions as

$$E\frac{d^3\sigma}{d^3p} = \frac{A}{\left(1+4m_p^2/s\right)^r}(1-\tilde{x})^q \exp\left[\frac{-Bp_T}{1+4m_p^2/s}\right], \quad (1)$$

where A, B, and r are constants, E is the particle energy, m_p is the proton mass, s is the Mandelstam variable giving the square of the total energy in the cm frame, and p_T is the transverse momentum of the produced meson. The other term is $\tilde{x} \equiv [x_F^2 + \frac{4}{s}(p_T^2 + m_\pi^2)]^{1/2}$, where x_F is the Feynman scaling variable and m_π is the pion mass. (Badhwar uses the symbol $x_\parallel^*$ in place of x_F). Also $q \equiv \frac{C_1+C_2p_T+C_3p_T^2}{\sqrt{1+4m_p^2/s}}$, where C_1, C_2, and C_3 are constants. All constants are listed in Table I. Other similar parametrizations, by Alper *et al.* [5], Ellis and Stroynowski [6], Mokhov and Striganov [7], and Carey *et al.* [8], are discussed elsewhere [1–8] and are not repeated here.

Norbury and Townsend [2] have recently compared the parametrizations of Badhwar, Stephens, and Golden [3], Alper *et al.* [5], Ellis and Stroynowski [6], Mokhov and Striganov [7], and Carey *et al.* [8] to the full set of proton-proton data recently measured [30] at a beam momentum of 158 GeV. In general, the parametrizations fit the data very well. Norbury and Townsend [2] also multiplied the proton-proton cross sections by a simple nuclear scaling factor and were also able to accurately fit the new proton-carbon data [31]. The fact

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TABLE I. Constants for the Badhwar parametrization [3,4]. Units for A, C_2, and C_3 are mb/GeV^2, GeV^{-1}, and GeV^{-2}, respectively, and other constants are dimensionless.

Particle	A	B	r	C_1	C_2	C_3
π^+	153	5.55	1	5.3667	−3.5	0.8334
π^-	127	5.3	3	7.0334	−4.5	1.667
π^0	140	5.43	2	6.1	3.3	0.6

that the proton-proton parametrizations were also able to be used for a proton-nucleus reaction holds out the hope that they can be used for nucleus-nucleus reactions in general. Given the success of the proton-carbon fit [2], one would certainly expect this to be the case at high energy. The question arises as to how low in energy these parametrizations might be taken. It would be very useful if they also worked in the intermediate energy region. This is tested in the present work. However, one would expect that high energy parametrizations, which are based on scaling hypotheses, may not work well in the intermediate energy region, and indeed, we find this to be true.

Experimental data for negative pion production in the Ar + KCl nuclear collisions at 800 MeV/nucleon have been measured by Nagamiya *et al.* [32]. They measured Lorentz-invariant differential cross sections as a function of transverse momentum and laboratory angle. The Badhwar parametrization [3] is scaled up to nucleus-nucleus collisions by multiplying the nucleon-nucleon cross section by $(A_P A_T)$, where A_P and A_T are the projectile and target nucleon numbers, respectively. This is then Lorentz transformed to the laboratory frame. The results are compared to the data of Nagamiya *et al.* [32] in Fig. 1 (solid lines). It can be seen that the scaled Badhwar parametrization fails to describe the data. Even if the scaling factor is changed to an arbitrary constant, the agreement with the spectral shapes is still very poor. The other parametrizations of Alper *et al.* [5], Ellis and Stroynowski [6], Mokhov and Striganov [7], and Carey *et al.* [8] have been treated in the same fashion as the Badhwar parametrization and compared to the data. Due to space considerations, we do not show the results here, but all of them also fail to describe the data, with comparisons of similar quality to the Badhwar comparison. As discussed previously, this failure of arithmetic parametrizations is not unexpected. They were developed to describe high energy nucleon-nucleon collisions in the ultrarelativistic limit, whereas the data of Nagamiya *et al.* [32] are for intermediate energy nucleus-nucleus collisions.

Several authors [32–35] have shown that a simple thermal model parametrization is able to describe both pion and kaon data in intermediate energy nucleus-nucleus collisions. In the nucleon-nucleon cm frame, this parametrization is given by [32]

$$E\frac{d^3\sigma}{d^3p} = N\exp(-T/E_0)\,, \qquad (2)$$

where T is the meson kinetic energy in the nucleon-nucleon center of momentum frame. (It is *not* the nucleus-nucleus cm frame.) The parameters N and E_0 are constants fitted to the data. These are different for protons, pions, and kaons. Some values of E_0 are listed in Refs. [32,33,35].

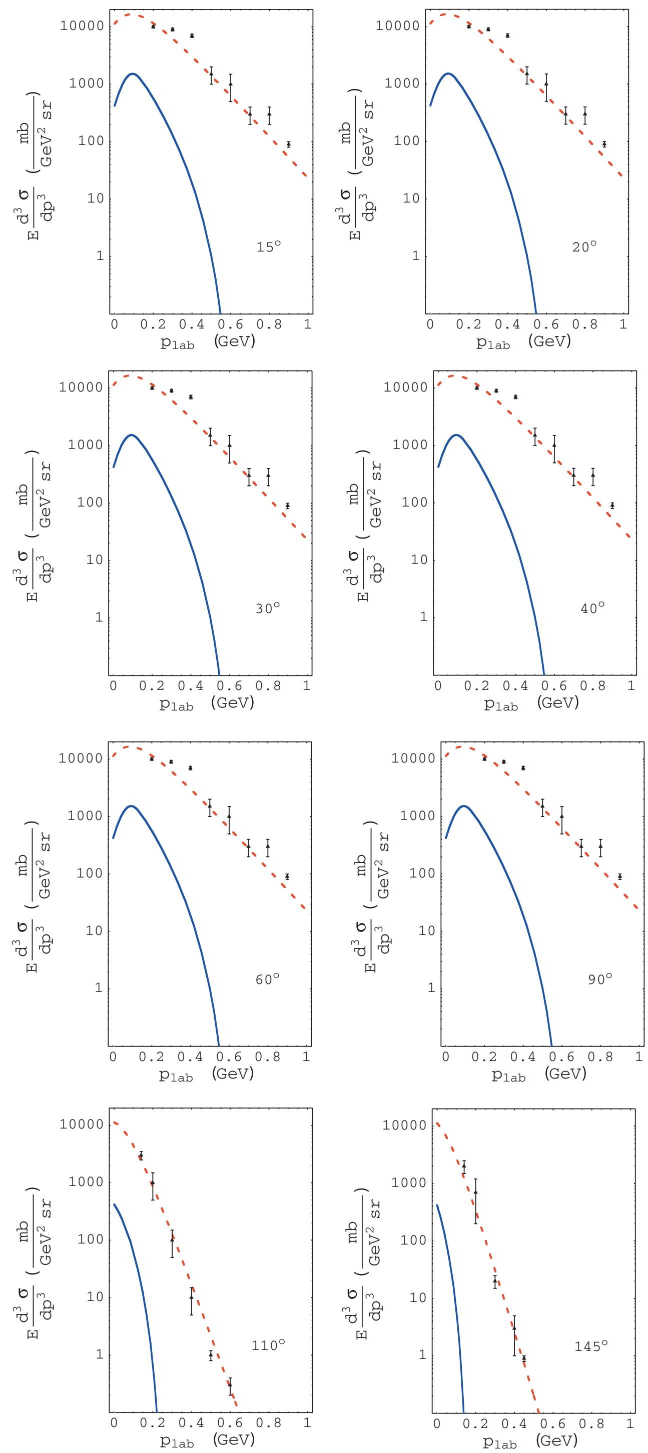

FIG. 1. (Color online) Inclusive π^- cross section for Ar + KCl collisions. Badhwar parametrization (solid line) multiplied by $(A_P A_T)$ fails to describe data [32] at 800 MeV/nucleon. Lab angles are indicated. Even if the multiplication factor is changed to an arbitrary constant, the theoretical shape does not match the shape of the data. Parametrizations of Refs. [5–8] are of similar poor quality. A thermal spectrum (dashed line) successfully describes data.

E_0 is different for different particles, such as pions and kaons. Equation (2) is characteristic of a thermal Boltzmann distribution and describes the "boiling" off of hadrons after nuclear equilibration has been reached. The pion thermal model parametrization, transformed to the laboratory frame, is also shown in Fig. 1 (dashed lines). The parameter N is fitted to the data with $N = 1.7 \times 10^4 \frac{\text{mb}}{\text{GeV}^2\ \text{sr}}$. The effective nuclear temperature, which is $E_0 = 66$ MeV, is taken from Ref. [32]. Comparison to experiment is very good. The rather complicated experimental distributions as a function of laboratory angle reveal themselves to be nothing more than a simple nucleon-nucleon center of momentum frame, thermal distribution appropriately transformed to the laboratory frame.

In Fig. 1 only a single temperature thermal fit is shown. Such thermal fits are also studied experimentally by Gosset *et al.* [36]. Hagedorn and Rafelski [37] first used a thermodynamic description of collisions using a statistical bootstrap model to calculate the temperature and average transverse momentum of the pion spectrum. Brockmann *et al.* [38] pointed out that the Hagedorn and Rafelski [37] model accurately predicted proton temperatures, but predicted pion temperatures that were much too high. Thus, Brockmann *et al.* [38] and Schwalb *et al.* [39] introduced a two-temperature model in which the pion spectra are fitted by two Boltzmann distributions, each with a different temperature. Such two-temperature fits are especially needed to describe heavy nuclear systems. Li and Bauer [40] provide a very nice discussion of the physics of such two-temperature models, where they point out that for heavy systems there are two phases corresponding to before and after the freeze-out time, which is the time after which most baryon collisions have ceased but excited baryons have not yet had time to decay [40]. Before freeze-out the dynamics are dominated by hot baryon-baryon collisions, with a high temperature Boltzmann distribution. Also produced are Δ and other resonances, most of which have not decayed but undergo equilibration. Brockmann *et al.* [38] and Bass *et al.* [41] link this second temperature to the Δ mass. After freeze-out when the system is cooler, the resonances decay and the resulting pions are characterized by a cooler Boltzmann distribution [40]. This two-step process [40] explains why two temperatures are needed for heavy systems, whereas only one-temperature fits work for light systems, in that a light system will have completely dissipated before the second temperature system equilibrates. Other experimental data supporting this can be found in Refs. [39] and [42]. Reviews of these and other models and data are provided by Wong [43] and Reisdorf *et al.* [44]. Poskanzer, Butler, and Hyde [45] and Zebelman *et al.* [46] discuss the modification of the Boltzmann spectra when Coulomb effects are considered between the outgoing charged pions and the residual nuclear system. The temperatures associated with the single-temperature fits for light systems and the two-temperature fits for heavy systems are also provided [36,38,40,41], as well as the mass dependence of the cross sections [44]. References [32,36,39,42,44] provide sufficient experimental data and parametrization constants that allow one to construct pion cross sections for use in heavy ion transport codes at intermediate energy. The cross section formulas are simple enough to allow transport codes to run quickly.

In summary, heavy ion transport requires accurate models describing hadron production in nucleus-nucleus collisions at high and intermediate energy. Nuclear models, such as the dual parton model [47], cascade model [48,49], thermal model [49,50], microscopic transport models [51,52], and molecular dynamics models [51,53], can be used as input into Monte Carlo codes, such as FLUKA [23] and MCNPX [54]. Alternatively, algebraic parametrizations can be used that significantly improve the speed at which space radiation and cosmic ray transport codes run [26]. The conclusions are as follows. The arithmetic parametrizations for nucleon-nucleon cross sections of references [1–3,9,10], when scaled by nuclear mass number factors, are suitable for transport code cross section inputs in the high energy region. However, such formulas fail to describe intermediate energy cross section data. Thermal model parametrizations [32,36,38–42,44] are more suitable for intermediate energy cross section inputs to transport codes. The main scientific advance of this Brief Report is that heavy ion transport codes using algebraic cross section formulas can use scaled nucleon-nucleon arithmetic parametrizations at high energy, but should use thermal model parametrizations at intermediate energy.

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