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Effect of medium dependent binding energies on inferring the temperatures and freeze-out density of disassembling hot nuclear matter from cluster yields

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We explore the abundance of light clusters in asymmetric nuclear matter at subsaturation density. With increasing density, binding energies and wave functions are modified due to medium effects. The method of Albergo, Costa, Costanzo, and Rubbino (ACCR) for determining the temperature and free nucleon density of a disassembling hot nuclear source from fragment yields is modified to include, in addition to Coulomb effects and flow, also effects of medium modifications of cluster properties, which become of importance when the nuclear matter density is above 10^{-3} fm^{-3}. We show how the analysis of cluster yields, to infer temperature and nucleon densities, is modified if the shifts in binding energies of in medium clusters are included. Although, at low densities, the temperature calculated from given yields changes only modestly if medium effects are taken into account, larger discrepancies are observed when the nucleon densities are determined from measured yields.

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I. INTRODUCTION

Understanding of nuclear matter at extreme conditions is one of key issues to clarify the problems in core-collapse supernovae as well as neutron stars and heavy-ion collisions. Altough heavy-ion reactions are often employed to explore the nuclear matter equation of state (EOS), careful theoretical work is needed to analyze the experimental signatures and to reconstruct the properties of hot and dense matter from the detected abundances and energy distributions of ejectiles or from correlation functions between different ejectiles produced in those reactions.

Among the interesting observables are the yield ratios of different fragments measured in such reactions. In many experiments one commonly observes light elements such as neutrons (n), protons (p), deuterons (d), tritons (t), ^{3}He (h), and ^{4}He (α) (see, for example, Ref. [1] and references therein). Larger clusters, typically with mass numbers $5 \leqslant A \leqslant 20$, are also observed and the production process of these fragments must be explained.

The decay of highly excited nuclear matter produced in heavy-ion collisions is a complex dynamic process that needs, in principle, a sophisticated treatment. One simple approach is the freeze-out concept in which the hot and dense matter in the initial stage is assumed to reach thermal equilibrium as long as reaction rates are high. With decreasing density, the reaction rates decrease and the equilibration process becomes suppressed. At that time the nuclear thermal and chemical equilibrium is frozen out. Often the description of the nuclear matter, in particular the distribution of clusters, is calculated within a statistical multifragmentation model assuming nuclear statistical equilibrium (NSE) [2,3]. Under the simplifying assumption that the final reaction product distribution is identical to the cluster distribution at the freeze-out point, the thermodynamic parameters such as temperature T and particle number densities, n_n and n_p for neutrons and protons, respectively, can be reconstructed from the observed abundances. A simple method for extracting the temperature of the fragmenting hot system was given by Albergo, Costa, Costanzo, and Rubbino (ACCR) [4]. The method is based on selecting double isotope (or isotone) ratios, R_2, such that the nucleon chemical potentials are eliminated, leading to a relation among R_2, T, and the binding energies of the isotopes (isotones). This method has been used in the analysis of a large number of experiments. See, for example, the early works of Refs. [5–7].

However, one has to be aware that the dynamic reaction processes do not cease abruptly, so the concept of a unique freeze-out time is only approximate. A single freeze-out time, independent of the species under consideration and their dynamical state, the flow and further parameters describing nonequilibrium effects, may not exist. In addition, secondary (post-freeze-out) decay will modify the original distribution that also contains excited states, and the decay products will be found in the final distribution. We point out that in Ref. [8], the ACCR method was modified to account for the screening due to the Coulomb interactions among fragments in the freeze-out volume by using the Wigner-Seitz approximation [9]. It was found that the corrections for the temperature are less than 20%, though for certain isotone double ratios it can be as large as 50%. In Ref. [10] the ACCR method was modified to account for the effect of radial collective flow. It was found that the effect on the extracted temperature is relatively small, but the increase in the freeze-out density can be significant for large flow energy. It was noted in Ref. [6] that an important improvement of the simple ACCR method resulted from taking into account postemission decay (secondary decay) processes of particles and, in particular, γ that modify the freeze-out yield ratios. Without this correction, different double ratios R_2 associated with selected sets of fragments (different thermometers) may result in significantly different temperature T. We will not discuss these issues further in this work; for a review see, for example, Ref. [11].

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If the freeze-out density is not very low, i.e., not at baryon number densities $n_B \lesssim 10^{-4}$ fm^{-3}, the NSE will be modified by medium effects. In this article, the shift of the binding energies of the light elements in hot and dense nuclear matter [12,13] is considered. The changes of the mass fractions of different nuclei due to medium effects complicates the determination of the temperature T and baryon number density $n_B = n_n + n_p$ from yields of the observed products. In the following we show that simple NSE approaches like the ACCR thermometer can be improved if in-medium effects are taken into account.

II. THE YIELD THERMOMETER

Within a quantum statistical approach to nuclear matter [12,13], using a cluster decomposition for the self-energy we obtain expressions for the total proton density

$$n_p(T, \mu_p, \mu_n) = \sum_{A,Z} Zn(A, Z) \tag{1}$$

and for the total neutron density

$$n_n(T, \mu_p, \mu_n) = \sum_{A,Z} (A - Z)n(A, Z). \tag{2}$$

Here, $n(A, Z)$ is the contribution of the A-nucleon cluster to the total nucleon density. Both Eqs. (1) and (2) may be considered a nuclear matter equation of state (EOS) that determines the nucleon densities n_τ as functions of the temperature and the neutron and proton chemical potentials, respectively, denoted by T, μ_n, and μ_p. Additional thermodynamic quantities such as free energy and other thermodynamic potentials are obtained by integration.

Starting with the ideal mixture of different species, where the interaction between the species is neglected, the number density of a cluster $n^{(0)}(A, Z)$ is given by

$$n^{(0)}(A, Z) = g_{A,Z} \int \frac{d^3p}{(2\pi)^3} f_{A,Z}\left[E^{(0)}_{A,Z}(p)\right], \tag{3}$$

with the (Fermi or Bose) distribution function

$$f_{A,Z}(E) = \frac{1}{e^{\frac{1}{T}[E - Z\mu_p - (A-Z)\mu_n]} - (-1)^A}. \tag{4}$$

In Eq. (3), $E^{(0)}_{A,Z}(p) = E^{(0)}_{A,Z} + \hbar^2 p^2/(2Am)$, where, in the noninteracting case considered here, $E^{(0)}_{A,Z}$ is the ground-state binding energy, $g_{A,Z}$ is the degeneracy factor of an isolated nucleus with mass number A and charge number Z, m is the average nucleon mass, and μ_n and μ_p are the chemical potentials of neutrons and protons, respectively. Note that, in general, excited states that are characterized by internal quantum numbers may occur in addition to $\{A, Z\}$. In that case, a summation over the excited states, including scattering states, should be carried out. The number density is assumed to be proportional to the cluster yield, observed after freeze-out. In the nondegenerate limit we have the prediction of the cluster yields within NSE models.

$$Y^{(0)}(A, Z) \propto n^{(0)}(A, Z) = g_{A,Z}\left(\frac{2\pi\hbar^2}{AmT}\right)^{-3/2} \times e^{-[E^{(0)}_{A,Z} - Z\mu_p - (A-Z)\mu_n]/T}. \tag{5}$$

Inserting $n^{(0)}(A, Z)$ for the cluster densities $n(A, Z)$, Eqs. (1) and (2) read

$$\begin{aligned} n^{(0)}_p(T, \mu_p, \mu_n) &= \sum_{A,Z} Zn^{(0)}(A, Z), \\ n^{(0)}_n(T, \mu_p, \mu_n) &= \sum_{A,Z} (A - Z)n^{(0)}(A, Z), \end{aligned} \tag{6}$$

for the total proton and neutron densities, respectively. They are approximations to the nuclear matter EOS, reflecting NSE.

Let us now consider the observed cluster yields $Y(A, Z)$ that are proportional to the number density fractions $n(A, Z)/n_B$ of the cluster $\{A, Z\}$. We introduce the (single) ratio of the observed cluster yields

$$R_{(AZ),(A'Z')} = \frac{Y(A, Z)}{Y(A', Z')}. \tag{7}$$

If we accept the concept of NSE, identifying the observed cluster yields with the predicted ones, Eq. (5), we can get an estimation for the temperature T and the chemical potentials μ_n, μ_p of nuclear matter produced in heavy-ion collisions in a fashion similar to that employing the well-known Saha equation in plasma physics [14]. Specifically, because the abundances of different bound states are determined by the temperature and the chemical potentials, observed yield ratios can be used to determine these parameters. A simple method to derive the temperature of the hot system was given by ACCR [4], assuming NSE (5) and selecting double isotope ratios such that the nucleon chemical potentials are eliminated. In particular, the H-He thermometer considers the double ratio $R^{(0)}_{\rm HHe}$ of cluster yields $Y^{(0)}$,

$$R^{(0)}_{\rm HHe} = \frac{Y^{(0)}(^2{\rm H})\, Y^{(0)}(^4{\rm He})}{Y^{(0)}(^3{\rm H})\, Y^{(0)}(^3{\rm He})} = \frac{3 \times 1}{2 \times 2}\left(\frac{2 \times 4}{3 \times 3}\right)^{3/2} \times e^{-[E^{(0)}_{^2{\rm H}} + E^{(0)}_{^4{\rm He}} - E^{(0)}_{^3{\rm H}} - E^{(0)}_{^3{\rm He}}]/T}, \tag{8}$$

where the degeneracy and mass factors are explicitly included. Identifying the double ratio

$$R_{\rm HHe} = \frac{R_{(^2{\rm H}),(^3{\rm H})}}{R_{(^3{\rm He}),(^4{\rm He})}} \tag{9}$$

of the observed cluster yields for $d(^2{\rm H})$, $t(^3{\rm H})$, $h(^3{\rm He})$, and $\alpha\,(^4{\rm He})$ with the prediction according to the NSE, $R_{\rm HHe} = R^{(0)}_{\rm HHe}$, we deduce the ACCR temperature $T^{(a)}_{\rm HHe}[= T^{(0)}_{\rm HHe}]$ corresponding to the observed double ratio $R_{\rm HHe}$ as

$$T^{(a)}_{\rm HHe} = \frac{14.325\ {\rm MeV}}{\ln[1.591 R_{\rm HHe}]}. \tag{10}$$

The constants 14.325 MeV $= -(-2.225 - 28.3 + 8.482 + 7.718)$ MeV and $1.591 = 9/\sqrt{32}$ reflect the ground-state binding energies, spins, and mass numbers of the ejectiles as given in Eq. (8).

Other combinations of isotopes can be used to construct double ratios of cluster yields where, within a simple NSE, the chemical potentials cancel out so that an ACCR temperature can be derived directly. Thus, thermometers based on the yields of other nuclei such as lithium or beryllium isotopes can be introduced. Similar approaches are used in hadron production to derive the temperature for the quark-gluon plasma phase transition [15] or in plasma physics [14,16] considering spectral line intensities of different ionization states of radiating atoms.

The advantage of the double ratio is that, within NSE, it does not contain the density, because the chemical potentials cancel. Therefore the temperature determination seems to be insensitive with respect to the determination of other parameters. These other parameters, in particular the chemical potentials, are observed if, in addition to the double ratios like $R_{\rm HHe}$, the single ratios $R_{(AZ),(A'Z')} = Y(A,Z)/Y(A',Z')$ of yields are considered.

There are some objections to inferring the parameter values of hot dense matter from the cluster yields. First, we have to take into account that collisions lead to initially inhomogeneous system evolving in time. Even assuming local thermal equilibrium, one has to separate the ejectiles arising from different sources. In efforts to do this, the H-He thermometer has been applied to the double ratio $R_{v_{\rm surf}}$ of cluster yields $Y(A,Z)$ for clusters with the same surface velocity [1]. In that case an additional factor $\sqrt{(9/8)}$ arises in the temperature equation when the number densities as a function of velocity are employed.

An important improvement of the simple NSE model was to take into account secondary decay processes that modify the freeze-out yield ratios. This has been considered in different articles. This correction is essential to reduce the differences of the ACCR temperatures obtained from different thermometers [6].

The simple NSE is based on a chemical picture considering a noninteracting, ideal mixture of different components, which is in chemical equilibrium due to reactive collisions as described by the mass action law. Such an approach is valid in the low-density limit, and related expressions such as virial expansions can be taken as a benchmark in that limit [17]. With increasing density, modifications arise that are based on taking the interactions between the different components into account. Thus, as the density increases, corrections to the ACCR approach to derive the temperatures of hot and dense matter are expected. In earlier work, the effects of the screening of the Coulomb interaction and of the flow on the freeze-out density and temperature of disassembling hot nuclei have been considered [10,11]. The effect of screening of the Coulomb interaction becomes of importance for heavy nuclei at densities near to the saturation density.

The main topic we address in this article is the required modification of the description of the matter as an ideal, noninteracting mixture of different components when densities are not low enough to justify this assumption. Despite the fact that the nucleon-nucleon interaction is short-ranged, the interaction between the free nucleons as well as nucleons bound in clusters is negligible only below about 10^{-3} times the nuclear saturation density, i.e., at baryonic densities $n_B \lesssim 10^{-4}$ fm^{-3}. An important question is the role of medium effects due to the nucleon-nucleon interactions. In fact, our work indicates that the concept of the simple NSE considering hot and dense nuclear matter as an ideal mixture of different clusters is not appropriate to describe disassembling hot matter at densities at and above approximately 1/10th of saturation density. We address this in the following section.

III. MEDIUM MODIFICATION OF CLUSTER PROPERTIES

Recent progress in the description of clusters in low-density nuclear matter [12,18–20] enables us to evaluate the abundance of deuterons, tritons, and helium nuclei in a microscopic approach, taking the influence of the medium into account. Within a quantum statistical approach to the many-particle system, we determine the single-particle spectral function, which allows calculation of the density of the nucleons as a function of T, μ_n, and μ_p. The main ingredient is the self-energy $\Sigma(1,z)$ that is treated in different approximations. The single-particle spectral function contains the single-nucleon quasiparticle contribution, $E^{\rm qu}(1) = E^{\rm qu}_{1,Z}(p)$ or $E^{\rm qu}_\tau(p)$, where τ denotes isospin (neutron or proton). The quasiparticle energy follows from the self-consistent solution of $E^{\rm qu}_\tau(p) = \hbar^2 p^2/(2m_\tau) + {\rm Re}\,\Sigma[p, E^{\rm qu}_\tau(p)]$.

Expressions for the single-nucleon quasiparticle energy $E^{\rm qu}_\tau(p)$ can be given by the Skyrme mean-field parametrization [21] or by more sophisticated approaches such as relativistic mean-field approaches [22] and relativistic Dirac-Brueckner Hartree Fock [23] calculations. In the effective mass approximation, the single-nucleon quasiparticle dispersion relation reads

$$E^{\rm qu}_\tau(p) = \Delta E^{\rm SE}_\tau(0) + \frac{\hbar^2}{2m^*_\tau} p^2 + \mathcal{O}(p^4)\,, \tag{11}$$

where the quasiparticle energies are shifted by $\Delta E^{\rm SE}_\tau(0)$ and m^*_τ denotes the effective mass of neutrons ($\tau = n$) or protons ($\tau = p$). Both quantities, $\Delta E^{\rm SE}_\tau(0)$ and m^*_τ, are functions of T, n_p, and n_n characterizing the surrounding matter. Empirical values for the effective mass near the saturation density are different from the nucleon mass. In the low-density region considered here, the effective mass may be replaced by the free nucleon mass. For calculating the yields, the quasiparticle shift $\Delta E^{\rm SE}_\tau(0)$ can be implemented in a renormalization of the corresponding chemical potentials.

In addition to the δ-like quasiparticle contribution, the contribution of the bound and scattering states can also be included in the single-nucleon spectral function by analyzing the imaginary part of $\Sigma(1,z)$. Within a cluster decomposition, A-nucleon T matrices appear in a many-particle approach. These T matrices describe the propagation of the A-nucleon cluster in nuclear matter. In this way, bound states contribute to the EOS, $n_\tau = n_\tau(T,\mu_n,\mu_p)$, see Refs. [13,24]. In the low-density limit, the propagation of the A-nucleon cluster is determined by the energy eigenvalues of the corresponding nucleus, and the simple EOS, Eqs. (1) and (2), results.

For the nuclei embedded in nuclear matter, an effective wave equation can be derived [12,13]. The A-particle wave function and the corresponding eigenvalues follow from solving the in-medium Schrödinger equation

$$\begin{aligned}&\left[E^{\rm qu}(1) + \cdots + E^{\rm qu}(A) - E^{\rm qu}_{A\nu}(p)\right]\psi_{A\nu p}(1\cdots A)\\ &\quad + \sum_{1'\cdots A'}\sum_{i<j}[1-\tilde f(i)-\tilde f(j)]V(ij,i'j')\\ &\quad \times \prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu p}(1'\cdots A') = 0\,.\end{aligned} \tag{12}$$

This equation contains the effects of the medium in the single-nucleon quasiparticle shifts as well as in the Pauli blocking terms.

The in-medium Fermi distribution function $\tilde{f}(1) = \{\exp[E^{\rm qu}(1)/T - \tilde{\mu}_1/T] + 1\}^{-1}$ contains the effective chemical potential $\tilde{\mu}_1$ that is determined by the total proton or neutron density, calculated in the quasiparticle approximation, $n_\tau = \Omega^{-1}\sum_1 \tilde{f}(1)\delta_{\tau_1,\tau}$. It describes the occupation of the phase space neglecting any correlations in the medium. In the low-density and nondegenerate limit ($\tilde{\mu}_\tau < 0$), assuming the effective mass approximation for the nucleon quasiparticle dispersion relation, we eliminate $\tilde{\mu}_\tau$ using

$$\tilde{f}_\tau(p) = \frac{1}{\exp\left[E_\tau^{\rm qu}(p)/T - \tilde{\mu}_\tau/T\right] + 1} \approx \frac{n_\tau}{2}\left(\frac{2\pi\hbar^2}{m_\tau^* T}\right)^{3/2} e^{-\frac{\hbar^2 p^2}{2m_\tau^* T}}. \qquad (13)$$

The solution of the in-medium Schrödinger equation (12) can be obtained in the low-density region by perturbation theory. In particular, the quasiparticle energy of the A-nucleon cluster follows as

$$E_{A,Z}^{\rm qu}(p) = E_{A,Z}^{(0)} + \frac{\hbar^2 p^2}{2Am} + \Delta E_{A,Z}^{\rm SE}(p) + \Delta E_{A,Z}^{\rm Pauli}(p). \qquad (14)$$

Additional contributions such as the Coulomb shift $\Delta E_{A,Z}^{\rm Coul}(p)$, which can be evaluated for dense matter in the Wigner-Seitz approximation [8,9,11,25], will not be considered here because the values of Z are small and the densities are low. The general formalism also allows us to describe pairing or quartetting, but this will not be done here. Disregarding the effects due to the change of the effective mass, the self-energy contribution to the quasiparticle shift is determined by the contribution of the single-nucleon shift

$$\Delta E_{A,Z}^{\rm SE}(0) = (A - Z)\Delta E_n^{\rm SE}(0) + Z\Delta E_p^{\rm SE}(0). \qquad (15)$$

Inserting the medium-dependent quasiparticle energies in the distribution functions $f_{A,Z}[E_{A,\nu}^{\rm qu}(p)]$, Eq. (4), this contribution to the quasiparticle shift can be included by renormalizing the chemical potentials μ_n and μ_p.

The most important effect on the calculation of the yields of light elements comes from the Pauli blocking terms in Eq. (12) in connection with the interaction potential. This contribution is restricted only to the bound states so that it may lead to the dissolution of the nuclei if the density of nuclear matter increases. The corresponding shift $\Delta E_{A,Z}^{\rm Pauli}(p)$ can be evaluated in perturbation theory provided that the interaction potential and the ground-state wave function are known. After angular averaging, the Pauli blocking shift can be approximated as

$$\Delta E_{A,Z}^{\rm Pauli}(p) \approx \Delta E_{A,Z}^{\rm Pauli}(0)\, e^{-\frac{\hbar^2 p^2}{2A^2 mT}}. \qquad (16)$$

The shift of the binding energy of light clusters at zero total momentum that is of first order in density [18,19] has been calculated recently [12]. In addition to neutrons (n) and protons (p), light elements deuterons ^{2}H, $\{A, Z\} = d$, tritons ^{3}H, $\{A, Z\} = t$, hellions ^{3}He, $\{A, Z\} = h$, and α particles ^{4}He, $\{A, Z\} = \alpha$ have been considered. The interaction potential and the nucleonic wave function of the few-nucleon system have been fitted to the binding energies and the root-mean-square (rms) radii of the corresponding nuclei. The following results (in MeV, fm) are obtained for the binding energy shifts.

$$\begin{aligned}
\Delta E_d^{\rm Pauli} &= \left\{\frac{38384}{\left(1+\frac{22.52}{T}\right)^{1/2}} - 0.39402 e^{0.049418\left(1+\frac{22.52}{T}\right)} \times \mathrm{Erfc}\left[0.2223\left(1+\frac{22.52}{T}\right)^{1/2}\right]\right\}\frac{n_p+n_n}{T^{3/2}},\\
\Delta E_t^{\rm Pauli} &= 3389.7\,[1 + 0.13347\,T]^{-3/2}\left(\frac{2}{3}n_p + \frac{4}{3}n_n\right),\\
\Delta E_h^{\rm Pauli} &= 3901.5\,[1 + 0.16455\,T]^{-3/2}\left(\frac{4}{3}n_p + \frac{2}{3}n_n\right),\\
\Delta E_\alpha^{\rm Pauli} &= 4716.0\,[1 + 0.09372\,T]^{-3/2}\left(n_p + n_n\right).
\end{aligned} \qquad (17)$$

These results describe only the linear shifts as functions of the nucleon densities. The differences between the values for $\Delta E_t^{\rm Pauli}$ and $\Delta E_h^{\rm Pauli}$ are mainly caused by different values of the rms radii for these two nuclei. With increasing density, higher-orders terms with respect to the densities also become relevant.

It can be shown [25] that the EOS can be evaluated as in the noninteracting case (3) given above, except that the number densities of clusters must be calculated with the quasiparticle energies,

$$n^{\rm qu}(A, Z) = g_{A,Z}\int \frac{d^3p}{(2\pi)^3} f_{A,Z}\left[E_{A,Z}^{\rm qu}(p)\right]. \qquad (18)$$

In the cluster-quasiparticle approximation, the EOS, Eqs. (1) and (2), reads

$$\begin{aligned}
n_p^{\rm qu}(T, \mu_p, \mu_n) &= \sum_{A,Z} Z n^{\rm qu}(A, Z),\\
n_n^{\rm qu}(T, \mu_p, \mu_n) &= \sum_{A,Z} (A - Z) n^{\rm qu}(A, Z),
\end{aligned} \qquad (19)$$

for the total proton and neutron density, respectively.

This result is an improvement of the NSE and allows for the smooth transition from the low-density limit up to the region of saturation density. The bound-state contributions to the EOS fade with increasing density because they merge with the continuum of scattering states. This improved NSE, however, does not contain the contribution of scattering states, in particular resonances appearing in the continuum of scattering states when bound states merge with the continuum. For the treatment of scattering states in the two-nucleon case, as well as the evaluation of the second virial coefficient, see Refs. [17,24]. We will also not consider the formation of heavy elements here. This limits the present results to the range of parameters T, n_n, and n_p, where the EOS is determined only by the light elements. For a more general approach to the EOS that takes also the contribution of heavier clusters into account see Ref. [25].

IV. IMPROVED THERMOMETER AND DENSITY DETERMINATION, INCLUDING MEDIUM EFFECTS

To show the effect of in-medium corrections, we start with a temperature T and densities n_n and n_p and calculate the corresponding yields $Y^{\rm qu}(A,Z)$, taking the in-medium shifts into account. Then we use these yields to infer the parameter values $T^{(a)}[=T^{(0)}]$, $n_n^{(a)}[=n_n^{(0)}]$, and $n_p^{(a)}[=n_p^{(0)}]$, using the ACCR relations that were derived neglecting in-medium corrections. In this way we obtain for given ratios of cluster yields $R_{(A,Z),(A',Z')}$, Eq. (7), $\{T, n_n, n_p\}$ that we identify as the values that would be derived from experiments if the medium effects are considered and those $\{T^{(a)}, n_n^{(a)}, n_p^{(a)}\}$, derived without taking the medium effects into account. Comparing these sets of the parameters we demonstrate how the medium modification of the binding energy of light nuclei, Eq. (18), can modify the results determined from the experimental yields of light clusters. The three ratios necessary to determine three thermodynamic parameters are derived here from the four yields of the light clusters $Y(^2\mathrm{H})$, $Y(^3\mathrm{H})$, $Y(^3\mathrm{He})$, and $Y(^4\mathrm{He})$.

We first compare results of the determination of the temperature T from cluster yields, if the in-medium quasiparticle shifts are taken into account, with the temperature $T^{(a)}$ determined from the same yields if medium effects are neglected. In particular, the temperature $T_{\rm HHe}$ is not related in a simple way to the double ratio $R_{\rm HHe}$, Eq. (9). Considering the yields in quasiparticle approximation, we have

$$R_{\rm HHe} = \frac{Y^{\rm qu}(^2\mathrm{H})\,Y^{\rm qu}(^4\mathrm{He})}{Y^{\rm qu}(^3\mathrm{H})\,Y^{\rm qu}(^3\mathrm{He})}. \tag{20}$$

If we take the yields $Y^{\rm qu}(A,Z) \propto n^{\rm qu}(A,Z)$, Eq. (18), in the nondegenerate case, we obtain the relation

$$T_{\rm HHe} = \frac{1}{\ln[1.591\,R_{\rm HHe}]}\big(14.325\,\mathrm{MeV} + \Delta \mathrm{E}_{\rm d}^{\rm Pauli} + \Delta E_\alpha^{\rm Pauli} - \Delta E_t^{\rm Pauli} - \Delta E_h^{\rm Pauli}\big), \tag{21}$$

The energy shifts are functions of temperature and densities so this relation has to be solved self-consistently.

However, neglecting in-medium corrections, we find from the same double ratio the apparent ACCR temperature $T_{\rm HHe}^{(a)}$ according to Eq. (10). Using, in the nondegenerate case, Eq. (5), the relation between both quantities is given by

$$T_{\rm HHe}^{(a)}(T,n_n,n_p) = \left[1 + \frac{\Delta E_t^{\rm Pauli} + \Delta E_h^{\rm Pauli} - \Delta E_d^{\rm Pauli} - \Delta E_\alpha^{\rm Pauli}}{E_t^{(0)} + E_h^{(0)} - E_d^{(0)} - E_\alpha^{(0)}}\right]^{-1} T_{\rm HHe}\,. \tag{22}$$

In the approximations considered here, the self-energy contributions to the shifts disappear, in a fashion similar to the chemical potentials. In Fig. 1 we show the ratio between the ACCR temperature $T^{(a)}$ and T as a function of the baryon density $n_B = n_p + n_n$ for various values of T.

Similarly, the densities can be estimated by considering single ratios $R_{(A,Z),(A',Z')}$, Eq. (7). If the shifts of the binding energies due to medium effects are neglected, we have

$$R_{(A,Z),(A',Z')}^{(a)} = \frac{g_{A,Z}A^{3/2}}{g_{A',Z'}A'^{3/2}} e^{-[E_{A,Z}^{(0)} - E_{A',Z'}^{(0)} - (Z-Z')\mu_p^{(a)} - (A-Z-A'+Z')\mu_n^{(a)}]/T^{(a)}}. \tag{23}$$

Assuming NSE and considering special combinations, we can obtain the chemical potentials of protons (μ_p) and neutrons (μ_n) from the triton to ^{4}He ratio or from the ^{3}He to ^{4}He ratio, respectively, as

$$\mu_p^{(a)} = -19.8\ \mathrm{MeV} + T^{(a)} \ln\left[\frac{3^{3/2}}{2^2}\frac{Y_\alpha}{Y_t}\right], \tag{24}$$

$$\mu_n^{(a)} = -20.6\ \mathrm{MeV} + T^{(a)} \ln\left[\frac{3^{3/2}}{2^2}\frac{Y_\alpha}{Y_h}\right]. \tag{25}$$

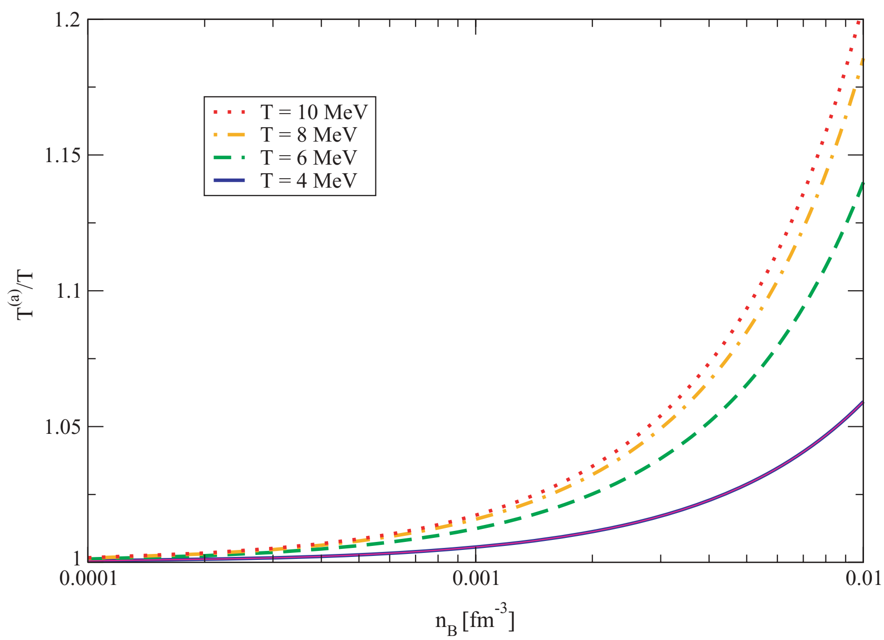

FIG. 1. (Color online) The ratio between the ACCR temperature $T^{(a)}$ (no medium effects) and T (including medium effects) as a function of the baryon density $n_B = n_p + n_n$ for various values of T.

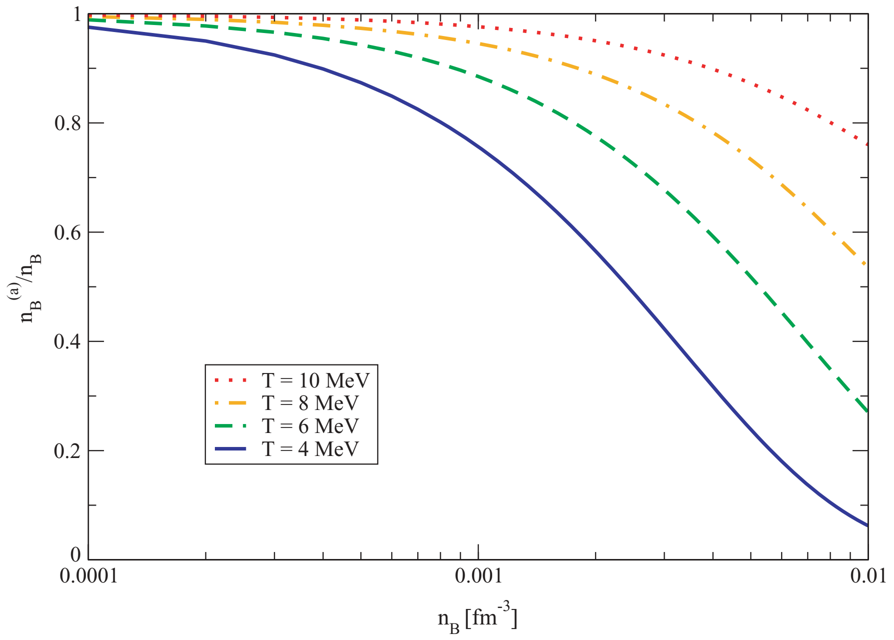

FIG. 2. (Color online) Similar to that described in the caption to Fig. 1 but for the ratio $n_B^{(a)}/n_B$.

This allows us to calculate the chemical potentials separately. Then, considering chemical equilibrium between the different clusters, the total proton and neutron densities are given by the mass action law, cf. Eqs. (1) and (2). Assuming NSE where in-medium corrections are neglected, we find from Eq. (6) the total densities $n_p^{(a)}[T^{(a)}, \mu_p^{(a)}, \mu_n^{(a)}]$ and $n_n^{(a)}[T^{(a)}, \mu_p^{(a)}, \mu_n^{(a)}]$ of protons and neutrons, respectively.

Taking into account the in-medium quasiparticle energy shifts of the nuclei, the relations are changed so that

$$\mu_p = -19.818 \text{ MeV} + \Delta E_\alpha^{\text{qu}} - \Delta E_t^{\text{qu}} + T \ln\left[\frac{3^{3/2}}{2^2}\frac{Y_\alpha}{Y_t}\right], \quad (26)$$

$$\mu_n = -20.582 \text{ MeV} + \Delta E_\alpha^{\text{qu}} - \Delta E_h^{\text{qu}} + T \ln\left[\frac{3^{3/2}}{2^2}\frac{Y_\alpha}{Y_h}\right], \quad (27)$$

where the temperature T is obtained taking the medium modifications of the energies of nuclei into account. Now, the total proton and neutron densities are calculated from the EOS, Eqs. (19), which contain medium-dependent quasicluster energy shifts.

To show the effect of these medium modifications, we start with given values for T, n_p, and n_n and calculate the cluster abundances solving Eqs. (19), taking the shifts into account and restricting our consideration to $A \leqslant 4$. This gives us certain values for the chemical potentials μ_p and μ_n. Obviously, within a self-consistent calculation we can reproduce not only the input quantities n_p and n_n from these values of μ_p and μ_n but also the single ratios for different yields, in particular $R_{^4\text{He},^3\text{H}}$, $R_{^4\text{He},^3\text{He}}$, and the double ratio R_{HHe}. Now, we consider this as input and determine within the simple NSE the ACCR values $T^{(a)}$, $\mu_p^{(a)}$, and $\mu_n^{(a)}$. In NSE, where medium shifts are neglected, we calculate the number densities $n^{(a)}(A, Z)$ of the nuclei, using the EOS, Eq. (6). Obviously, the single and double ratios given above are reproduced. However, not only will the temperature $T^{(a)}$ differ from the input value T, but also the total proton density $n_p^{(a)}$ and the total neutron density $n_n^{(a)}$ will deviate from the input values n_p and n_n, respectively.

In Fig. 2 we show the ratio between the ACCR baryon density $n_B^{(a)} = n_p^{(a)} + n_n^{(a)}$ and $n_B = n_p + n_n$ as a function of n_B for various values of T.

V. DISCUSSION AND CONCLUSIONS

The assumption of NSE provides a simple means to estimate the thermodynamic parameters of nuclear matter at freeze-out from the observed yields of nuclei. This approach is applicable as long as the interaction between the clusters can be neglected. However, the thermometers and chemical potentials are no longer correctly scaled when the shifts of the binding energies due to the interaction with the surrounding matter become of relevance. The derivation of the thermodynamic parameters from the measured yields has to be carried out in a self-consistent manner because the binding energies, which determine the yields, are themselves dependent on the temperatures and densities.

Analyzing empirical data, the use of the ACCR method can only give a first approximation to the temperature and the density. Taking these first estimations, the shift of the binding energies of the clusters can be estimated. With these modified energies, the next iteration deriving the values of the parameters from the measured yields can be made, and a self-consistent solution is expected after a sufficient number of iterations. Alternatively one can also produce tables for yields taking the medium shifts into account, so the optimal values of the parameters are obtained by interpolating within the table to identify the values of the parameters that best correspond to the measured yields.

Comparing the values of the parameters obtained in the full calculation, with inclusion of medium effects on the yields with those deduced in the ACCR approach, we find that moderate deviations in the temperature arise for densities larger than 0.0001 fm^{-3}. Determination of the densities is more sensitive to the medium effects.

The shift of the binding energies has been given in first order of the density, and higher-order terms in the density are expected to contribute if the density increases. Starting with baryonic densities near 10^{-2} fm^{-3}, the composition has to be calculated with momentum-dependent shifts instead of the rigid shifts considered here, and then the temperature is found from the ratio of the mass fractions after the composition is calculated in a self-consistent way. One has to perform the full momentum integration instead of considering a rigid shift as given at $P = 0$, when the shifts depend on the center-of-mass momentum of the cluster. The results given here are applicable at densities that are not too high, i.e., up to 0.01 fm^{-3}.

In conclusion we point out that the fragment yields from hot and dense nuclear matter produced in heavy-ion collisions can be used to infer temperatures and proton/neutron densities of the early stages of the expanding hot matter. The assumption of thermal equilibrium can be only a first approach to this nonequilibrium process. To determine the yield of the different clusters, a simple statistical model neglecting all medium effects, i.e., treating it as an ideal mixture of noninteracting nuclei, is not applicable when the density is larger than 0.0001 fm^{-3}. Self-energy and Pauli blocking will lead to energy shifts, which have to be taken into account to reconstruct the thermodynamic parameters from measured yields. The success of the simple ACCR method to derive the values for the temperature can be understood from a partial compensation of the effect of the energy shifts so reasonable values for the temperature are obtained also at relatively high densities. More care must be taken in inferring densities from the data. It should be mentioned that similar questions have to be considered when hadron production is investigated at the quark-gluon phase transition.

Cross-checks can be performed to see to what extent the approach given here is consistent. Hitherto we considered only the yields of d, t, h, and α, and the corresponding ratios are reflected by the temperature and the chemical potentials of the neutrons and protons. The determination of the yields of additional clusters will allow for a comparison between predictions and experimental data.

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